Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

Appendix A

Answers

1. 9709_m21_ms_12 Q: 2

Question	Answer	Marks	Guidance
	$u = 2x - 3$ leading to $u^4 - 3u^2 - 4\ [=0]$	M1	Or $u = (2x-3)^2$ leading to $u^2 - 3u - 4\ [=0]$
	$(u^2 - 4)(u^2 + 1)\ [=0]$	M1	Or $(u-4)(u+1)\ [=0]$
	$2x - 3 = [\pm]2$	A1	
	$x = \frac{1}{2}, \frac{5}{2}$ **only**	A1	
		4	

2. 9709_s21_ms_11 Q: 6

Question	Answer	Marks	Guidance
	$(2k-3)x^2 - kx - (k-2) = 3x - 4$	*M1	Equating curve and line
	$(2k-3)x^2 - (k+3)x - (k-6)[=0]$	DM1	Forming a 3-term quadratic
	$(k+3)^2 + 4(2k-3)(k-6)[=0]$	DM1	Use of discriminant (dependent on **both** previous M marks)
	$9k^2 - 54k + 81[=0]$ [leading to $k^2 - 6k + 9 = 0$]	M1	Simplifying and solving *their* 3-term quadratic in k
	$k = 3$	A1	
	Alternative method for Question 6		
	$(2k-3)x^2 - kx - (k-2) = 3x - 4$	*M1	Equating curve and line
	$2(2k-3)x - k = 3 \Rightarrow x = \frac{k+3}{4k-6}$ or $k = \frac{3+6x}{4x-1}$	DM1	Differentiating and solving for x or k
	Either $(2k-3)\left(\frac{k+3}{4k-6}\right)^2 - k\left(\frac{k+3}{4k-6}\right) - (k-2) = 3\left(\frac{k+3}{4k-6}\right) - 4$ **Or** $4x\left(\frac{3x^2+3x-6}{2x^2-x-1}\right) - 6x - \left(\frac{3x^2+3x-6}{2x^2-x-1}\right) = 3$	DM1	Substituting *their* x into equation or *their* $k = \frac{3x^2+3x-6}{2x^2-x-1}$ or $k = \frac{3x+6}{2x+1}$ into derivative equation (dependent on **both** previous M marks)
	$9k^2 - 54k + 81[=0]$ [leading to $k^2 - 6k + 9 = 0$]	M1	Simplifying and solving *their* 3-term quadratic in k (or solving for x)
	$k = 3$	A1	
			SC If M0, B1 for differentiating, equating to 3 and solving for x or k
		5	

3. 9709_s21_ms_12 Q: 1

Question	Answer	Marks	Guidance
(a)	$(4x-3)^2$ or $(4x+(-3))^2$ or $a=-3$	B1	$k(4x-3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery.
	$+1$ or $b=1$	B1	
		2	
(b)	[For one root] $k=1$ or '*their b*'	B1 FT	Either by inspection or solving or from $24^2 - 4 \times 16 \times (10-k) = 0$ WWW
	[Root or $x=$] $\frac{3}{4}$ or 0.75	B1	**SC B2** for correct final answer WWW.
		2	

4. 9709_s20_ms_11 Q: 5

(a)	$x(mx+c)=16 \to mx^2+cx-16=0$	**B1**
	Use of $b^2-4ac = c^2+64m$	**M1**
	Sets to 0 $\to m = \frac{-c^2}{64}$	**A1**
		3
(b)	$x(-4x+c)=16$ Use of $b^2-4ac \to c^2-256$	**M1**
	$c>16$ and $c<-16$	**A1 A1**
		3

5. 9709_s19_ms_13 Q: 1

	Answer	Mark	Partial Marks
(i)	$\left[(x-2)^2\right]$ $[+4]$	**B1 DB1**	2nd B1 dependent on 2 inside bracket
		2	
(ii)	$(x-2)^2<5 \to -\sqrt{5}<x-2$ and/or $x-2<\sqrt{5}$	**M1**	Allow e.g. $x-2<\pm\sqrt{5}$, $x-2=\pm\sqrt{5}$ and decimal equivalents for √5 For M1, ft from *their*(i). Also allow √13 instead of √5 for clear slip
	$2-\sqrt{5}<x<2+\sqrt{5}$	**A1A1**	A1 for each inequality – allow two separate statements but there must be 2 inequalities for x. Non-hence methods, if completely correct, score SC 1/3. Condone $\leqslant$
		[3]	

6. 9709_s18_ms_13 Q: 1

	Answer	Mark	Partial Marks
	[3] $\left[(x-2)^2\right]$ [–5]	**B1B1B1**	OR $a=3$, $b=-2$, $c=-5$. 1st mark is dependent on the form $(x+a)^2$ following 3
		3	

7. 9709_w18_ms_11 Q: 1

	Answer	Mark	Partial Marks
	$(4x^{½}-3)(x^{½}-2)$ oe soi Alt: $4x+6=11\sqrt{x} \Rightarrow 16x^2-73x+36$	M1	Attempt solution for $x^{½}$ or sub $u = x^{½}$
	$x^{½}=3/4$ or 2 $(16x-9)(x-4)$	A1	Reasonable solutions for $x^{½}$ implies M1 (x = 2, 3/4, M1A0)
	$x = 9/16$ oe or 4	A1	Little or no working shown scores SCB3, spotting one solution, B0
		3	

8. 9709_m17_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$(3k)^2-4\times2\times k$	M1	Attempt b^2-4ac
	$9k^2-8k>0$ soi Allow $9k^2-8k\geqslant 0$	A1	Must involve correct inequality. Can be implied by correct answers
	0, 8/9 soi	A1	
	$k<0$, $k>8/9$ (or 0.889)	A1	Allow $(-\infty, 0)$, $(8/9, \infty)$
	Total:	4	

9. 9709_s16_ms_11 Q: 6

	Answer	Mark	Partial Marks
(a)	$y = 2x^2 - 4x + 8$ Equates with $y = mx$ and selects a, b, c Uses $b^2 = 4ac$ $\rightarrow m = 4$ or -12.	M1 M1 A1 [3]	 Equate + solution or use of dy/dx Use of discriminant for both.
(b) (i)	$f(x) = x^2 + ax + b$ Eqn of form $(x-1)(x-9)$ $\rightarrow a = -10, b = 9$ (or using 2 sim eqns **M1 A1**)	M1 A1 [2]	Any valid method allow $(x+1)(x+9)$ for **M1** must be stated
(ii)	Calculus or $x = \frac{1}{2}(1 + 9)$ by symmetry $\rightarrow (5, -16)$	M1 A1 [2]	Any valid method

10. 9709_w16_ms_11 Q: 1

	Answer	Mark		Partial Marks
(i)	$(x+3)^2-7$	B1B1	[2]	For $a = 3$, $b = -7$
(ii)	1, −7 seen $x>1$, $x<-7$ oe	B1 B1	[2]	$x>1$ or $x<-7$ Allow $x\leqslant -7, x\geqslant 1$ oe

11. 9709_s15_ms_13 Q: 1

	Answer	Mark	Partial Marks
	$2(x-3)^2-11$	**B1B1B1** **[3]**	For 2, $(x-3)^2$, −11. Or a=2, b= 3, c= 11

12. 9709_w15_ms_13 Q: 3

	Answer	Mark	Partial Marks
(i)	$[3]\left[(x-1)^2\right][-1]$	**B1B1B1** [3]	
(ii)	$f'(x)=3x^2-6x+7$	**B1**	Ft *their* **(i)** + 5
	$=3(x-1)^2+4$	**B1**√	
	>0 hence increasing	**DB1** [3]	Dep B1√ unless other valid reason

13. 9709_m22_ms_12 Q: 5

Question	Answer	Marks	Guidance
(a)	$2[\{(x-2)^2\}\ \{+3\}]$	**B1 B1**	B1 for $a = 2$, B1 for $b = 3$. $2(x-2)^2+6$ gains B1B0
		2	
(b)	{Translation} $\begin{pmatrix}\{2\}\\\{3\}\end{pmatrix}$ OR {Stretch} {*y* direction} {factor 2}	**B2,1,0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
	{Stretch} {*y* direction} {factor 2} OR {Translation} $\begin{pmatrix}\{2\}\\\{6\}\end{pmatrix}$	**B2,1,0**	B2 for fully correct, B1 with two elements correct. {} indicates different elements.
		4	

14. 9709_m22_ms_12 Q: 9

Question	Answer	Marks	Guidance
(a)	$\left[x^{\frac{1}{2}}=\right]\dfrac{4\pm\sqrt{16-4}}{2}=2\pm\sqrt{3}$	**M1 A1**	OE. Answer must come from formula or completing square. If M0A0 scored then **SC B1** for $2\pm\sqrt{3}$ only.
	$[x=]\left(2\pm\sqrt{3}\right)^2$	**M1**	Attempt to square *their* $2\pm\sqrt{3}$
	$7+4\sqrt{3}$, $7-4\sqrt{3}$	**A1**	Accept $7\pm4\sqrt{3}$ or $a=7, b=\pm4, c=3$ **SC B1** instead of second M1A1 for correct final answer only.
	Alternative method for question 9(a)		
	$-4x^{\frac{1}{2}}+1=0$ leading to $(x+1)^2=16x$ leading to $x^2-14x+1=0$	***M1 A1**	OE
	$x=\dfrac{14\pm\sqrt{196-4}}{2}$	**DM1**	Attempt to solve for x
	$7+4\sqrt{3}$, $7-4\sqrt{3}$	**A1**	**SC B1** instead of second M1A1 for correct final answer only.
		4	
(b)	$[\text{gh}(x)=]\ m\left(x^{\frac{1}{2}}-2\right)^2+n$	**M1**	SOI
	$[\text{gh}(x)=]\ m\left(x-4x^{\frac{1}{2}}+4\right)+n\ \equiv\ x-4x^{\frac{1}{2}}+1$	**A1**	SOI
	$m=1,\ n=-3$	**A1 A1**	WWW
		4	

15. 9709_m21_ms_12 Q: 5

Question	Answer	Marks	Guidance
(a)	(Stretch) (factor 3 in y direction **or** parallel to the y-axis)	**B1 B1**	
	(Translation) $\begin{pmatrix}4\\0\end{pmatrix}$	**B1 B1**	Allow Translation 4 (units) in x direction. N.B. Transformations can be given in either order.
		4	
(b)	$[y =]\ 3f(x-4)$	**B1 B1**	B1 for 3 , B1 for $(x-4)$ with no extra terms.
		2	

16. 9709_m21_ms_12 Q: 7

Question	Answer	Marks	Guidance
(a)	$[f(x)=](x+1)^2+2$	**B1 B1**	Accept $a=1, b=2$.
	Range [of f is (y)] $\geqslant 2$	**B1FT**	OE. Do not allow $x \geqslant 2$, FT on *their b*.
		3	
(b)	$y=(x+1)^2+2$ leading to $x=[\pm]\sqrt{y-2}-1$	**M1**	Or by using the formula. Allow one sign error.
	$f^{-1}(x)=-\sqrt{x-2}-1$	**A1**	
		2	

Question	Answer	Marks	Guidance
(c)	$2(x^2+2x+3)+1=13$	**B1**	Or using a correct completed square form of f(x).
	$2x^2+4x-6[=0]$ leading to $(2)(x-1)(x+3)[=0]$	**B1**	Or $x=1, x=-3$ using formula or completing square. Must reach 2 solutions.
	$x=-3$ **only**	**B1**	
		3	

17. 9709_s21_ms_11 Q: 9

Question	Answer	Marks	Guidance
(a)	Range of f is f(x) $\geqslant -4$	**B1**	Allow y, f or 'range' or $[-4, \infty)$
		1	
(b)	$y=(x-2)^2-4 \Rightarrow (x-2)^2=y+4 \Rightarrow x-2=+\sqrt{(y+4)}$ or $\pm\sqrt{(y+4)}$	**M1**	May swap variables here
	$[f^{-1}(x)]=\sqrt{(x+4)}+2$	**A1**	
		2	
(c)	$(x-2)^2-4=-\frac{5}{3}x+2 \Rightarrow x^2-4x+4-4=-\frac{5}{3}x+2\ [\Rightarrow x^2-\frac{7}{3}x-2=0]$	**M1**	Equating and simplifying to a 3-term quadratic
	$(3x+2)(x-3)[=0]$ or $\dfrac{7\pm\sqrt{7^2-4(3)(-6)}}{6}$ OE	**M1**	Solving quadratic
	$x=3$ only	**A1**	
		3	

Question	Answer	Marks	Guidance
(d)	$f^{-1}(12) = 6$	M1	Substitute 12 into *their* $f^{-1}(x)$ and evaluate
	$g(f^{-1}(12)) = 6a + 2$	M1	Substitute *their* '6' into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	M1	Substitute the result into $g(x)$ and = 62
	$6a^2 + 2a - 60$ [= 0]	M1	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	A1	
	Alternative method for Question 9(d)		
	$g(f^{-1}(x)) = a\left(\sqrt{x+4}+2\right)+2$ or $gg(x) = a(ax+2)+2$	M1	Substitute *their* $f^{-1}(x)$ or $g(x)$ into $g(x)$
	$g(g(f^{-1}(x))) = a\left(a\left(\sqrt{x+4}+2\right)+2\right)+2$	M1	Substitute the result into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	M1	Substitute 12 and = 62
	$6a^2 + 2a - 60$ [= 0]	M1	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	A1	
		5	

18. 9709_s21_ms_12 Q: 2

Question	Answer	Marks	Guidance
(a)	Translation $\begin{pmatrix}1\\0\end{pmatrix}$	B1	Allow shift and allow by 1 in x-direction or [parallel to/on/in/along/against] the x-axis or horizontally. 'Translation by 1 to the right' only, scores B0
	Stretch	B1	Stretch. **SC B2** for amplitude doubled.
	Factor 2 in y-direction	B1	With/by **factor** 2 in y-direction or [parallel to/on/in/along/against] the y-axis or vertically or with x axis invariant 'With/by factor 2 upwards' only, scores B0. Accept SF as an abbreviation for scale factor.
		3	**Note:** Transformations can be in either order
(b)	[$-\sin 6x$][$+ 15x$] or [$\sin(-6x)$][$+ 15x$] OE	B1 B1	Accept an unsimplified version. ISW. B1 for each correct component – square brackets indicate each required component.
			If B0, **SC B1** for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right)+\frac{5}{3}x$
		2	

19. 9709_s21_ms_12 Q: 5

Question	Answer	Marks	Guidance
(a)	$ff(x) = 2(2x^2 + 3)^2 + 3$	M1	Condone = 0.
	$8x^4 + 24x^2 + 21$	A1	ISW if correct answer seen. Condone = 0.
		2	
(b)	$8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19$ [= 0]	M1	Equating $34x^3 + 19$ to *their* 3-term ff(x) and collect all terms on one side condone ± sign errors.
	$8x^4 - 10x^2 + 2$[= 0]	A1	
	$[2](x^2 - 1)(4x^2 - 1)$	M1	Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem.
	$\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to}\right] x = 1 \text{ or } x = \frac{1}{2}$	A1	If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or *their* ax^4 otherwise M0A0 due to calculator use. Condone ± 1, $\pm\frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$.
		4	

20. 9709_s21_ms_13 Q: 6

Question	Answer	Marks	Guidance
(a)	$f(x) = (x-1)^2 + 4$	B1	
	$g(x) = (x+2)^2 + 9$	B1	
	$g(x) = f(x+3) + 5$	B1 B1	B1 for each correct element. Accept $p = 3, q = 5$
		4	

Question	Answer	Marks	Guidance
(b)	Translation or Shift	B1	
	$\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ or acceptable explanation	B1 FT	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ & $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ FT from *their* $f(x+p)+q$ *or their* $f(x) \to g(x)$ Do not accept $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$
		2	

21. 9709_s21_ms_13 Q: 8

Question	Answer	Marks	Guidance
(a)	$\left[\text{fg}(x)=\right]1/(2x+1)^2-1$	B1	SOI
	$1/(2x+1)^2-1=3$ leading to $4(2x+1)^2=1$ or $\frac{1}{(2x+1)}=[\pm]2$ or $16x^2+16x+3=0$	M1	Setting $\text{fg}(x)=3$ and reaching a stage before $2x+1=\pm½$ or reaching a 3 term quadratic in x
	$2x+1=\pm½$ or $2x+1=-½$ or $(4x+1)(4x+3)[=0]$	A1	Or formula or completing square on quadratic
	$x=-\frac{3}{4}$ only	A1	
	Alternative method for Question 8(a)		
	$x^2-1=3$	M1	
	$\text{g}(x)=-2$	A1	
	$\frac{1}{(2x+1)}=-2$	M1	
	$x=-\frac{3}{4}$ only	A1	
		4	

Question	Answer	Marks	Guidance
(b)	$y=\frac{1}{(2x+1)^2}-1$ leading to $(2x+1)^2=\frac{1}{y+1}$ leading to $2x+1=[\pm]\frac{1}{\sqrt{y+1}}$	*M1	Obtain $2x+1$ or $2y+1$ as the subject
	$x=[\pm]\frac{1}{2\sqrt{y+1}}-\frac{1}{2}$	DM1	Make $x(\text{or }y)$ the subject
	$-\frac{1}{2\sqrt{x+1}}-\frac{1}{2}$	A1	OE e.g. $-\frac{\sqrt{x+1}}{2x+2}-\frac{1}{2}$, $-\left(\sqrt{\frac{-x}{4x+4}+\frac{1}{4}}+\frac{1}{2}\right)$
		3	

22. 9709_w21_ms_11 Q: 8

Question	Answer	Marks	Guidance
(a)	$\left\{-3(x-2)^2\right\}$ $\{+14\}$	B1 B1	B1 for each correct term; condone $a = 2$, $b = 14$.
		2	
(b)	$[k =]$ 2	B1	Allow $[x] \leqslant 2$.
		1	

Question	Answer	Marks	Guidance
(c)	[Range is] [y] $\leqslant -13$	B1	Allow [f(x)] $\leqslant -13$, [f] $\leqslant -13$ but NOT $x \leqslant -13$.
		1	
(d)	$y=-3(x-2)^2+14$ leading to $(x-2)^2=\frac{14-y}{3}$	M1	Allow $\frac{y-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$x=2(\pm)\sqrt{\frac{14-y}{3}}$	A1	Allow $\frac{y-14}{-3}$.
	$[\mathrm{f}^{-1}(x)]=2-\sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root only.
	Alternative method for question 8(d)		
	$x=-3(y-2)^2+14$ leading to $(y-2)^2=\frac{14-x}{3}$	M1	Allow $\frac{x-14}{-3}$. Allow 1 error in rearrangement if x, y on opposite sides.
	$=2(\pm)\sqrt{\frac{14-x}{3}}$	A1	Allow $\frac{x-14}{-3}$.
	$[\mathrm{f}^{-1}(x)]=2-\sqrt{\frac{14-x}{3}}$	A1	OE. Allow $\frac{x-14}{-3}$. Must be x on RHS; must be negative square root only.
		3	

Question	Answer	Marks	Guidance
(e)	[g(x) =] $\left\{-3(x+3-2)^2\right\}+\{14+1\}$	B2, 1, 0	OR $\left\{-3(x+3)^2\right\}+\left\{12(x+3)\right\}+\{3\}$
	$\mathrm{g}(x)=-3x^2-6x+12$	B1	
		3	

23. 9709_w21_ms_12 Q: 2

Question	Answer	Marks	Guidance
(a)	Stretch with [scale factor] either $\pm$ 2 or $\pm\frac{1}{2}$	B1	
	Scale factor $\frac{1}{2}$ in the x-direction	B1	
	Translation $\begin{pmatrix}0\\-3\end{pmatrix}$ or translation of 3 units in negative y-direction	B1	
		3	
(b)	$(10, 9)$	B1 B1	B1 for each correct co-ordinate.
		2	

24. 9709_w21_ms_12 Q: 3

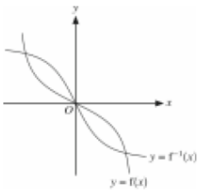
Question	Answer	Marks	Guidance
(a)	$\mathrm{f}(5)=[2]$ and $\mathrm{f}(\textit{their}\,2)=[5]$ OR $\mathrm{ff}(5)=\left[\frac{2+3}{2-1}\right]$ OR $\dfrac{\frac{x+3}{x-1}+3}{\frac{x+3}{x-1}-1}$ and an attempt to substitute $x=5$.	M1	Clear evidence of applying f twice with $x = 5$.
	5	A1	
		2	

Question	Answer	Marks	Guidance
(b)	$\frac{x+3}{x-1}=y \Rightarrow x+3=xy-y$ OR $\frac{y+3}{y-1}=x \Rightarrow y+3=xy-x$	*M1	Setting f(*x*) = *y* or swapping *x* and *y*, clearing of fractions and expanding brackets. Allow ± sign errors.
	$xy-x=y+3 \Rightarrow x=\frac{y+3}{y-1}$ OE OR $y+3=xy-x \Rightarrow y=\left[\frac{x+3}{x-1}\right]$ OE	DM1	Finding *x* or *y* = . Allow ± sign errors.
	$[\mathrm{f}^{-1}(x) \text{ or } y]=\frac{x+3}{x-1}$	A1	OE e.g. $1+\frac{4}{x-1}$ etc. Must be a function of *x*, cannot be *x* = .
		3	

25. 9709_w21_ms_13 Q: 1

Question	Answer	Marks	Guidance
	{Reflection} {[in the] *x*-axis} or {Stretch of scale factor -1} {parallel to y-axis}	*B1 DB1	{} indicate how the B1 marks should be awarded throughout.
	Then {Translation} $\left\{\begin{pmatrix}0\\3\end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the positive *y*-direction. **N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
	Alternative method for question 1		
	{Translation} $\left\{\begin{pmatrix}0\\-3\end{pmatrix}\right\}$	B1 B1	Or Translation 3 units in the negative *y*-direction.
	Then {Reflection} {in the *x*-axis} or {Stretch of scale factor -1} {parallel to *y*-axis}	*B1 DB1	**N.B.** If order reversed a maximum of 3 out of 4 marks awarded.
		4	

26. 9709_w21_ms_13 Q: 6

Question	Answer	Marks	Guidance
(a)		B1	A reflection of the given curve in *y* = *x* (the line *y* = *x* can be implied by position of curve).
		1	

Question	Answer	Marks	Guidance
(b)	$y=\frac{-x}{\sqrt{4-x^2}}$ leading to $x^2=y^2\left(4-x^2\right)$	*M1	Squaring and clearing the fraction. Condone one error in squaring $-x$ or y
	$x^2\left(1+y^2\right) = 4y^2$	DM1	OE. Factorisation of the new subject with order of operations correct. Condone sign errors.
	$x=(\pm)\frac{2y}{\sqrt{1+y^2}}$	DM1	$x=(\pm)\sqrt{\left(\frac{4y^2}{(1+y^2)}\right)}$ OE is acceptable for this mark. Isolating the new subject. Order of operations correct. Condone sign errors.
	$\mathrm{f}^{-1}(x)=\frac{-2x}{\sqrt{1+x^2}}$	A1	Selecting the correct square root. Must not have fractions in numerator or denominator.
		4	
(c)	1 or $a=1$	B1	Do not allow $x=1$ or $-1<x<1$
		1	
(d)	$[\mathrm{fg}(x)=\mathrm{f}(2x)=]\ \frac{-2x}{\sqrt{4-4x^2}}$	B1	Allow $\frac{-2x}{\sqrt{4-(2x)^2}}$ or any correct unsimplified form.
	$\mathrm{fg}(x)=\frac{-x}{\sqrt{1-x^2}}$ or $\frac{-x}{1-x^2}\sqrt{1-x^2}$ or $\frac{x}{x^2-1}\sqrt{1-x^2}$	B1	Result of cancelling 2 in numerator and denominator.
		2	

27. 9709_m20_ms_12 Q: 2

	Answer	Mark	Partial Marks
	[Stretch] [factor 2, x direction (or y-axis invariant)]	*B1 DB1	
	[Translation or Shift] [1 unit in y direction] **or** [Translation/Shift] $\left[\begin{pmatrix}0\\1\end{pmatrix}\right]$	B1B1	Accept transformations in either order. Allow (0, 1) for the vector
		4	

28. 9709_m20_ms_12 Q: 9

	Answer	Mark	Partial Marks
(a)	$\left[2(x+3)^2\right]$ $[-7]$	B1B1	Stating $a=3, b=-7$ gets B1B1
		2	
(b)	$y=2(x+3)^2-7 \to 2(x+3)^2=y+7 \to (x+3)^2=\frac{y+7}{2}$	M1	First 2 operations correct. Condone sign error or with *x/y* interchange
	$x+3=(\pm)\sqrt{\frac{y+7}{2}} \to x=(\pm)\sqrt{\frac{y+7}{2}}-3 \to \ \mathrm{f}^{-1}(x)=-\sqrt{\frac{x+7}{2}}-3$	A1FT	FT on *their a* and *b*. Allow $y = ...$
	Domain: $x \geqslant -5$ **or** $\geqslant -5$ **or** $[-5, \infty)$	B1	Do not accept $y=\ldots,\ f(x)=\ldots, f^{-1}(x)=\ldots$
		3	
(c)	$\mathrm{fg}(x) = 8x^2-7$	B1FT	SOI. FT on *their* –7 from part **(a)**
	$8x^2-7=193 \to x^2=25 \to x=-5$ only	B1	
	Alternative method for question 9(c)		
	$\mathrm{g}(x) = \mathrm{f}^{-1}(193) \to \ 2x-3=-\sqrt{100}-3$	M1	FT on *their* $\mathrm{f}^{-1}(x)$
	$x=-5$ only	A1	
		2	
(d)	(Largest *k* is) $-\frac{1}{2}$	B1	Accept $-\frac{1}{2}$ or $k \leqslant -\frac{1}{2}$
		1	

29. 9709_s20_ms_11 Q: 6

(a)	$3(3x+b)+b=9x+4b \to 10=18+4b$	M1
	$b=-2$	A1
	Either $\mathrm{f}(14) = 2$ **or** $\mathrm{f}^{-1}(x) = 2(x + a)$ etc.	M1
	$a=5$	A1
		4
(b)	$\mathrm{gf}(x) = 3\left(\frac{1}{2}x-5\right)-2$	M1
	$\mathrm{gf}(x) = \frac{3}{2}x-17$	A1
		2

30. 9709_s20_ms_12 Q: 5

(a)	$\mathrm{ff}(x) = a-2(a-2x)$	M1
	$\mathrm{ff}(x) = 4x-a$	A1
	$\mathrm{f}^{-1}(x) = \frac{a-x}{2}$	M1 A1
		4
(b)	$4x-a=\frac{a-x}{2} \to 9x=3a$	M1
	$x=\frac{a}{3}$	A1
		2

31. 9709_s20_ms_13 Q: 3

(a)	$(y)=f(-x)$	B1
		1
(b)	$(y)=2f(x)$	B1
		1
(c)	$(y)=f(x+4)-3$	B1 B1
		2

32. 9709_s20_ms_13 Q: 9

(a)	$\left[(x-2)^2\right][-1]$	B1 B1
		2
(b)	Smallest $c = 2$ (**FT** on *their* part **(a)**)	B1FT
		1
(c)	$y=(x-2)^2-1 \rightarrow (x-2)^2=y+1$	*M1
	$x=2(\pm)\sqrt{y+1}$	DM1
	$(f^{-1}(x))=2+\sqrt{x+1}$ for $x>8$	A1
		3
(d)	$gf(x)=\frac{1}{(x-2)^2-1+1}=\frac{1}{(x-2)^2}$ OE	B1
	Range of gf is $0<gf(x)<\frac{1}{9}$	B1 B1
		3

33. 9709_w20_ms_11 Q: 11

	Answer	Mark	Partial Marks
(a)	$fg(x) = (2x+1)^2+3$	B1	OE
		1	
(b)	$y=(2x+1)^2+3 \rightarrow 2x+1=(\pm)\sqrt{y-3}$	M1	1st two operations. Allow one sign error or *x*/*y* interchanged
	$x=(\pm)\frac{1}{2}\left(\sqrt{y-3}\ -1\right)$	M1	OE 2nd two operations. Allow one sign error or *x*/*y* interchanged
	$\left(fg^{-1}(x)=\right)\ \frac{1}{2}\left(\sqrt{x-3}\ -1\right)$ for $(x)>3$	A1 B1	Allow $(3, \infty)$
		4	
(c)	$gf(x) = 2(x^2+3)+1$	B1	SOI
	$(2x+1)^2+3-3=2(x^2+3)+1 \rightarrow 2x^2+4x-6\ (=0)$	*M1	Express as 3-term quadratic
	$(2)(x+3)(x-1)\ (=0)$	DM1	Or quadratic formula or completing the square
	$x=1$	A1	
		4	

34. 9709_w20_ms_12 Q: 5

	Answer	Mark	Partial Marks
(a)	0	B1	
		1	
(b)	$(f^{-1}(x)) = \frac{x+2}{4}$, $(g^{-1}(x)) = \frac{4-x}{x}$ or $\frac{4}{x}-1$	B1 B1	OE. Sight of correct inverses.
	$x^2 + 6x - 16 (= 0)$	B1	Equating inverses and simplifying.
	$(x+8)$ and $(x-2)$	M1	Correct attempt at solution of *their* 3-term quadratic-factorising, completing the square or use of formula.
	$(x =)$ 2 or −8	A1	Do not accept answers obtained with no method shown.
		5	

35. 9709_w20_ms_13 Q: 1

	Answer	Mark	Partial Marks
(a)	$\left[(x+3)^2\right]$ $[-4]$	B1 B1	
		2	
(b)	[Translation or shift] $\begin{pmatrix}-3\\-4\end{pmatrix}$	B1 B1 FT	Accept [translation/shift] $\begin{pmatrix}-\textit{their } a\\ \textit{their } b\end{pmatrix}$ OR translation –3 units in x-direction and (translation) –4 units in y-direction.
		2	

36. 9709_w20_ms_13 Q: 6

	Answer	Mark	Partial Marks
(a)	$y = \frac{2x}{3x-1} \rightarrow 3xy - y = 2x \rightarrow 3xy - 2x = y$ (or $-y = 2x - 3xy$)	*M1	For 1st two operations. Condone a sign error
	$x(3y-2) = y \rightarrow x = \frac{y}{3y-2}$ (or $x = \frac{-y}{2-3y}$)	DM1	For 2nd two operations. Condone a sign error
	$\left(f^{-1}(x)\right) = \frac{x}{3x-2}$	A1	Allow $\left(f^{-1}(x)\right) = \frac{-x}{2-3x}$
		3	
(b)	$\left[\frac{2(3x-1)+2}{3(3x-1)}\right] = \left[\frac{6x}{3(3x-1)} = \frac{2x}{3x-1}\right]$	B1 B1	AG, WWW First B1 is for a correct single unsimplified fraction. An intermediate step needs to be shown. Equivalent methods accepted.
		2	
(c)	$(f(x)) > \frac{2}{3}$	B1	Allow $(y) > \frac{2}{3}$. Do not allow $x > \frac{2}{3}$
		1	

37. 9709_m19_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	$\left[(x-2)^2\right]+[3]$	**B1 DB1**	2nd B1 dependent on ±2 in 1st bracket
		2	
(ii)	Largest *k* is 2 Accept $k \leqslant 2$	**B1**	Must be in terms of *k*
		1	
(iii)	$y=(x-2)^2+3 \Rightarrow x-2=(\pm)\sqrt{y-3}$	**M1**	
	$\Rightarrow f^{-1}(x)=2-\sqrt{x-3}$ for $x>4$	**A1B1**	
		3	
(iv)	$gf(x)=\dfrac{2}{x^2-4x+7-1}=\dfrac{2}{(x-2)^2+2}$	**B1**	Either form
	Since $f(x)>4 \Rightarrow gf(x)<2/3$ (or since $x<1$ etc)	**M1A1**	2/3 in answer implies M1 www
	range of gf(*x*) is $0<gf(x)(<2/3)$	**B1**	Accept $0<y<2/3$, (0, 2/3) but $0<x<2/3$ is SCM1A1B0
		4	

38. 9709_s19_ms_11 Q: 5

	Answer	Mark	Partial Marks
(i)	$-2(x-3)^2+15$ $(a=-3, b=15)$	**B1 B1**	Or seen as $a=-3$, $b=15$ B1 for each value
		2	
(ii)	$(f(x) \leqslant)$ 15	**B1**	**FT** for $(\leqslant)$ their "*b*" Don't accept (3,15) alone
		1	
(iii)	$gf(x)=2(-2x^2+12x-3)+5=-4x^2+24x-6+5$	**B1**	
	$gf(x)+1=0 \rightarrow -4x^2+24x=0$	**M1**	
	$x=0$ or 6	**A1**	Forms and attempts to solve a quadratic Both answers given.
		3	

39. 9709_s19_ms_12 Q: 7

	Answer	Mark	Partial Marks
(i)	$(f^{-1}(x)) = \dfrac{x+2}{3}$ oe	B1	
	$y = \dfrac{2x+3}{x-1} \rightarrow (x-1)y = 2x+3 \rightarrow x(y-2) = y+3$	M1	Correct method to obtain $x =$, (or $y =$, if interchanged) but condone $+/-$ sign errors
	$(g^{-1}(x)$ or $y) = \dfrac{x+3}{x-2}$ oe $\left(eg\,\dfrac{5}{x-2}+1\right)$	A1	Must be in terms of x
	$x \neq 2$ only	B1	**FT** for value of x from their denominator = 0
		4	
(ii)	$\left(fg(x)=\right)\ \dfrac{3(2x+3)}{x-1} - 2\ (=\dfrac{7}{3})$	B1	
	$18x + 27 = 13x - 13$ or $3(4x + 11) = 7(x - 1)$ $(5x = -40)$	M1	Correct method from their $fg = \dfrac{7}{3}$ leading to a linear equation and collect like terms. Condone omission of $2(x-1)$.
	Alternative method for question 7(ii)		
	$(f^{-1}(\dfrac{7}{3})) = \dfrac{13}{9}$	B1	
	$\dfrac{2x+3}{x-1} = \dfrac{13}{9} \rightarrow 9(2x + 3) = 13(x - 1)\ (\rightarrow 5x = -40)$	M1	Correct method from $g(x) =$ their $\dfrac{13}{9}$ leading to a linear equation and collect like terms.
	$x = -8$	A1	
		3	

40. 9709_s19_ms_13 Q: 4

	Answer	Mark	Partial Marks
(i)	Max(***a***) is 8	B1	Allow $a = 8$ or $a \leqslant 8$
	Min(***b***) is 24	B1	Allow $b = 24$ or $b \geqslant 24$
		2	SCB1 for 8 and 24 seen
(ii)	$gf(x) = \dfrac{96}{x-1} - 4$ **or** $gf(x) = \dfrac{100-4x}{x-1}$	B1	$2\left(\dfrac{48}{x-1}\right) - 4$ is insufficient Apply ISW
		1	
(iii)	$y = \dfrac{96}{x-1} - 4 \rightarrow y+4 = \dfrac{96}{x-1} \rightarrow x-1 = \dfrac{96}{y+4}$	M1	**FT** from *their*(**ii**) provided (**ii**) involves algebraic fraction. Allow sign errors
	$(gf)^{-1}(x) = \dfrac{96}{x+4} + 1$	A1	OR $\dfrac{100+x}{x+4}$. Must be a function of x. Apply ISW
		2	

41. 9709_w19_ms_11 Q: 7

	Answer	Mark	Partial Marks
(i)	Range of f is $0 < f(x) < 3$	B1B1	OE. Range cannot be defined using x
	Range of g is $g(x) > 2$	B1	OE
		3	
(ii)	$\left(fg(x)=\right)\frac{3}{2\left(\frac{1}{x}+2\right)+1}=\frac{3x}{2+5x}$	B1B1	Second B mark implies first B mark
		2	
(iii)	$y=\frac{3x}{2+5x} \rightarrow 2y+5xy=3x \rightarrow 3x-5xy=2y$	M1	Correct order of operations
	$x(3-5y)=2y \rightarrow x=\frac{2y}{3-5y}$	M1	Correct order of operations
	$\left((fg)^{-1}(x)\right)=\frac{2x}{3-5x}$	A1	
		3	

42. 9709_w19_ms_13 Q: 2

	Answer	Mark	Partial Marks
	$(y=)\left[(x-3)^2\right][-2]$	*B1 DB1	DB1 dependent on 3 in 1st bracket
	$x-3=(\pm)\sqrt{y+2}$ **or** $y-3=(\pm)\sqrt{x+2}$	M1	Correct order of operations
	$\left(g^{-1}(x)\right)=3+\sqrt{x+2}$	A1	Must be in terms of x
	Domain (of g^{-1}) is $(x) > -1$	B1	Allow $(-1, \infty)$. Do not allow $y > -1$ or $g(x) > -1$ or $g^{-1}(x) > -1$
		5	

43. 9709_s18_ms_12 Q: 7

	Answer	Mark	Partial Marks
(i)	$25 - 2(x + 3)^2$	B1 B1	Mark expression if present: B1 for 25 and B1 for $-2(x+3)^2$. If no expression award $a = 25$ B1 and $b = 3$ B1.
		2	
(ii)	$(-3, 25)$	B1FT	FT from answers to (i) or by calculus
		1	
(iii)	$(k) = -3$ also allow x or $k \geqslant -3$	B1FT	FT from answer to (i) or (ii) **NOT** $x = -3$
		1	

	Answer	Mark	Partial Marks
(iv)	**EITHER**		
	$y = 25 - 2(x+3)^2 \rightarrow 2(x+3)^2 = 25 - y$	*M1	Makes their squared term containing x the subject or equivalent with x/y interchanged first. Condone errors with +/- signs.
	$x+3 = (\pm)\sqrt{½(25-y)}$	DM1	Divide by ±2 and then square root allow ±.
	OR		
	$y = 7 - 2x^2 - 12x \rightarrow 2x^2 + 12x + y - 7 \ (= 0)$	*M1	Rearranging equation of the curve.
	$x = \dfrac{-12 \pm \sqrt{12^2 - 8(y-7)}}{4}$	DM1	Correct use of their 'a, b and c' in quadratic formula. Allow just + in place of ±.
	$g^{-1}(x) = \sqrt{\left(\dfrac{25-x}{2}\right)} - 3$ oe isw if substituting $x = -3$	A1	± gets A0. Must now be a function of x. Allow $y =$
		3	

44. 9709_s18_ms_13 Q: 10

	Answer	Mark	Partial Marks
(i)	Smallest value of c is 2. Accept 2, $c = 2$, $c \geqslant 2$. Not in terms of x	B1	Ignore superfluous working, e.g. $\dfrac{d^2y}{dx^2} = 2$
		1	
(ii)	$y = (x-2)^2 + 2 \rightarrow x - 2 = (\pm)\sqrt{y-2} \rightarrow x = (\pm)\sqrt{y-2} + 2$	M1	Order of operations correct. Allow sign errors
	$f^{-1}(x) = \sqrt{x-2} + 2$	A1	Accept $y = \sqrt{x-2} + 2$
	Domain of f^{-1} is $x \geqslant 6$. Allow $\geqslant 6$.	B1	Not $f^{-1}(x) \geqslant 6$. Not $f(x) \geqslant 6$. Not $y \geqslant 6$
		3	
(iii)	$\left[(x-2)^2 + 2 - 2\right]^2 + 2 = 51$ SOI Allow 1 term missing for M mark Or $\left(x^2 - 4x + 6\right)^2 - 4\left(x^2 - 4x + 6\right) + 6 = 51$	M1A1	ALT. $f(x) = f^{-1}(51)$ (M1) $= \sqrt{51-2} + 2$ (A1)
	$(x-2)^4 = 49$ or $(x^2 - 4x + 4)^2 = 49$ OR $x^4 - 8x^3 + 24x^2 - 32x - 33 = 0$ often implied by next line	A1	$(x-2)^2 + 2 = \sqrt{49} + 2$ OR $f(x) = 9$
	$(x-2)^2 = (\pm)7$ OR $x^2 - 4x - 3 = 0$. Ignore $x^2 - 4x + 11 = 0$	A1	$(x-2)^2 = 7$ OR $x = f^{-1}(9)$
	$x = 2 + \sqrt{7}$ only CAO $x = 2 + \sqrt[4]{49}$ scores 3/5	A1	$x = 2 + \sqrt{7}$
		5	

45. 9709_w18_ms_11 Q: 11

	Answer	Mark	Partial Marks
(a)(i)	[Greatest value of a is] 3	**B1**	Must be in terms of a. Allow $a < 3$. Allow $a \leqslant 3$
		1	
(a)(ii)	Range is $y > -1$	**B1**	Ft on *their a*. Accept any equivalent notation
	$y = (x-3)^2 - 1 \to (x-3)^2 = 1 + y \to x = 3(\pm)\sqrt{1+y}$	**M1**	Order of operations correct. Allow sign errors
	$f^{-1}(x) = 3 - \sqrt{1+x}$ cao	**A1**	
		3	
(b)(i)	gg(2x) $= \left[(2x-3)^2 - 3\right]^2$	**B1**	
	$(2x-3)^4 - 6(2x-3)^2 + 9$	**B1**	
		2	
(b)(ii)	$\left[16x^4 - 96x^3 + 216x^2 - 216x + 81\right] + \left[(-24x^2 + 72x - 54) + 9\right]$ $16x^4 - 96x^3 + 192x^2 - 144x + 36$	**B4,3,2,1,0**	
		4	

46. 9709_w18_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$2x^2 - 12x + 7 = 2(x-3)^2 - 11$	**B1 B1**	Mark full expression if present: B1 for $2(x-3)^2$ and B1 for -11. If no clear expression award $a = -3$ and $b = -11$.
		2	
(ii)	Range (of f or y) $\geqslant$ **'their – 11'**	**B1FT**	FT for their 'b' or start again. Condone **>**. Do **NOT** accept $x >$ or $\geqslant$
		1	
(iii)	(k =) –"their a" also allow ***x* or *k* $\leqslant$ 3**	**B1FT**	FT for their "a" or start again using $\frac{dy}{dx} = 0$. Do **NOT accept *x* = 3.**
		1	
(iv)	$y = 2(x-3)^2 - 11 \to y + 11 = 2(x-3)^2$ $\frac{y+11}{2} = (x-3)^2$	***M1**	Isolating their $(x-3)^2$, condone -11.
	$x = 3 + \sqrt{\left(\frac{y+11}{2}\right)}$ or $3 - \sqrt{\left(\frac{y+11}{2}\right)}$	**DM1**	Other operations in correct order, allow ± at this stage. Condone – 3.
	($g^{-1}(x)$ or y) $= 3 - \sqrt{\left(\frac{x+11}{2}\right)}$	**A1**	needs '–'. x and y could be interchanged at the start.
		3	

47. 9709_w18_ms_13 Q: 11

	Answer	Mark	Partial Marks
(i)	$[2]\left[(x-3)^2\right][-7]$	**B1B1B1**	
		3	
(ii)	Largest value of k is 3. Allow (k =) 3.	**B1**	Allow $k \leqslant 3$ but not $x \leqslant 3$ as final answer.
		1	

	Answer	Mark	Partial Marks
(iii)	$y=2(x-3)^2-7 \to (x-3)^2 = ½(y+7)$ or with *x/y* transposed	M1	Ft *their a, b, c*. Order of operations correct. Allow sign errors
	$x=3\pm\sqrt{½(y+7)}$ Allow 3 + √ or 3 –√ or with *x/y* transposed	DM1	Ft *their a, b, c*. Order of operations correct. Allow sign errors
	$f^{-1}(x)=3-\sqrt{½(x+7)}$	A1	
	(Domain is *x*) $\geqslant$ *their* -7	B1FT	Allow other forms for interval but if variable appears must be *x*
		4	
(iv)	$x+3\leqslant 1$. Allow $x + 3 = 1$	M1	Allow $x+3\leqslant k$
	largest p is -2 . Allow (p =) -2	A1	Allow $p\leqslant -2$ but not $x\leqslant -2$ as final answer.
	$fg(x)=f(x+3)=2x^2-7$ cao	B1	
		3	

48. 9709_m17_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	gf(*x*) = $3(2x^2+3)+2=6x^2+11$	B1	AG
	fg(*x*) $=2(3x+2)^2+3$ Allow $18x^2+24x+11$	B1	ISW if simplified incorrectly. Not retrospectively from **(ii)**
	Total:	2	
(ii)	$y=2(3x+2)^2+3 \Rightarrow 3x+2=(\pm)\sqrt{(y-3)/2}$ oe	M1	Subtract 3;divide by 2;square root. Or *x/y* interchanged. Allow $\frac{\sqrt{y-3}}{2}$ for 1st M
	$\Rightarrow x=(\pm)\frac{1}{3}\sqrt{(y-3)/2}-\frac{2}{3}$ oe	M1	Subtract 2; divide by 3; Indep. of 1st M1. Or *x/y* interchanged.
	$\Rightarrow (fg)^{-1}(x)=\frac{1}{3}\sqrt{(x-3)/2}-\frac{2}{3}$ oe	A1	Must be a function of *x*. Allow alt. method $g^{-1}f^{-1}(x)$ OR $18\left(x+\frac{2}{3}\right)^2+3 \Rightarrow (fg)^{-1}(x)=\sqrt{\frac{x-3}{18}}-\frac{2}{3}$
	Solve *their* $(fg)^{-1}(x)\geqslant 0$ or attempt range of fg	M1	Allow range $\geqslant 3$ for M only. Can be implied by correct answer or $x > 11$
	Domain is $x\geqslant 11$	A1	
	Total:	5	

	Answer	Mark	Partial Marks
(iii)	$6(2x)^2+11=2(3x+2)^2+3$	M1	Replace *x* with 2*x* in gf and equate to *their* fg(*x*) from (i). Allow $\underline{12}x^2+11=$
	$6x^2-24x=0$ oe	A1	Collect terms to obtain correct quadratic expression.
	$x=0$, 4	A1	Both required
	Total:	3	

49. 9709_s17_ms_11 Q: 9

	Answer	Mark	Partial Marks
	$f: x \mapsto \frac{2}{3-2x}$ $g: x \mapsto 4x+a,$		
(i)	$y=\frac{2}{3-2x} \to y(3-2x)=2 \to 3-2x=\frac{2}{y}$	**M1**	Correct first 2 steps
	$\to 2x=3-\frac{2}{y} \to f^{-1}(x)=\frac{3}{2}-\frac{1}{x}$	**M1 A1**	Correct order of operations, any correct form with $f(x)$ or $y=$
	Total:	**3**	
(ii)	$gf(-1)=3$ $f(-1)=\frac{2}{5}$	**M1**	Correct first step
	$\frac{8}{5}+a=3 \to a=\frac{7}{5}$	**M1 A1**	Forms an equation in a and finds a, OE
			(or $\frac{8}{3-2x}+a=3$, **M1** Sub and solves **M1**, **A1**)
	Total:	**3**	
(iii)	$g^{-1}(x)=\frac{x-a}{4}=f^{-1}(x)$	**M1**	Finding $g^{-1}(x)$ and equating to their $f^{-1}(x)$ even if $a=7/5$
	$\to x^2-x(a+6)+4(=0)$	**M1**	Use of b^2-4ac on a quadratic with a in a coefficient
	Solving $(a+6)^2=16$ or $a^2+12a+20(=0)$	**M1**	Solution of a 3 term quadratic
	$\to a=-2$ or -10	**A1**	
	Total:	**4**	

50. 9709_s17_ms_13 Q: 9

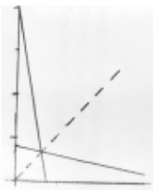
	Answer	Mark	Partial Marks
(i)	$(3x-1)^2+5$	**B1B1B1**	First 2 marks dependent on correct $(ax+b)^2$ form. OR $a=3,\ b=-1,\ c=5$ e.g. from equating coefs
	Total:	**3**	
(ii)	Smallest value of p is 1/3 seen. (Independent of (i))	**B1**	Allow $p \geqslant 1/3$ or $p=1/3$ or 1/3 seen. But not in terms of x.
	Total:	**1**	
(iii)	$y=(3x-1)^2+5 \Rightarrow 3x-1=(\pm)\sqrt{y-5}$	**B1 FT**	OR $y=9\left(x-\frac{1}{3}\right)^2+5 \Rightarrow (y-5)/9=\left(x-\frac{1}{3}\right)^2$ (Fresh start)
	$x=(\pm)\frac{1}{3}\sqrt{y-5}+\frac{1}{3}$ OE	**B1 FT**	Both starts require 2 operations for each mark. FT for *their* values from part (i)
	$f^{-1}(x)=\frac{1}{3}\sqrt{x-5}+\frac{1}{3}$ OE domain is $x \geq their 5$	**B1B1 FT**	Must be a function of x and $\pm$ removed. Domain must be in terms of x. Note: $\sqrt{y-5}$ expressed as $\sqrt{y}-\sqrt{5}$ scores Max **B0B0B0B1** [See below for general instructions for different starts]
	Total:	**4**	
(iv)	$q<5$ CAO	**B1**	
	Total:	**1**	
Alt (iii)	For start $(ax-b)^2+c$ or $a(x-b)^2+c$ ($a \neq 0$) ft for their a, b, c For start $(x-b)^2+c$ ft but award only **B1** for 3 correct operations For start $a(bx-c)^2+d$ ft but award **B1** for first2 operations correct and **B1** for the next 3 operations correct		

51. 9709_w17_ms_11 Q: 9

	Answer	Mark	Partial Marks
(i)	$gg(x) = g(2x - 3) = 2(2x - 3) - 3 = 4x - 9$	**M1A1**	
		2	
(ii)	$y = \frac{1}{x^2 - 9} \rightarrow x^2 = \frac{1}{y} + 9$ OE	**M1**	Invert; add 9 to both sides or with *x/y* interchanged
	$f^{-1}(x) = \sqrt{\frac{1}{x} + 9}$	**A1**	
	Attempt soln of $\sqrt{\frac{1}{x} + 9} > 3$ or attempt to find range of f. $(y > 0)$	**M1**	
	Domain is $x > 0$ CAO	**A1**	May simply be stated for **B2**
		4	

	Answer	Mark	Partial Marks
(iii)	*EITHER:* $\frac{1}{(2x-3)^2 - 9} = \frac{1}{7}$	**(M1**	
	$(2x-3)^2 = 16$ or $4x^2 - 12x - 7 = 0$	**A1**	
	$x = 7/2$ or $-1/2$	**A1**	
	$x = 7/2$ only	**A1)**	
	OR: $g(x) = f^{-1}\left(\frac{1}{7}\right)$	**(M1**	
	$g(x) = 4$	**A1**	
	$2x - 3 = 4$	**A1**	
	$x = 7/2$	**A1)**	
		4	

52. 9709_w17_ms_12 Q: 2

	Answer	Mark	Partial Marks
(i)	$\frac{4-x}{5}$	**B1**	OE
	Equate a valid attempt at f^{-1} with f, or with *x*, or f with *x* $\rightarrow \left(\frac{2}{3}, \frac{2}{3}\right)$ **or** (0.667, 0.667)	**M1, A1**	Equating and an attempt to solve as far *x* =. Both coordinates.
		3	
(ii)		**B1**	Line $y = 4 - 5x$ – must be straight, through approximately (0,4) and intersecting the positive *x* axis near (1,0) as shown.
		B1	Line $y = \frac{4-x}{5}$ – must be straight and through approximately (0, 0.8). No need to see intersection with *x* axis.
		B1	A line through (0,0) and the point of intersection of a pair of straight lines with negative gradients. This line must be at 45° unless scales are different in which case the line must be labelled *y=x*.
		3	

53. 9709_w17_ms_13 Q: 6

	Answer	Mark	Partial Marks
(i)	$y=\frac{2}{x^2-1} \Rightarrow x^2=\frac{2}{y}+1$ OE	M1	
	$x=(\pm)\sqrt{\frac{2}{y}+1}$ OE	A1	With or without x/y interchanged.
	$f^{-1}(x)=-\sqrt{\frac{2}{x}+1}$ OE	A1	Minus sign obligatory. Must be a function of x.
		3	

	Answer	Mark	Partial Marks
(ii)	$\left(\frac{2}{x^2-1}\right)^2+1=5$	B1	
	$\frac{2}{x^2-1}=(\pm)2$ OE OR $x^4-2x^2=0$ OE $x^2-1=(\pm)1 \Rightarrow x^2=2$ (or 0) $x=-\sqrt{2}$ or -1.41 only	B1	Condone $x^2=0$ as an additional solution
		4	

54. 9709_m16_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	$2a+4b=8$ $2a^2+3a+4b=14$ $2a^2+3a+(8-2a)=14 \to (a+2)(2a-3)=0$ $a=-2$ or $3/2$ $b=3$ or $5/4$	**M1** **A1** **M1** **A1** **A1** **[5]**	Substitute in –2 and –3 Sub linear into quadratic & attempt solution If A0A0 scored allow SCA1 for either $(-2,3)$ or (3/2, 5/4)
(ii)	$y=\left(x-\frac{1}{2}\right)^2-\frac{13}{4}$ Attempt completing of square $x-\frac{1}{2}=(\pm)\sqrt{y+\frac{13}{4}}$ oe $f^{-1}(x)=\frac{1}{2}-\sqrt{x+\frac{13}{4}}$ oe Domain of f^{-1} is $(x)\geqslant -13/4$	**M1A1** **DM1** **A1** **B1**√ **[5]**	Allow with x/y transposed Allow with x/y transposed Allow y =..... Must be a function of x Allow >, $-13/4\leqslant x\leqslant\infty$, $\left[-\frac{13}{4},\infty\right]$ etc

55. 9709_s16_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$f: x \mapsto 10-3x$, $g: x \mapsto \frac{10}{3-2x}$,		
	$ff(x) = 10-3(10-3x)$	**B1**	Correct unsimplified expression
	$gf(2) = \frac{10}{3-2(10-3(2))}$ $(=-2)$	**B1**	Correct unsimplified expression with 2 in for x
	$x=2$	**B1** [3]	

56. 9709_s16_ms_12 Q: 11

	Answer	Mark	Partial Marks
	$f : x \mapsto 6x - x^2 - 5$		
(i)	$6x - x^2 - 5 \leqslant 3$		
	$\rightarrow x^2 - 6x + 8 \geqslant 0$	M1	$\pm(6x - x^2 - 8) =, \leqslant, \geqslant 0$ and attempts to solve
	$\rightarrow x = 2, x = 4$	A1	Needs both values whether =2, <2, >2
	$x \leqslant 2, x \geqslant 4$ condone < and/or >	A1 [3]	Accept all recognisable notation.
(ii)	Equate $mx + c$ and $6x - x^2 - 5$ Use of "$b^2 - 4ac$"	M1 DM1	Equates, sets to 0. Use of discriminant with values of $a.b.c$ independent of x.
	$4c = m^2 - 12m + 16$. **AG**	A1	= (0) must appear before last line.
	OR		
	$\frac{dy}{dx} = 6 - 2x = m \rightarrow x = \left(\frac{6-m}{2}\right)$	M1	Equates $\frac{dy}{dx}$ to m and rearrange
	$m\left(\frac{6-m}{2}\right) + c = 6\left(\frac{6-m}{2}\right) - \left(\frac{6-m}{2}\right)^2 - 5$	M1	Equates $mx + c$ and $6x - x^2 - 5$ and substitutes for x
	$4c = m^2 - 12m + 16$. **AG**	A1 [3]	
(iii)	$6x - x^2 - 5 = 4 - (x - 3)^2$	B1 B1 [2]	4 B1 – $(x - 3)^2$ B1
(iv)	$k = 3$.	B1√ [1]	√ for "b".
(v)	$g^{-1}(x) = \sqrt{4 - x} + 3$	M1 A1 [2]	Correct order of operations. $\pm\sqrt{4-x} + 3$ M1A0 $\sqrt{x-4} + 3$ M1A0 $\sqrt{4-y} + 3$ M1A0

57. 9709_s16_ms_13 Q: 10

	Answer	Mark	Partial Marks
(i)	$2(ax^2+b)+3=6x^2-21$ $a=3,\ b=-12$	**M1** **A1A1** [3]	
(ii)	$3x^2-12\geqslant 0$ or $6x^2-21\geqslant 3$ $x\leqslant -2$ i.e. (max) $q=-2$	**M1** **A1** [2]	Allow = or $\leqslant$ or > or <. Ft from *their a, b* Must be in terms of q (eg $q\leqslant -2$)
(iii)	$y\geqslant 6(-3)^2-21\Rightarrow$ range is $(y)\geqslant 33$	**B1** [1]	Do not allow $y>33$.Accept all other notations e.g. [33, ∞) or [33, ∞]
(iv)	$y=6x^2-21\Rightarrow x=(\pm)\sqrt{\dfrac{y+21}{6}}$ $(\mathrm{fg})^{-1}(x)=-\sqrt{\dfrac{x+21}{6}}$ Domain is $x\geqslant 33$	**M1** **A1** **B1**√ [3]	 Allow y =..... . Must be a function of x ft from *their* part (**iii**) but x essential

58. 9709_w16_ms_11 Q: 8

	Answer	Mark		Partial Marks
(i)	fg(x) = $5x$ Range of fg is $y\geqslant 0$ oe	**M1A1** **B1**	[3]	only Accept $y>0$
(ii)	$y=4/(5x+2)\Rightarrow x=(4-2y)/5y$ oe $\mathrm{g}^{-1}(x)=(4-2x)/5x$ oe 0, 2 with no incorrect inequality $0<x\leqslant 2$ oe, c.a.o.	**M1** **A1** **B1,B1** **B1**	[5]	Must be a function of x

59. 9709_w16_ms_13 Q: 8

	Answer	Mark		Partial Marks
(i)	$(2x+3)^2+1$ Cannot score retrospectively in **(iii)**	**B1B1B1**	[3]	For $a=2,\ b=3,\ c=1$
(ii)	$g(x)=2x+3$ cao	**B1**	[1]	In **(ii)**,**(iii)** Allow if from $4\left(x+\frac{3}{2}\right)^2+1$
(iii)	$y=(2x+3)^2+1 \Rightarrow 2x+3=(\pm)\sqrt{y-1}$ or ft from **(i)**	**M1**		Or with *x/y* transposed.
	$x=(\pm)\frac{1}{2}\sqrt{y-1}-\frac{3}{2}$ or ft from **(i)**	**M1**		Or with *x/y* transposed Allow sign errors.
	$(fg)^{-1}(x)=\frac{1}{2}\sqrt{x-1}-\frac{3}{2}$ cao Note alt. method $g^{-1}f^{-1}$	**A1**		Must be a function of *x*. Allow *y* = ...
	Domain is $(x)>10$	**B1**	[4]	Allow (10, ∞), $10<x<\infty$ etc. but not with *y* or f or g involved. Not $\geqslant 10$
	ALT. method for first 3 marks:			
	Trying to obtain $g^{-1}\left[f^{-1}(x)\right]$	***M1**		
	$g^{-1}=\frac{1}{2}(x-3),\ f^{-1}=\sqrt{x-1}$	**DM1**		Both required
	A1 for $\frac{1}{2}\sqrt{x-1}-\frac{3}{2}$	**A1**		

60. 9709_s15_ms_12 Q: 11

	Answer	Mark	Partial Marks
	$f: x \mapsto 2x^2-6x+5$		
(i)	$2x^2-6x+5-p=0$ has no real roots	M1	Sets to 0 with *p* on LHS.
	Uses $b^2-4ac \rightarrow 36-8(5-p)$	DM1	Uses discriminant.
	Sets to 0 $\rightarrow p<\frac{1}{2}$	A1 [3]	co – must be "<", not "⩽".
(ii)	$2x^2-6x+5=2\left(x-\frac{3}{2}\right)^2+\frac{1}{2}$	3 × B1 [3]	co
(iii)	Range of g $\frac{1}{2}\leqslant g(x)\leqslant 13$	B1√ B1 [2]	√ on **(ii)** co from sub of $x=4$
	$h: x\mapsto 2x^2-6x+5$ for $k\leqslant x\leqslant 4$		
(iv)	Smallest $k=\frac{3}{2}$	B1√ [1]	√ on **(ii)**
(v)	$h(x)=2\left(x-\frac{3}{2}\right)^2+\frac{1}{2}$	M1	Using comp square form to try and get *x* as subject or *y* if transposed.
	Order of operations $\pm\frac{1}{2}$, ÷2, √, $\pm\frac{3}{2}$	DM1	Order must be correct
	→ Inverse = $\frac{3}{2}+\sqrt{\left(\frac{x}{2}-\frac{1}{4}\right)}$	A1 [3]	co (without ±)

61. 9709_s15_ms_13 Q: 6

	Answer	Mark	Partial Marks
(i)	Attempt to find $(f^{-1})^{-1}$	**M1**	
	$2xy = 1 - 5x$ or $\frac{1}{2x} = y + \frac{5}{2}$ Allow 1 sign error	**A1**	Or with *x*/*y* transposed.
	$x = \frac{1}{2y+5}$ oe Allow 1 sign error (total)	**A1**	Or with *x*/*y* transposed. Allow $x = \frac{\frac{1}{2}}{y+\frac{5}{2}}$.
	$(f(x)) = \frac{1}{2x+5}$ for $x \geqslant -\frac{9}{4}$ **(Allow** $-\frac{9}{4} \leqslant x \leqslant \infty$)	**A1 B1** **[5]**	Allow $\frac{\frac{1}{2}}{x+\frac{5}{2}}$. Condone $x > \frac{-9}{4}$, $(\frac{-9}{4}, \infty)$ (etc.)
(ii)	$\mathbf{f}^{-1}\left(\frac{1}{x}\right) = \frac{1-\frac{5}{x}}{\frac{2}{x}}$	**M1**	Reasonable attempt to find $\mathbf{f}^{-1}\left(\frac{1}{x}\right)$.
	$\frac{x-5}{2}$ or $\frac{1}{2}x - \frac{5}{2}$	**A1** **[2]**	

62. 9709_w15_ms_11 Q: 9

	Answer	Mark	Partial Marks
(i)	$-(1)(x-3)^2 + 4$	**B1B1B1** [3]	
(ii)	Smallest (*m*) is 3	**B1**√ [1]	Accept $m \geqslant 3$, $m = 3$. **Not** $x \geqslant 3$. Ft *their b*
(iii)	$(x-3)^2 = 4 - y$	**M1**	Or x/y transposed. Ft *their a*, *b*, *c*
	Correct order of operations	**M1**	
	$f^{-1}(x) = 3 + \sqrt{4-x}$ cao	**A1**	Accept $y =$ if clear
	Domain is $x \leqslant 0$	**B1** [4]	

63. 9709_w15_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$f : x \mapsto 3x + 2$, $g : x \mapsto 4x - 12$	**B1**	
	$f^{-1}(x) = \frac{x-2}{3}$	**B1**	
	$gf(x) = 4(3x + 2) - 12$	**M1**	Equates, collects terms, +soln
	Equate $\rightarrow$ $x = \frac{2}{7}$	**A1** [4]	

64. 9709_w15_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	$f : x \to x^2 + ax + b$, $x^2 + 6x - 8 = (x+3)^2 - 17$ or $2x + 6 = 0 \to x = -3 \to y = -17$ $\to$ Range $f(x) \geqslant -17$	B1 B1 B1√ [3]	B1 for $(x + 3)^2$. B1 for – 17 or B1 for $x = -3$, B1 $y = -17$ Following through visible method.
(ii)	$(x - k)(x + 2k) = 0$ $\equiv x^2 + 5x + b = 0$ $\to k = 5$ $\to b = -2k^2 = -50$	M1 A1 A1 [3]	Realises the link between roots and the equation comparing coefficients of x
(iii)	$(x+a)^2 + a(x+a) + b = a$ Uses $b^2 - 4ac \to 9a^2 - 4(2a^2 + b - a)$ $\to a^2 < 4(b - a)$	M1 DM1 A1 [3]	Replaces "x" by "$x + a$" in 2 terms Any use of discriminant

65. 9709_w15_ms_13 Q: 8

	Answer	Mark	Partial Marks
(i)	$3x + 1 \leqslant -1$ (Accept $3x + 1 = -1, 3a + 1 = -1$) $x \leqslant -2/3 \Rightarrow$ largest value of a is $-2/3$ (in terms of a)	M1 A1 [2]	Do not allow gf in (i) to score in (iii) Accept $a \leqslant -2/3$ and $a = -2/3$
(ii)	$fg(x) = 3(-1 - x^2) + 1$ $fg(x) + 14 = 0 \Rightarrow 3x^2 = 12$ oe (2 terms) $x = -2$ only	B1 B1 B1 [3]	No marks in this part for gf used
(iii)	$gf(x) = -1 - (3x+1)^2$ oe $gf(x) \leqslant -50 \Rightarrow (3x+1)^2 \geqslant 49$ (Allow $\leqslant$ *or* = $3x + 1 \geqslant 7$ *or* $3x + 1 \leq -7$ (one sufficient) www $x \leqslant -8/3$ only www	B1 M1 A1 A1 [4]	No marks in this part for fg used OR attempt soln of $9x^2 + 6x - 48 + / \leqslant / \geqslant 0$ OR $x - 2 \geqslant$ *or* $3x + 8 \leqslant 0$(one suffic)

66. 9709_m22_ms_12 Q: 2

Question	Answer	Marks	Guidance
	$x^2 + 2cx + 4 = 4x + c$ leading to $x^2 + 2cx - 4x + 4 - c \; [=0]$	*M1	Equate ys and move terms to one side of equation.
	$b^2 - 4ac = (2c-4)^2 - 4(4-c)$	DM1	Use of discriminant with *their* correct coefficients.
	$[4c^2 - 16c + 16 - 16 + 4c =] \; 4c^2 - 12c$	A1	
	$b^2 - 4ac > 0$ leading to $(4)c(c-3) > 0$	M1	Correctly apply '> 0' considering both regions.
	$c < 0, \; c > 3$	A1	Must be in terms of c. **SC B1** instead of M1A1 for $c \leqslant 0, c \geqslant 3$
		5	

67. 9709_m22_ms_12 Q: 6

Question	Answer	Marks	Guidance
(a)	$(x+1)^2+(3x-22)^2=85$	**M1**	OE. Substitute equation of line into equation of circle.
	$10x^2-130x+400\ [=0]$	**A1**	Correct 3-term quadratic
	$[10](x-8)(x-5)$ leading to $x=8$ or 5	**A1**	Dependent on factors or formula or completing of square seen.
	(8, 4), (5, –5)	**A1**	If M1A1A0A0 scored, then **SC B1** for correct final answer only.
		4	
(b)	Mid-point of $AB = \left(6\frac{1}{2}, -\frac{1}{2}\right)$	**M1**	Any valid method
	Use of $C = (-1, 2)$	**B1**	SOI
	$r^2=\left(-1-6\frac{1}{2}\right)^2+\left(2+\frac{1}{2}\right)^2$	**M1**	Attempt to find r^2. Expect $r^2=62\frac{1}{2}$.
	Equation of circle is $(x+1)^2+(y-2)^2=62\frac{1}{2}$	**A1**	OE.
		4	

68. 9709_m21_ms_12 Q: 4

Question	Answer	Marks	Guidance
	$x^2+kx+6=3x+k$ leading to $x^2+x(k-3)+(6-k)\ [=0]$	**M1**	Eliminate y and form 3-term quadratic.
	$(k-3)^2-4(6-k)[>0]$	**M1**	OE. Apply b^2-4ac.
	$k^2-2k-15[>0]$	**A1**	Form 3-term quadratic.
	$(k+3)(k-5)[>0]$	**A1**	Or $k=-3,\ 5$ from use of formula or completing square.
	$k<-3,\quad k>5$	**A1 FT**	Or any correct alternative notation, do not allow $\leqslant, \geqslant$. FT for *their* outside regions.
		5	

69. 9709_m21_ms_12 Q: 8

Question	Answer	Marks	Guidance
(a)	Centre of circle is (4, 5)	**B1 B1**	
	$r^2=(7-4)^2+(1-5)^2$	**M1**	OE. Either using *their* centre and A or C **or** using A and C and dividing by 2.
	$r=5$	**A1 FT**	FT on *their* (4, 5) if used.
	Equation is $(x-4)^2+(y-5)^2=25$	**A1**	OE. Allow 5^2 for 25.
		5	
(b)	Gradient of radius = $\dfrac{9-5}{7-4}=\dfrac{4}{3}$	**B1 FT**	FT for use of *their* centre.
	Equation of tangent is $y-9=-\dfrac{3}{4}(x-7)$	**B1**	or $y=\dfrac{-3x}{4}+\dfrac{57}{4}$
		2	

70. 9709_s21_ms_11 Q: 10

Question	Answer	Marks	Guidance
(a)	When $y=0$ $x^2-4x-77=0$ [$\Rightarrow (x+7)(x-11)=0$ or $(x-2)^2=81$]	M1	Substituting $y=0$
	So x-coordinates are −7 and 11	A1	
		2	

Question	Answer	Marks	Guidance
(b)	Centre of circle C is (2, −3)	B1	
	Gradient of AC is $-\frac{1}{3}$ or Gradient of BC is $\frac{1}{3}$	M1	For either gradient (M1 sign error, M0 if x-coordinate(s) in numerator)
	Gradient of tangent at A is 3 or Gradient of tangent at B is −3	M1	For either perpendicular gradient
	Equations of tangents are $y=3x+21$, $y=-3x+33$	A1	For either equation
	Meet when $3x+21=-3x+33$	M1	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection (2, 27)	A1	
	Alternative method for Question 10(b)		
	Implicit differentiation: $2y\frac{dy}{dx}$ seen	B1	
	$2x-4+2y\frac{dy}{dx}+6\frac{dy}{dx}=0$	M1	Fully differentiated $=0$ with at least one term involving y differentiated correctly
	Gradient of tangent at A is 3 or Gradient of tangent at B is −3	M1	For either gradient
	Equations of tangents are $y=3x+21$, $y=-3x+33$	A1	For either equation
	Meet when $3x+21=-3x+33$	M1	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection (2, 27)	A1	
		6	

71. 9709_s21_ms_12 Q: 6

Question	Answer	Marks	Guidance
	Gradient AB = $\frac{1}{2}$	B1	SOI
	Lines meet when $-2x+4=\frac{1}{2}(x-8)+3$ Solving as far as $x=$	*M1	Equating given perpendicular bisector with the line through (8, 3) using *their* gradient of AB (but not -2) and solving. Expect $x=2$, $y=0$.
	Using mid-point to get as far as $p=$ or $q=$	DM1	Expect $\frac{8+p}{2}=2$ or $\frac{3+q}{2}=0$
	$p=-4$, $q=-3$	A1	Allow coordinates of B are (−4, −3).
	Alternative method for Question 6		
	Gradient AB = $\frac{1}{2}$	B1	SOI
	$\frac{q-3}{p-8}=\frac{1}{2}$ [leading to $2q=p-2$], $\frac{q+3}{2}=-2\left(\frac{8+p}{2}\right)+4$ [leading to $q=-11-2p$]	*M1	Equating gradient of AB with *their* gradient of AB (but not -2) and using mid-point in equation of perpendicular bisector.
	Solving simultaneously *their* 2 linear equations	DM1	Equating and solving 2 correct equations as far as $p=$ or $q=$.
	$p=-4$, $q=-3$	A1	Allow coordinates of B are (−4, −3).

Question	Answer	Marks	Guidance
	Alternative method for Question 6		
	Gradient AB = $\frac{1}{2}$	B1	
	$\frac{q-3}{p-8}=\frac{1}{2}$ $[\text{leading to } p=2q+2]$, $y-\frac{q+3}{2}=-2(x-(q+5))$ $\left[\text{leading to } y=-2x+\frac{5q+23}{2}\right]$	*M1	Equating gradient of *AB* with *their* gradient of *AB* (but not -2) and using mid-point in equation of perpendicular bisector.
	their $\frac{5q+23}{2}=4 \Rightarrow q=$	DM1	Equating and solving as far as q or p =
	$p=-4$, $q=-3$	A1	Allow coordinates of B are $(-4, -3)$.
		4	

72. 9709_s21_ms_12 Q: 7

Question	Answer	Marks	Guidance
(a)	$(5-1)^2+(11-5)^2=52$ or $\frac{11-5}{5-1}$	M1	For substituting (1,5) into circle equation or showing gradient $=\frac{3}{2}$.
	For both circle equation and gradient, and proving line is perpendicular and stating that *A* lies on the circle	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$(x-5)^2+(y-11)^2=52$ and $y-5=-\frac{2}{3}(x-1)$	M1	Both equations seen and attempt to solve. May see $y=-\frac{2}{3}x+\frac{17}{3}$
	Solving simultaneously to obtain $(y-5)^2=0$ or $(x-1)^2=0 \Rightarrow 1$ root or tangent or discriminant $=0 \Rightarrow 1$ root or tangent	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$\frac{dy}{dx}=\frac{10-2x}{2y-22}=\frac{10-2}{10-22}$	M1	Attempting implicit differentiation of circle equation and substitute $x=1$ and $y=5$.
	Showing gradient of circle at A is $-\frac{2}{3}$	A1	Clear reasoning.
		2	
(b)	Centre is $(-3, -1)$	B1 B1	B1 for each correct co-ordinate.
	Equation is $(x+3)^2+(y+1)^2=52$	B1 FT	FT *their* centre, but not if either (1, 5) or (5, 11). Do not accept $\sqrt{52^2}$.
		3	

73. 9709_s21_ms_13 Q: 3

Question	Answer	Marks	Guidance
	$x^2-4x+3=mx-6$ leading to $x^2-x(4+m)+9$	***M1**	Equating and gathering terms. May be implied on the next line.
	b^2-4ac leading to $(4+m)^2-4\times 9$	**DM1**	SOI. Use of the discriminant with *their* a, b and c
	$4+m=\pm 6$ or $(m-2)(m+10)=0$ leading to $m=2$ or -10	**A1**	Must come from $b^2-4ac=0$ SOI
	Substitute both *their m* values into *their* equation in line 1	**DM1**	
	$m=2$ leading to $x=3$; $m=-10$ leading to $x=-3$	**A1**	
	(3, 0), (–3, 24)	**A1**	Accept 'when $x=3$, $y=0$; when $x=-3$, $y=24$' If final A0A0 scored, **SC B1** for one point correct WWW
	Alternative method for Question 3		
	$\frac{dy}{dx}=2x-4\rightarrow 2x-4=m$	***M1**	
	$x^2-4x+3=(2x-4)x-6$	**DM1**	
	$x^2-4x+3=2x^2-4x-6\rightarrow 9=x^2\rightarrow x=\pm 3$	**A1**	
	$y=0,\ 24$ or (3, 0), (–3, 24)	**A1**	
	Substitute both *their x* values into *their* equation in line 1	**DM1**	Or substitute both *their* (x,y) into $y=mx-6$
	When $x=3$, $m=2$; when $x=-3$, $m=-10$	**A1**	If A0, DM1, A0 scored, **SC B1** for one point correct WWW
		6	

74. 9709_s21_ms_13 Q: 10

Question	Answer	Marks	Guidance
(a)	Gradient of $AB=-\frac{3}{5}$, gradient of $BC=\frac{5}{3}$ or lengths of all 3 sides or vectors	**M1**	Attempting to find required gradients, sides or vectors
	$m_{ab}m_{bc}=-1$ or Pythagoras or $\overrightarrow{AB}.\overrightarrow{BC}=0$ or $\cos ABC=0$ from cosine rule	**A1**	WWW
		2	
(b)	Centre = mid-point of $AC=(2,4)$	**B1**	
		1	

Question	Answer	Marks	Guidance
(c)	$\left(x - their\ x_c\right)^2 + \left(y - their\ y_c\right)^2 \left[= r^2\right] or \left(their\ x_c - x\right)^2 + \left(their\ y_c - y\right)^2 = \left[r^2\right]$	M1	Use of circle equation with *their* centre
	$(x-2)^2 + (y-4)^2 = 17$	A1	Accept $x^2 - 4x + y^2 - 8y + 3 = 0$ OE
		2	
(d)	$\left(\frac{x+3}{2}, \frac{y+0}{2}\right) = (2,4)$ or **BE** = 2**BD** = $2\begin{pmatrix}-1\\4\end{pmatrix}$ Or Equation of *BE* is $y = -4(x-3)$ or $y - 4 = -4(x-2)$ leading to $y = -4x + 12$ Substitute equation of *BE* into circle and form a 3-term quadratic.	M1	Use of mid-point formula, vectors, steps on a diagram May be seen to find *x* coordinate at *E*
	$(x, y) = (1, 8)$ or **OE** = $\begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\8\end{pmatrix} = \begin{pmatrix}1\\8\end{pmatrix}$	A1	$E = (1, 8)$ Accept without working for both marks **SC B2**
	Gradient of *BD*, *m*, = –4 or gradient $AC = \frac{1}{4}$ = gradient of tangent	B1	Or gradient of *BE* = -4
	Equation of tangent is $y - 8 = ¼(x - 1)$ OE	M1 A1	For M1, equation through *their* E or (1, 8) (not, *A*, *B* or *C*) and with gradient $\frac{-1}{their - 4}$
		5	

75. 9709_w21_ms_11 Q: 2

Question	Answer	Marks	Guidance
	$kx^2 + 2x - k = kx - 2$ leading to $kx^2 + (-k+2)x - k + 2\ [=0]$	*M1	Eliminate *y* and form 3-term quadratic. Allow 1 error.
	$(-k+2)^2 - 4k(-k+2)$	DM1	Apply $b^2 - 4ac$; allow 1 error but *a*, *b* and *c* must be correct for *their* quadratic.
	$5k^2 - 12k + 4$ or $(-k+2)(-k+2-4k)$	A1	May be shown in quadratic formula.
	$(-k+2)(-5k+2)$	DM1	Solving a 3-term quadratic in *k* (all terms on one side) by factorising, use of formula or completing the square. Factors must expand to give *their* coefficient of k^2.
	$\frac{2}{5} < k < 2$	A1	WWW, accept two separate correct inequalities. If M0 for solving quadratic, **SC B1** can be awarded for correct final answer.
		5	

76. 9709_w21_ms_11 Q: 7

Question	Answer	Marks	Guidance
(a)	$r^2\left[=(5-2)^2+(7-5)^2\right]=13$	**B1**	$r^2=13$ or $r=\sqrt{13}$
	Equation of circle is $(x-5)^2+(y-2)^2=13$	**B1 FT**	OE. FT on *their* 13 but LHS must be correct.
		2	
(b)	$(x-5)^2+\ (5x-10-2)^2=13$	**M1**	Substitute $y=5x-10$ into *their* equation.
	$26x^2-130x+156\ [=0]$	**A1 FT**	OE 3-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26](x-2)(x-3)\ [=0]$	**M1**	Solve 3-term quadratic in x by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of x^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.

Question	Answer	Marks	Guidance
(b)	**Alternative method for question 7(b)**		
	$\left(\frac{y+10}{5}-5\right)^2+(y-2)^2=13$	**M1**	Substitute $x=\frac{y+10}{5}$ into *their* equation.
	$\frac{26y^2}{25}-\frac{26y}{5}\ [=0]$	**A1 FT**	OE 2-term quadratic with all terms on one side. FT on *their* circle equation.
	$[26]y(y-5)\ [=0]$	**M1**	Solve 2-term quadratic in y by factorising, using formula or completing the square. Factors must expand to give *their* coefficient of y^2.
	(2, 0), (3, 5)	**A1 A1**	Coordinates must be clearly paired; A1 for each correct point. A1 A0 available if two x or y values only. If M0 for solving quadratic, **SC B2** can be awarded for correct coordinates, **SC B1** if two x or y values only.
	$(AB)^2=(3-2)^2+(5-0)^2$	**M1**	SOI. Using *their* points to find length of AB.
	$AB=\sqrt{26}$	**A1**	ISW. Dependent on final M1 only.
		7	

77. 9709_w21_ms_13 Q: 9

Question	Answer	Marks	Guidance
(a)	$x^2+(2x+5)^2=20$ leading to $x^2+4x^2+20x+25=20$	M1	Substitute $y=2x+5$ and expand bracket.
	$(5)(x^2+4x+1)[=0]$	A1	3-term quadratic.
	$x=\dfrac{-4\pm\sqrt{16-4}}{2}$	M1	OE. Apply formula or complete the square.
	$A=(-2+\sqrt{3},1+2\sqrt{3})$	A1	Or 2 correct x values.
	$B=(-2-\sqrt{3},1-2\sqrt{3})$	A1	Or all values correct. **SC B1** all 4 values correct in surd form without working. **SC B1** all 4 values correct in decimal form from correct formula or completion of the square
	$AB^2=their(x_2-x_1)^2+their(y_2-y_1)^2$	M1	Using *their* coordinates in a correct distance formula. Condone one sign error in x_2-x_1 or y_2-y_1
	$\left[AB^2=48+12\right.$ leading to $\left.\right]AB=\sqrt{60}$	A1	OE. CAO. Do not accept decimal answer. Answer must come from use of surd form in distance formula.
		7	

Question	Answer	Marks	Guidance
(b)	$x^2+m^2(x-10)^2=20$	*M1	Finding equation of tangent and substituting into circle equation.
	$x^2(m^2+1)-20m^2x+20(5m^2-1)\ [=0]$	DM1	OE. Brackets expanded and all terms collected on one side of the equation.
	$[b^2-4ac=]400m^4-80(m^2+1)(5m^2-1)$	M1	Using correct coefficients from *their* quadratic equation.
	$400m^4-80(5m^4+4m^2-1)=0\ \rightarrow(-80)(4m^2-1)=0$	A1	OE. Must have '=0' for A1.
	$m=\pm\dfrac{1}{2}$	A1	
	Alternative method for question 9(b)		
	Length, l of tangent, is given by $l^2=10^2-20$	M1	
	$l=\sqrt{80}$	A1	
	$\tan\alpha=\dfrac{\sqrt{20}}{\sqrt{80}}=\dfrac{1}{2}$	M1 A1	Where α is the angle between the tangent and the x-axis.
	$m=\pm\dfrac{1}{2}$	A1	
		5	

78. 9709_m20_ms_12 Q: 12

	Answer	Mark	Partial Marks
(a)	Centre = (2, –1)	B1	
	$r^2 = \left[2-(-3)\right]^2 + \left[-1-(-5)\right]^2$ or $\left[2-7\right]^2 + \left[-1-3\right]^2$ OE	M1	OR $\frac{1}{2}\left[(-3-7)^2 + (-5-3)^2\right]$ OE
	$(x-2)^2 + (y+1)^2 = 41$	A1	Must not involve surd form **SCB3** $(x+3)(x-7)+(y+5)(y-3)=0$
		3	
(b)	Centre = *their* (2, –1) + $\begin{pmatrix}8\\4\end{pmatrix}$ = (10, 3)	B1FT	SOI FT on *their* (2, –1)
	$(x-10)^2 + (y-3)^2 =$ *their* 41	B1FT	FT on *their* 41 even if in surd form **SCB2** $(x-5)(x-15)+(y+1)(y-7)=0$
		2	
(c)	Gradient *m* of line joining centres = $\frac{4}{8}$ OE	B1	
	Attempt to find mid-point of line.	M1	Expect (6, 1)
	Equation of *RS* is $y-1=-2(x-6)$	M1	Through *their* (6, 1) with gradient $\frac{-1}{m}$
	$y=-2x+13$	A1	AG
	Alternative method for question 12(c)		
	$(x-2)^2 + (y+1)^2 - 41 = (x-10)^2 + (y-3)^2 - 41$ OE	M1	
	$x^2 - 4x + 4 + y^2 + 2y + 1 = x^2 - 20x + 100 + y^2 - 6y + 9$ OE	A1	Condone 1 error **or** errors caused by 1 error in the first line
	$16x + 8y = 104$	A1	
	$y=-2x+13$	A1	AG
		4	
(d)	$(x-10)^2 + (-2x+13-3)^2 = 41$	M1	Or eliminate y between C_1 and C_2
	$x^2 - 20x + 100 + 4x^2 - 40x + 100 = 41 \rightarrow 5x^2 - 60x + 159 = 0$	A1	AG
		2	

79. 9709_s20_ms_11 Q: 10

(a)	Centre is (3, 1)	**B1**
	Radius = 5 (Pythagoras)	**B1**
	Equation of C is $(x-3)^2+(y-1)^2=25$ (**FT** on *their* centre)	**M1** **A1FT**
		4
(b)	Gradient from (3, 1) to (7, 4) = ¾ (this is the normal)	**B1**
	Gradient of tangent $= -\frac{4}{3}$	**M1**
	Equation is $y-4=-\frac{4}{3}(x-7)$ or $3y+4x=40$	**M1A1**
		4
(c)	*B* is centre of line joining centres → (11, 7)	**B1**
	Radius = 5 New equation is $(x-11)^2+(y-7)^2=25$ (**FT** on coordinates of B)	**M1** **A1FT**
		3

80. 9709_s20_ms_12 Q: 6

(a)	$2x^2+kx+k-1 = 2x+3 \rightarrow 2x^2+(k-2)x+k-4=0$	**M1**
	Use of $b^2-4ac=0 \rightarrow (k-2)^2=8(k-4)$	**M1**
	$k=6$	**A1**
		3
(b)	$2x^2+2x+1 = 2\left(x+\frac{1}{2}\right)^2+1-\frac{1}{2}$ $a=\frac{1}{2}, b=\frac{1}{2}$	**B1 B1**
	vertex $\left(-\frac{1}{2},\frac{1}{2}\right)$ (**FT** on *a* and *b* values)	**B1FT**
		3

81. 9709_s20_ms_12 Q: 11

(a)	Express as $(x-4)^2+(y+2)^2 = 16+4+5$	M1
	Centre $C(4, -2)$	A1
	Radius = $\sqrt{25} = 5$	A1
		3
(b)	$P(1,2)$ to $C(4, -2)$ has gradient $-\frac{4}{3}$ (**FT** on coordinates of C)	B1FT
	Tangent at P has gradient = $\frac{3}{4}$	M1
	Equation is $y-2=\frac{3}{4}(x-1)$ or $4y = 3x + 5$	A1
		3
(c)	Q has the same coordinate as P $y = 2$	B1
	Q is as far to the right of C as P $x = 3 + 3 + 1 = 7$ Q (7, 2)	B1
		2
(d)	Gradient of tangent at $Q = -\frac{3}{4}$ by symmetry (**FT** from part (**b**))	B1FT
	Eqn of tangent at Q is $y-2=-\frac{3}{4}(x-7)$ or $4y + 3x = 29$	M1
	$T(4, \frac{17}{4})$	A1
		3

82. 9709_s20_ms_13 Q: 1

	$3x^2+2x+4=mx+1 \rightarrow 3x^2+x(2-m)+3\ (=0)$	B1
	$(2-m)^2-36$ SOI	M1
	$(m + 4)(m - 8)$ (>/= 0) **or** $2 - m$ >/= 6 and $2 - m$ </= –6 OE	A1
	$m < -4$, $m > 8$ WWW	A1
	Alternative method for question 1	
	$\frac{dy}{dx} = 6x + 2 \rightarrow m = 6x + 2 \rightarrow 3x^2+2x+4=(6x+2)x+1$	M1
	$x=\pm 1$	A1
	$m=\pm 6+2 \rightarrow m=8$ or -4	A1
	$m < -4$, $m > 8$ WWW	A1
		4

83. 9709_w20_ms_11 Q: 1

	Answer	Mark	Partial Marks
	$2x^2+5=mx-3 \rightarrow 2x^2-mx+8\ (=0)$	B1	Form 3-term quadratic
	m^2-64	M1	Find b^2-4ac.
	$-8<m<8$	A1	Accept (-8, 8) and equality included
		3	

84. 9709_w20_ms_11 Q: 9

	Answer	Mark	Partial Marks
(a)	$m_{AB} = \frac{4-2}{-1-3} = -\frac{1}{2}$	B1	
	Equation of tangent is $y-2=2(x-3)$	B1 FT	(3, 2) with *their* gradient $-\frac{1}{m_{AB}}$
		2	
(b)	$AB^2 = 4^2 + 2^2 = 20$ or $r^2 = 20$ or $r = \sqrt{20}$ or $AB = \sqrt{20}$	B1	
	Equation of circle centre *B* is $(x-3)^2 + (y-2)^2 = 20$	M1 A1	FT *their* 20 for M1
		3	
(c)	$(x-3)^2 + (2x-6)^2 = \textit{their } 20$	M1	Substitute *their* $y-2=2x-6$ into *their* circle, centre *B*
	$5x^2 - 30x + 25 = 0$ or $5(x-3)^2 = 20$	A1	
	$[(5)(x-5)(x-1)$ or $x-3=\pm 2]$ $x = 5, 1$	A1	
		3	

85. 9709_w20_ms_12 Q: 3

	Answer	Mark	Partial Marks
	$2x^2 + m(2x+1) - 6x - 4 (= 0)$	*M1	y eliminated and all terms on one side with correct algebraic steps. Condone ± errors
	Using b^2-4ac on $2x^2 + x(2m-6) + m - 4 \ (= 0)$	DM1	Any use of discriminant with their *a*, *b* and *c* identified correctly.
	$4m^2 - 32m + 68$ **or** $2m^2 - 16m + 34$ **or** $m^2 - 8m + 17$	A1	
	$(2m-8)^2 + k$ **or** $(m-4)^2 + k$ **or** minimum point $(4, k)$ **or** finds $b^2-4ac \ (= -4, -16, -64)$	DM1	OE. Any valid method attempted on their 3-term quadratic
	$(m-4)^2 + 1$ oe + always > 0 → 2 solutions for all values of *m* **or** Minimum point (4,1) + (fn) always > 0 → 2 solutions for all values of *m* **or** $b^2-4ac < 0$ + no solutions → 2 solutions for the original equation for all values of *m*	A1	Clear and correct reasoning and conclusion without wrong working.
		5	

86. 9709_w20_ms_12 Q: 9

	Answer	Mark	Partial Marks
(a)	$r=\sqrt{(6^2+3^2)}$ or $r^2=45$	**B1**	Sight of r = 6.7 implies B1
	$(x-5)^2+(y-1)^2=r^2$ or $x^2-10x+y^2-2y=r^2-26$	**M1**	Using centre given and *their* radius or r in correct formula
	$(x-5)^2+(y-1)^2=45$ or $x^2-10x+y^2-2y=19$	**A1**	Do not allow $\left(\sqrt{45}\right)^2$ for r^2
		3	
(b)	*C* has coordinates (11, 4)	**B1**	
	0.5	**B1**	OE, Gradient of *AB*, *BC* or *AC*.
	Grad of CD $=-2$	**M1**	Calculation of gradient needs to be shown for this M1.
	($\frac{1}{2}\times-2=-1$) then states + perpendicular → hence shown or tangent	**A1**	Clear reasoning needed.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	0.5	**B1**	OE, Gradient of *AB*, *BC* or *AC*.
	Gradient of the perpendicular is −2 → Equation of the perpendicular is $y-4=-2(x-11)$	**M1**	Use of $m_1m_2=-1$ with *their* gradient of *AB*, *BC* or *AC* and correct method for the equation of the perpendicular. Could use *D*(5, 16) instead of *C*(11,4).
	Checks *D*(5, 16) **or** checks gradient of *CD* and then states *D* lies on the line **or** *CD* has gradient −2→ hence shown or tangent	**A1**	Clear check and reasoning needed. Checks that the other point lies on the line or checks gradient.
(b)	**Alternative method for question 9(b)**		
	C has coordinates (11, 4) **or** Gradient of *AB*, *BC* or *AC* = 0.5	**B1**	Only one of *AB*, *BC* or *AC* needed.
	Equation of the perpendicular is $y-4=-2(x-11)$	**B1**	Finding equation of *CD*.
	$(x-5)^2+(-2x+26-1)^2=45\rightarrow(x^2-22x+121=0)$	**M1**	Solving simultaneously with the equation of the circle.
	$(x-11)^2=0$ or $b^2-4ac=0$ → repeated root → hence shown or tangent	**A1**	Must state repeated root.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding $CD=\sqrt{180}$ and $BD=\sqrt{225}$	**B1**	OE. Calculated from the co-ordinates of *B*, *C* & *D* without using *r*.
	Checking (their BD)^2 – (their CD)^2 is the same as (their r)^2	**M1**	
	∴ Pythagoras valid ∴ perpendicular → hence shown or tangent	**A1**	Triangle *ACD* could be used instead.
	Alternative method for question 9(b)		
	C has coordinates (11, 4)	**B1**	
	Finding vectors $\overline{AC}$ and $\overline{CD}$ **or** $\overline{BC}$ and $\overline{CD}$ (= $\begin{pmatrix}6\\3\end{pmatrix}$ *and* $\begin{pmatrix}-6\\12\end{pmatrix}$ **or** $\begin{pmatrix}12\\6\end{pmatrix}$ *and* $\begin{pmatrix}-6\\12\end{pmatrix}$)	**B1**	Must be correct pairing.
	Applying the scalar product to one of these pairs of vectors	**M1**	Accept *their* $\overline{AC}$ and $\overline{CD}$ or *their* $\overline{BC}$ and $\overline{CD}$
	Scalar product = 0 then states ∴ perpendicular → hence shown or tangent	**A1**	
		4	
(c)	*E* (−1, 4)	**B1 B1**	WWW B1 for each coordinate Note: Equation of DE which is $y=2x+6$ may be used to find *E*
		2	

87. 9709_w20_ms_13 Q: 4

	Answer	Mark	Partial Marks
	$3x^2-4x+4=mx+m-1 \rightarrow 3x^2-(4+m)x+(5-m)\ (=0)$	M1	3-term quadratic
	$b^2-4ac=(4+m)^2-4\times3\times(5-m)$	M1	Find b^2-4ac for *their* quadratic
	$m^2+20m-44$	A1	
	$(m+22)(m-2)$	A1	Or use of formula or completing square. This step must be seen
	$m>2\ ,\ m<-22$	A1	Allow $x>2\ ,\ x<-22$
		5	

88. 9709_m19_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$4x^{1/2}=x+3\rightarrow$ $(x^{1/2})^2-4x^{1/2}+3\ (=0)$ **OR** $16x=x^2+6x+9$	M1	Eliminate y from the 2 equations and then: **Either** treat as quad in $x^{1/2}$ **OR** square both sides and RHS is 3-term
	$x^{1/2}=1$ or 3 $x^2-10x+9\ (=0)$	A1	If in 1st method $x^{1/2}$ becomes x, allow only M1 unless subsequently squared
	$x=1$ or 9	A1	
	$y=4$ or 12	A1ft	Ft from *their* x values If the 2 solutions are found by trial substitution B1 for the first coordinate and B3 for the second coordinate
	$AB^2=(9-1)^2+(12-4)^2$	M1	
	$AB=\sqrt{128}$ or $8\sqrt{2}$ oe or 11.3	A1	
		6	
(ii)	$dy/dx=2x^{-1/2}$	B1	
	$2x^{-1/2}=1$	M1	Set *their* derivative = *their* gradient of AB and attempt to solve
	(4, 8)	A1	Alternative method without calculus: $M_{AB}=1$, tangent is $y=mx+c$ where m = 1 and meets $y=4x^{1/2}$ when $4x^{1/2}=x+c$. This is a quadratic with $b^2=4ac$, so $16-4\times1\times c=0$ so c = 4 B1 Solving $4x^{1/2}=x+4$ gives $x=4$ and $y=8$ M1A1
		3	
(iii)	Equation of normal is $y-8=-1(x-4)$	M1	Equation through *their* T and with gradient –1/*their* gradient of AB. Expect $y=-x+12$,
	Eliminate y (or x) $\rightarrow -x+12=x+3$ or $y-3=12-y$	M1	May use *their* equation of AB
	(4½, 7½)	A1	
		3	

89. 9709_s19_ms_11 Q: 2

	Answer	Mark	Partial Marks
(i)	Eliminates x or $y \rightarrow y^2-4y+c-3=0$ or $x^2+(2c-16)x+c^2-48=0$	M1	Eliminates x or y completely to a quadratic
	Uses $b^2=4ac \rightarrow 4c-28=0$	M1	Uses discriminant = 0. (c the only variable) Any valid method (may be seen in part **(i)**)
	$c=7$	A1	
	Alternative method for question 2(i)		
	$\frac{dy}{dx}=\frac{1}{2\sqrt{(x+3)}}=\frac{1}{4}$	M1	
	Solving	M1	
	$c=7$	A1	
		3	
(ii)	Uses $c=7$, $y^2-4y+4=0$	M1	Ignore (1,–2), c=-9
	(1, 2)	A1	
		2	

90. 9709_s19_ms_11 Q: 4

	Answer	Mark	Partial Marks
(i)	Gradient of $AB=-½ \rightarrow$ Gradient of $BC=2$	M1	Use of $m_1.m_2=-1$ for correct lines
	Forms equation in h $\frac{3h-2}{h}=2$	M1	Uses normal line equation or gradients for h.
	$h=2$	A1	
	Alternative method for question 4(i)		
	Vectors AB.BC=0	M1	Use of vectors AB and BC
	Solving	M1	
	$h=2$	A1	
	Alternative method for question 4(i)		
	Use of Pythagoras to find 3 lengths	M1	
	Solving	M1	
	$h=2$	A1	
		3	
(ii)	y coordinate of D is 6, (3 × 'their' h) $\frac{6-0}{x-4}=2 \rightarrow x=7 \rightarrow D\ (7, 6)$	B1	**FT**
	Vectors: AD.AB=0	M1 A1	Must use y = 6 Realises the y values of C and D are equal. Uses gradient or line equation to find x.
		3	

91. 9709_s19_ms_12 Q: 2

	Answer	Mark	Partial Marks
	Midpoint of AB is (5, 1)	**B1**	Can be seen in working, accept $\left(\frac{10}{2}, \frac{2}{2}\right)$.
	$m_{AB} = -½$ oe	**B1**	
	C to (5, 1) has gradient 2	***M1**	Use of $m_1 \times m_2 = -1$.
	Forming equation of line ($y = 2x - 9$)	**DM1**	Using their perpendicular gradient and their midpoint to form the equation.
	C (0, −9) **or** $y = -9$	**A1**	
		5	

92. 9709_s19_ms_13 Q: 7

	Answer	Mark	Partial Marks
(i)	$D = (5, 1)$	**B1**	
		1	
(ii)	$(x-5)^2 + (y-1)^2 = 20$ **oe**	**B1**	**FT** on *their D*. Apply ISW, oe but not to contain square roots
		1	
(iii)	$(x-1)^2 + (y-3)^2 = (9-x)^2 + (y+1)^2$ **soi**	**M1**	Allow 1 sign slip For M1 allow with √ signs round both sides but sides must be equated
	$x^2 - 2x + 1 + y^2 - 6y + 9 = x^2 - 18x + 81 + y^2 + 2y + 1$	**A1**	
	$y = 2x - 9$ www **AG**	**A1**	
	Alternative method for question 7(iii)		
	grad. of $AB = -½ \rightarrow$ grad of perp bisector $= \frac{-1}{-½}$	**M1**	
	Equation of perp. bisector is $y - 1 = 2(x - 5)$	**A1**	
	$y = 2x - 9$ www **AG**	**A1**	
		3	
(iv)	Eliminate y (or x) using equations in **(ii)** and **(iii)**	***M1**	To give an (unsimplified) quadratic equation
	$5x^2 - 50x + 105$ (= 0) **or** $5(x-5)^2 = 20$ **or** $5y^2 - 10y - 75$ (= 0) **or** $5(y-1)^2 = 80$	**DM1**	Simplify to one of the forms shown on the right (allow arithmetic slips)
	$x = 3$ and 7, **or** $y = -3$ and 5	**A1**	
	(3, −3), (7, 5)	**A1**	Both pairs of x & y correct implies A1A1. SC B2 for no working
		4	

93. 9709_w19_ms_11 Q: 3

	Answer	Mark	Partial Marks
	$\frac{dy}{dx} = 6x^2 - 10x - 3$	**B1**	
	At $x = 2$, $\frac{dy}{dx} = 24 - 20 - 3 = 1 \rightarrow a = 1$	**M1** **A1**	
	$6 = 2 + b \rightarrow b = 4$	**B1FT**	Substitute $x = 2$, $y = 6$ in $y = (\textit{their } a)x + b$
	$6 = 16 - 20 - 6 + c \rightarrow c = 16$	**B1**	Substitute $x = 2$, $y = 6$ into equation of curve
		5	

94. 9709_w19_ms_11 Q: 6

	Answer	Mark	Partial Marks
	Equation of line is $y = mx - 2$	B1	**OR**
	$x^2 - 2x + 7 = mx - 2 \rightarrow x^2 - x(2+m) + 9 = 0$	M1	
	Apply $b^2 - 4ac(=0) \rightarrow (2+m)^2 - 4\times 9\ (=0)$	*M1	
	$m = 4$ or -8	A1	
	$m = 4 \rightarrow x^2 - 6x + 9 = 0 \rightarrow x = 3$ $m = -8 \rightarrow x^2 + 6x + 9 = 0 \rightarrow x = -3$	DM1	
	(3, 10), (–3, 22)	A1A1	
	Alternative method for question 6		
	$\frac{dy}{dx} = 2x - 2$	B1	
	$2x - 2 = m$	M1	
	$x^2 - 2x + 7 = (2x-2)x - 2 = 2x^2 - 2x - 2$	M1	
	$x^2 - 9 = 0 \rightarrow x = \pm 3$	A1	
	(3, 10), (–3, 22)	A1A1	
	When $x = 3$, $m = 4$; when $x = -3$, $m = -8$	A1	
		7	

95. 9709_w19_ms_12 Q: 2

	Answer	Mark	Partial Marks
	Attempt to find the midpoint M	M1	
	(1, 4)	A1	
	Use a gradient of $\pm\frac{2}{3}$ and *their* M to find the equation of the line.	M1	
	Equation is $y - 4 = -\frac{2}{3}(x - 1)$	A1	AEF
	Alternative method for question 2		
	Attempt to find the midpoint M	M1	
	(1, 4)	A1	
	Replace 1 in the given equation by c and substitute *their* M	M1	
	Equation is $y - 4 = -\frac{2}{3}(x - 1)$	A1	AEF
		4	

96. 9709_w19_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$f(x) = g(x) \rightarrow 2x^2 + 6x + 1 + k\ (= 0)$	*M1	Forms a quadratic with all terms on same side.
	Use of $b^2 = 4ac$	DM1	Uses the discriminant = 0.
	$(k =)\ 3½$	A1	OE, WWW
	Alternative method for question 9(i)		
	$4x + 8 = 2\ (\rightarrow x = -1½)$	*M1	Differentiating, equating gradients and solving to give $x =$
	Substitutes *their* x value into either $2x^2 + 6x + 1 + k = 0$ OR into the curve to find $y\left(= \frac{-13}{2}\right)$ then both values into the line.	DM1	Substituting appropriately for *their* x and proceeding to find a value of k.
	$(k =)\ 3½$	A1	OE, WWW
		3	
(ii)	$2x^2 + 6x - 8\ (< 0)$	M1	Forms a quadratic with all terms on same side
	-4 and 1	A1	
	$-4 < x < 1$	A1	CAO
		3	
(iii)	$(g^{-1}(x)) = \frac{x-1}{2}$	B1	Needs to be in terms of x.
	$(g^{-1}f(x)) = \frac{2x^2 + 8x + 1 - 1}{2} = 0 \rightarrow (2x^2 + 8x = 0) \rightarrow x =$	M1	Substitutes f into g^{-1} and attempts to solve it = 0 as far as $x =$
	0, –4	A1	CAO
		3	
(iv)	$2(x + 2)^2 - 7$	B1 B1	or $a = +2$, $b = -7$
	(Least value of $f(x)$ or $y =$) -7 or $\geqslant -7$	B1FT	FT for *their* b from a correct form of the expression.
		3	

97. 9709_w19_ms_13 Q: 6

	Answer	Mark	Partial Marks
(i)	$3kx - 2k = x^2 - kx + 2 \rightarrow x^2 - 4kx + 2k + 2\ (= 0)$	B1	kx terms combined correctly-implied by correct $b^2 - 4ac$
	Attempt to find $b^2 - 4ac$	M1	Form a quadratic equation in k
	1 and $-\frac{1}{2}$	A1	SOI
	$k > 1,\ k < -\frac{1}{2}$	A1	Allow $x > 1,\ x < -1/2$
		4	
(ii)	$y = 3x - 2, \quad y = -\frac{3}{2}x + 1$	M1	Use of *their* k values (twice) in $y = 3kx - 2k$
	$3x - 2 = -\frac{3}{2}x + 1$ OR $y + 2 = 2 - 2y$	M1	Equate *their* tangent equations OR substitute $y = 0$ into both lines
	$x = \frac{2}{3}, \rightarrow y = 0$ in one or both lines	A1	Substitute $x = \frac{2}{3}$ in one or both lines
		3	

98. 9709_m18_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$\frac{1}{\sqrt{3}}=\frac{2}{x}$ or $y-2=\frac{-1}{\sqrt{3}}x$	M1	OE, Allow $y-2=\frac{+1}{\sqrt{3}}x$. Attempt to express $\tan\frac{\pi}{6}$ *or* $\tan\frac{\pi}{3}$ exactly is required or the use of $1/\sqrt{3}$ *or* $\sqrt{3}$
	$(x=)2\sqrt{3}$	A1	OE
		2	
(ii)	Mid-point (a, b) = (½ *their* **(i)**, 1)	B1FT	Expect ($\sqrt{3}$, 1)
	Gradient of AB leading to gradient of bisector, m	M1	Expect $-1/\sqrt{3}$ leading to $m=\sqrt{3}$
	Equation is $y - their\,b = m(x - their\,a)$ OE	DM1	Expect $y-1=\sqrt{3}\left(x-\sqrt{3}\right)$
	$y=\sqrt{3}x-2$ OE	A1	
		4	

99. 9709_m18_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$1+cx=cx^2-3x \rightarrow cx^2-x(c+3)-1\ (=0)$	M1	Multiply throughout by x and rearrange terms on one side of equality
	Use $b^2-4ac\left[=(c+3)^2+4c=c^2+10c+9 \text{ or } (c+5)^2-16\right]$	M1	Select their correct coefficients which must contain 'c' twice Ignore = 0, < 0, >0 etc. at this stage
	(Critical values) –1, –9	A1	SOI
	$c\leqslant -9,\ \ c\geqslant -1$	A1	
		4	

	Answer	Mark	Partial Marks
(ii)	Sub their c to obtain a quadratic $\left[c=-1\rightarrow -x^2-2x-1(=0)\right]$	M1	
	$x=-1$	A1	
	Sub their c to obtain a quadratic $\left[c=(-9 \rightarrow -9x^2+6x-1(=0)\right]$	M1	
	$x=1/3$	A1	[Alt 1: $dy/dx=-1/x^2=c$, when $c=-1, x=\pm 1, c=-9, x=\pm\frac{1}{3}$ Give M1 for equating the gradients, A1 for all four answers and M1A1 for checking and eliminating] [Alt 2: $dy/dx=-1/x^2=c$ leading to $1/x-1/x^2=(-1/x^2)(x)-3$ Give M1 A1 at this stage and M1A1 for solving]
		4	

100. 9709_s18_ms_11 Q: 5

	Answer	Mark	Partial Marks
(i)	Eqn of AC $y=-\frac{1}{2}x+4$ (gradient must be $\Delta y/\Delta x$)	M1A1	Uses gradient and a given point for equa. CAO
	Gradient of $OB=2 \rightarrow y=2x$ (If y missing only penalise once)	M1 A1	Use of $m_1m_2=-1$, answers only ok.
		4	

	Answer	Mark	Partial Marks
(ii)	Simultaneous equations → ((1.6, 3.2))	M1	Equate and solve for M1 and reach ⩾1 solution
	This is mid-point of OB. → B (3.2, 6.4)	M1 A1	Uses mid-point. CAO
	or		
	Let coordinates of B (h, k) $OA = AB \rightarrow h^2 = 8k - k^2$ $OC = BC \rightarrow k^2 = 16h - h^2 \rightarrow (3.2, 6.4)$		M1 for both equations, M1 for solving with $y = 2x$
	or		
	gradients $\left(\frac{k-4}{h} \times \frac{k}{h-8} = -1\right)$		M1 for gradient product as –1, M1 solving with $y = 2x$
	or		
	Pythagoras: $h^2 + (k-4)^2 + (h-8)^2 + k^2 = 4^2 + 8^2$		M1 for complete equation, M1 solving with $y = 2x$
		3	

101. 9709_s18_ms_11 Q: 9

	Answer	Mark	Partial Marks
	$f : x \mapsto \frac{x}{2} - 2, \quad g : x \mapsto 4 + x - \frac{x^2}{2}$		
(i)	$4 + x - \frac{x^2}{2} = \frac{x}{2} - 2 \rightarrow x^2 - x - 12 = 0$	M1	Equates and forms 3 term quadratic
	→ (4, 0) and (−3, −3.5) Trial and improvement, B3 all correct or B0	A1 A1	A1 For both x values or a correct pair. A1 all.
		3	
(ii)	$f(x) > g(x)$ for $x > 4$, $x < -3$	B1, B1	B1 for each part. Loses a mark for ⩽ or ⩾.
		2	
(iii)	$fg(x) = 2 + \frac{x}{2} - \frac{x^2}{4} - 2 \left(= \frac{x}{2} - \frac{x^2}{4}\right)$	B1	CAO, any correct form
	i.e. $-\frac{1}{4}((x-1)^2 - 1)$ or $\frac{dy}{dx} = \frac{1}{2} - \frac{2x}{4} = 0 \rightarrow x = 1$	M1 A1	Completes the square or uses calculus. First A1 is for $x = 1$ or completed square form
	$\rightarrow y = \frac{1}{4} \rightarrow$ Range of fg $\leqslant \frac{1}{4}$,	A1	CAO, OE e.g. $y \leqslant \frac{1}{4}$, $[-\infty, \frac{1}{4})$ etc.
		4	
(iv)	Calculus or completing square on 'h' → $x = 1$	M1	May use a sketch or $-\frac{b}{2a}$
	$k = 1$ (accept $k \geqslant 1$)	A1	Complete method. CAO
		2	

102. 9709_s18_ms_12 Q: 2

	Answer	Mark	Partial Marks
(i)			A complete method as far as finding a set of values for k by:
	Either $(x-3)^2 + k - 9 > 0$, $k - 9 > 0$		Either completing the square and using '*their* $k - 9$' > or ⩾ 0 OR
	or $2x - 6 = 0 \rightarrow (3, k - 9)$, $k - 9 > 0$	M1	Differentiating and setting to 0, using '*their* $x=3$' to find y and using '*their* $k - 9$' > or ⩾0 OR
	or $b^2 < 4ac$ oe → $36 < 4k$		Use of discriminant < or ⩽ 0. Beware use of > and incorrect algebra.
	→ $k > 9$ Note: not ⩾	A1	T&I leading to (or no working) correct answer 2/2 otherwise 0/2.
		2	

	Answer	Mark	Partial Marks
(ii)	**EITHER**		
	$x^2-6x+k=7-2x\rightarrow x^2-4x+k-7$ (= 0)	*M1	Equates and collects terms.
	Use of $b^2-4ac=0$ $(16-4(k-7)=0)$	DM1	Correct use of discriminant = 0, involving k from a 3 term quadratic.
	OR		
	$2x-6=-2\rightarrow x=2$ $(y=3)$	*M1	Equates their $\frac{dy}{dx}$ to ± 2, finds a value for x.
	(*their* 3) or 7– 2(*their* 2) = (*their* 2)^2 – 6(*their* 2) + k	DM1	Substitutes their value(s) into the appropriate equation.
	$\rightarrow k=11$	A1	
		3	

103. 9709_s18_ms_12 Q: 8

	Answer	Mark	Partial Marks
	EITHER		
	Gradient of bisector $=-\frac{3}{2}$	B1	
	gradient $AB=\frac{5h-h}{4h+6-h}$	*M1	Attempt at $\frac{y-step}{x-step}$
	Either $\frac{5h-h}{4h+6-h}=\frac{2}{3}$ or $-\frac{4h+6-h}{5h-h}=-\frac{3}{2}$	*M1	Using $m_1m_2=-1$ appropriately to form an equation.
	OR		
	Gradient of bisector $=-\frac{3}{2}$	B1	
	Using gradient of AB and A, B or midpoint $\rightarrow \frac{2}{3}x+\frac{h}{3}=y$ oe	*M1	Obtain equation of AB using gradient from $m_1m_2=-1$ and a point.
	Substitute co-ordinates of one of the other points	*M1	Arrive at an equation in h.
	h = 2	A1	
	Midpoint is $\left(\frac{5h+6}{2},3h\right)$ or (8, 6)	B1FT	Algebraic expression or FT for numerical answer from '*their h*'
	Uses midpoint and '*their h*' with $3x+2y=k$	DM1	Substitutes '*their midpoint*' into $3x+2y=k$. If $y=-\frac{3}{2}x+c$ is used (expect $c=18$) the method mark should be withheld until they ×2.
	$\rightarrow k=36$ soi	A1	
		7	

104. 9709_s18_ms_13 Q: 6

	Answer	Mark	Partial Marks
(i)	Gradient, m, of $AB = \frac{3k+5-(k+3)}{k+3-(-3k-1)}$ OE $\left(= \frac{2k+2}{4k+4}\right)$ $= \frac{1}{2}$	M1A1	Condone omission of brackets for M mark
		2	
(ii)	Mid-pt = $[\frac{1}{2}(-3k-1+k+3), \frac{1}{2}(3k+5+k+3)] =$ $\left(\frac{-2k+2}{2}, \frac{4k+8}{2}\right)$ SOI	B1B1	B1 for $\frac{-2k+2}{2}$, B1 for $\frac{4k+8}{2}$ (ISW) or better, i.e. $(-k+1, 2k+4)$
	Gradient of perpendicular bisector is $\frac{-1}{\textit{their } m}$ SOI Expect –2	M1	Could appear in subsequent equation and/or could be in terms of k
	Equation: $y-(2k+4)=-2[x-(-k+1)]$ OE	DM1	Through *their* mid-point and with *their* $\frac{-1}{m}$ (now numerical)
	$y+2x=6$	A1	Use of numerical k in **(ii)** throughout scores SC2/5 for correct answer
		5	

105. 9709_w18_ms_11 Q: 2

	Answer	Mark	Partial Marks
	$x^2+bx+5=x+1 \to x^2+x(b-1)+4\ (=0)$	M1	Eliminate x or y with all terms on side of an equation
	$(b^2-4ac=)\ (b-1)^2-16$	M1	
	b associated with –3 & +5 or $b-1$ associated with ± 4	A1	$(x-2)^2=0\ or\ (x+2)^2=0, x=\pm 2, b-1=\pm 4$ (M1A1) Association can be an equality or an inequality
	$b \geqslant 5, b \leqslant -3$	A1	
		4	

106. 9709_w18_ms_11 Q: 3

	Answer	Mark	Partial Marks
(i)	Gradient of $AB = -3/4$	B1	Accept $-3a/4a$
	$y=-\frac{3}{4}x$ oe	B1FT	Answer must not include a. Ft on *their* numerical gradient
		2	
(ii)	$(4a)^2+(3a)^2=(10/3)^2$ soi	M1	May be unsimplified
	$25a^2=100/9$ oe	A1	
	$a = 2/3$	A1	
		3	

107. 9709_w18_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$2x+\frac{12}{x}=k-x$ or $y=2(k-y)+\frac{12}{k-y}$ → 3 term quadratic.	*M1	Attempt to eliminate y (or x) to form a 3 term quadratic. Expect $3x^2-kx+12$ or $3y^2-5ky+(2k^2+12)$ (= 0)
	Use of $b^2-4ac \to k^2-144<0$	DM1	Using the discriminant, allow $\leqslant$, = 0; expect 12 and −12
	$\mathbf{-12<k<12}$	A1	Do **NOT accept** $\leqslant$. Separate statements OK.
		3	
(ii)	Using $k=15$ in their 3 term quadratic	M1	From (i) or restart. Expect $3x^2-15x+12$ or $3y^2-75y+462$ (= 0)
	$x=1,4$ or $y=11, 14$	A1	Either pair of x or y values correct..
	(1, 14) and (4, 11)	A1	Both pairs of coordinates
		3	
(iii)	Gradient of $AB=-1\to$ Perpendicular gradient = +1	B1FT	Use of $m_1m_2=-1$ to give +1 or ft from their A and B.
	Finding their midpoint using their (1, 14) and (4, 11)	M1	Expect (2½, 12½)
	Equation: $\mathbf{y-12½=(x-2½)}$ [$y=x+10$]	A1	Accept correct unsimplified and isw
		3	

108. 9709_w18_ms_13 Q: 4

	Answer	Mark	Partial Marks
(i)	Gradient, m, of AB = 3/4	B1	
	Equation of BC is $y-4=\frac{-4}{3}(x-3)$	M1A1	Line through (3, 4) with gradient $\frac{-1}{m}$ (M1). (Expect $y=\frac{-4}{3}x+8$)
	$x=6$	A1	Ignore any y coordinate given.
		4	
(ii)	$(AC)^2=7^2+1^2 \to AC=7.071$	M1A1	M mark for $\sqrt{(their6+/-1)^2+1}$.
		2	

109. 9709_w18_ms_13 Q: 9

	Answer	Mark	Partial Marks
(i)	For *their* 3-term quad a recognisable application of b^2-4ac	M1	Expect $2x^2-x(3+k)+1-k^2\ (=0)$ oe for the 3-term quad.
	$(b^2-4ac=)\ (3+k)^2-4(2)(1-k^2)$ oe	A1	Must be correct. Ignore any RHS
	$9k^2+6k+1$	A1	Ignore any RHS
	$(3k+1)^2\geqslant 0$ Do not allow > 0. Hence curve and line meet. **AG**	A1	Allow $(9)\left(k+\frac{1}{3}\right)^2\geqslant 0$. Conclusion required.
	ALT Attempt solution of 3-term quadratic	M1	
	Solutions $x=k+1$, ½$(1-k)$	A1A1	
	Which exist for all values of k. Hence curve and line meet. **AG**	A1	
		4	

	Answer	Mark	Partial Marks
(ii)	$k = -1/3$	B1	**ALT** $dy/dx = 4x - 3 \Rightarrow 4x - 3 = k$
	Sub (one of) *their* $k = -⅓$ into either line 1 → $2x^2 - \frac{8}{3}x + \frac{8}{9}\ (=0)$ Or into the derivative of line 1 → $4x\ -(3+k)(=0)$	M1	Sub $k = 4x - 3$ into line 1 → $2x^2 - x(4x) + 1 - (4x-3)^2\ (=0)$
	$x = 2/3$ Do not allow unsubstantiated $\left(\frac{2}{3}, -\frac{1}{9}\right)$ following $k = -\frac{1}{3}$	A1	$x = 2/3$, $y = -1/9$ (both required) [from $-18x^2 + 24x - 8$ (=0) oe]
	$y = -1/9$ Do not allow unsubstantiated $\left(\frac{2}{3}, -\frac{1}{9}\right)$ following $k = -\frac{1}{3}$	A1	$k = -1/3$
		4	

110. 9709_s17_ms_12 Q: 2

	Answer	Mark	Partial Marks
(i)	Gradient = 1.5 Gradient of perpendicular = −⅔	B1	
	Equation of AB is $y - 6 = -⅔(x+2)$ Or $3y + 2x = 14$ oe	M1 A1	Correct use of straight line equation with a changed gradient and (− 2, 6), the (− (− 2)) must be resolved for the **A1** ISW.
			Using $y = mx + c$ gets **A1** as soon as c is evaluated.
	Total:	3	
(ii)	Simultaneous equations → Midpoint (1, 4)	M1	Attempt at solution of simultaneous equations as far as x =, or y =.
	Use of midpoint or vectors → B (4, 2)	M1A1	Any valid method leading to x, or to y.
	Total:	3	

111. 9709_s17_ms_13 Q: 3

	Answer	Mark	Partial Marks
	EITHER Elim y to form 3-term quad eqn in $x^{1/3}$ (or u or y or even x)	(M1	Expect $x^{2/3} - x^{1/3} - 2(=0)$ or $u^2 - u - 2\ (=0)$ etc.
	$x^{1/3}$ (or u or y or x) = 2, −1	*A1	Both required. But $\underline{x}$ = 2,−1 and not then cubed or cube rooted scores **A0**
	Cube solution(s)	DM1	Expect x = 8, − 1. Both required
	(8, 3), (−1,0)	A1)	
	OR Elim x to form quadratic equation in y	(M1	Expect $y + 1 = (y-1)^2$
	$y^2 - 3y = 0$	*A1	
	Attempt solution	DM1	Expect y = 3, 0
	(8, 3), (−1,0)	A1)	
	Total:	4	

112. 9709_s17_ms_13 Q: 8

	Answer	Mark	Partial Marks
(i)	$(b-1)/(a+1)=2$	**M1**	OR Equation of AP is $y-1=2(x+1)\to y=2x+3$
	$b=2a+3$ CAO	**A1**	Sub $x=a$, $y=b\to b=2a+3$
	Total:	**2**	
(ii)	$AB^2=11^2+2^2=125$ oe	**B1**	Accept $AB=\sqrt{125}$
	$(a+1)^2+(b-1)^2=125$	**B1 FT**	FT on *their* 125.
	$(a+1)^2+(2a+2)^2=125$	**M1**	Sub from part (i) → quadratic eqn in a (or possibly in $b\to b^2-2b-99=0$)
	$(5)(a^2+2a-24)=0\to \text{eg}(a-4)(a+6)=0$	**M1**	Simplify and attempt to solve
	$a=4$ or -6	**A1**	
	$b=11$ or -9	**A1**	OR (4, 11), (–6, –9) If **A0A0**, SR1 for either (4, 11) or (–6, –9)
	Total:	**6**	

113. 9709_w17_ms_11 Q: 6

	Answer	Mark	Partial Marks
(i)	Mid-point of $AB=(3, 5)$	**B1**	Answers may be derived from simultaneous equations
	Gradient of $AB=2$	**B1**	
	Eqn of perp. bisector is $y-5=-½(x-3)\to 2y=13-x$	**M1A1**	AG For **M1** FT from mid-point and gradient of AB
		4	
(ii)	$-3x+39=5x^2-18x+19\to (5)(x^2-3x-4)(=0)$	**M1**	Equate equations and form 3-term quadratic
	$x=4$ or -1	**A1**	
	$y=4½$ or 7	**A1**	
	$CD^2=5^2+2½^2\to CD=\sqrt{\frac{125}{4}}$	**M1A1**	Or equivalent integer fractions ISW
		5	

114. 9709_w17_ms_13 Q: 2

	Answer	Mark	Partial Marks
	$ax+3a=-\frac{2}{x}\to ax^2+3ax+2\ (=0)$	***M1**	Rearrange into a 3-term quadratic.
	Apply $b^2-4ac>0$ SOI	**DM1**	Allow $\geqslant$. If no inequalities seen, **M1** is implied by 2 correct final answers in a or x.
	$a<0,\ a>\frac{8}{9}$ (or 0.889) OE	**A1 A1**	For final answers accept $0>a>\frac{8}{9}$ but not $\leqslant, \geqslant$.
		4	

115. 9709_m16_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	Mid-point of $AB = (7, 3)$ soi Grad. of $AB = -2 \rightarrow$ grad of perp. bisector = 1/2 soi Eqn of perp. bisector is $y-3=\frac{1}{2}(x-7)$	**B1** **M1** **A1** **[3]**	 Use of $m_1m_2=-1$
(ii)	Eqn of CX is $y-2=-2(x-1)$ $\frac{1}{2}x-\frac{1}{2}=-2x+4$ $x = 9/5$, $y = 2/5$ $BX^2=7.2^2+1.4^2$ soi $BX = 7.33$	**M1** **DM1** **A1** **M1** **A1** **[5]**	Using their original gradient and (1,2) Solve simultaneously dependent on both previous M's

116. 9709_s16_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	A (0, 7), B (8, 3) and C ($3k$, k) m of AB is $-½$ oe. Eqn of AB is $y = -½x + 7$ Let $x = 3k$, $y = k$ ***k* = 2.8 oe** **OR** $\frac{7-k}{0-3k}=\frac{3-k}{8-3k}$ $\rightarrow 20k=56 \rightarrow k=2.8$ **OR** $\frac{7-k}{0-3k}=\frac{7-3}{0-8}$ $\rightarrow 20k=56 \rightarrow k=2.8$	 **B1** **M1** **M1** **A1** **M1A1** **DM1A1** **M1A1** **DM1A1** [4]	 Using A,B or C to get an equation Using C or A,B in the equation Using A,B & C to equate gradients Simplifies to a linear or 3 term quadratic = 0. Using A,B and C to equate gradients Simplifies to a linear or 3 term quadratic = 0.
(ii)	M(4, 5) Perpendicular gradient = 2. Perp bisector has eqn $y-5=2(x-4)$ Let $x = 3k$, $y = k$ $k=\frac{3}{5}$ oe **OR** $(0-3k)^2 + (7-k)^2 = (8-3k)^2 + (3-k)^2$ $-14k + 49 = 73 - 54k \rightarrow 40k = 24 \rightarrow k=0.6$	**B1** **M1** **M1** **A1** **M1A1** **DM1A1** [4]	anywhere in (ii) Use of $m_1m_2=-1$ soi Forming eqn using their M and their "*perpendicular m*" Use of Pythagoras. Simplifies to a linear or 3 term quadratic = 0.

117. 9709_s16_ms_13 Q: 11

	Answer	Mark	Partial Marks
(i)	$AB^2 = 6^2 + 7^2 = 85,\ BC^2 = 2^2 + 9^2 = 85$ (→ isosceles)	**B1B1**	Or $AB = BC = \sqrt{85}$ etc
	$AC^2 = 8^2 + 2^2 = 68$	**B1**	
	$M = (2, -2)$ or $BM^2 = (\sqrt{85})^2 - (½\sqrt{68})^2$	**B1**	Where M is mid-point of AC
	$BM = \sqrt{2^2 + 8^2} = \sqrt{68}$ or $\sqrt{85 - 17} = \sqrt{68}$	**B1**	
	Area $\Delta ABC = \frac{1}{2}\sqrt{68}\sqrt{68} = 34$	**B1** [6]	
(ii)	Gradient of $AB = 7/6$	**B1**	
	Equation of AB is $y + 1 = \frac{7}{6}(x + 2)$	**M1**	Or $y - 6 = \frac{7}{6}(x - 4)$
	Gradient of $CD = -6/7$	**M1**	
	Equation of CD is $y + 3 = \frac{-6}{7}(x - 6)$	**M1**	
	Sim Eqns $2 = \frac{-6}{7}x + \frac{36}{7} - \frac{7}{6}x - \frac{14}{6}$	**M1**	
	$x = \frac{34}{85} = \frac{2}{5}$ oe	**A1** [6]	

118. 9709_w16_ms_11 Q: 4

	Answer	Mark		Partial Marks
(i)	$C = (4, -2)$	**B1**		
	$m_{AB} = -1/2 \rightarrow m_{CD} = 2$	**M1**		Use of $m_1 m_2 = -1$ on their m_{AB}
	Equation of CD is $y + 2 = 2(x - 4)$ oe	**M1**		Use of *their* C and m_{CD} in a line equation
	$y = 2x - 10$	**A1**	[4]	
(ii)	$AD^2 = (14 - 0)^2 + (-7 - (-10))^2$	**M1**		Use *their* D in a correct method
	$AD = 14.3$ or $\sqrt{205}$	**A1**	[2]	

119. 9709_w16_ms_12 Q: 3

	Answer	Mark		Partial Marks
(i)	$2x^2 - 6x + 5 > 13$ $2x^2 - 6x - 8 (> 0)$ $(x =) -1$ and 4. $x > 4$, $x < -1$	 **M1** **A1** **A1**	[3]	 Sets to 0 + attempts to solve Both values required Allow all recognisable notation.
(ii)	$2x^2 - 6x + 5 = 2x + k$ $\rightarrow 2x^2 - 8x + 5 - k (= 0)$ Use of $b^2 - 4ac$ $\rightarrow -3$ **OR** $\frac{dy}{dx} = 4x - 6$ $4x - 6 = 2$ $x = 2$ $x = 2 \rightarrow y = 1$ Using *their* (2,1) in $y = 2x + k$ or $y = 2x^2 - 6x + 5$ $\rightarrow k = -3$	 **M1*** **DM1** **A1** **M1*** **DM1** **A1**	[3] [3]	 Equates and sets to 0. Use of discriminant Sets (their $\frac{dy}{dx}$) = 2 Uses *their* x = 2 and *their* y = 1

120. 9709_w16_ms_12 Q: 5

	Answer	Mark		Partial Marks
	$A(a, 0)$ and $B(0, b)$ $a^2 + b^2 = 100$ M has coordinates $\left(\frac{a}{2}, \frac{b}{2}\right)$ M lies on $2x + y = 10$ $\rightarrow a + \frac{b}{2} = 10$ Sub $\rightarrow a^2 + (20 - 2a)^2 = 100$ or $\left(10 - \frac{b}{2}\right)^2 + b^2 = 100$ $\rightarrow a = 6, b = 8.$	**B1** **M1*** **B1**√ **M1*** **DM1** **A1**	 [6]	soi Uses Pythagoras with their A & B. √ on their A and B. Subs into given line, using their M, to link a and b. Forms quadratic in a or in b. cao

121. 9709_w16_ms_13 Q: 1

	Answer	Mark		Partial Marks
	$kx^2 - 3x = x - k \Rightarrow kx^2 - 4x + k (= 0)$ $(-4)^2 - 4(k)(k)$ soi $k > 2$, $k < -2$ cao Allow $(2, \infty)$ etc. Allow $2<k<-k$	**M1** **M1** **A1**	 [3]	Eliminate y and rearrange into 3-term quad $b^2 - 4ac$.

122. 9709_w16_ms_13 Q: 6

	Answer	Mark		Partial Marks
(i)	$\frac{2+x}{2}=n \Rightarrow x=2n-2$ $\frac{m+y}{2}=-6 \Rightarrow y=-12-m$	**B1** **B1**	[2]	No MR for (½(2+*n*), ½(*m* – 6)) Expect $(2n-2, -12-m)$
(ii)	Sub *their* *x*, *y* into $y=x+1 \rightarrow -12-m=2n-2+1$ $\frac{m+6}{2-n}=-1$ oe Not nested in an equation Eliminate a variable $m=-9,\ n=-1$	**M1*** **B1** **DM1** **A1A1**	[5]	Expect $m+2n=-11$ Expect $m-n=-8$ Note: other methods possible

123. 9709_s15_ms_11 Q: 6

	Answer	Mark	Partial Marks
(i)	$y-2t=-2(x-3t)(y+2x=8t)$ Set *x* to 0 $\rightarrow B(0, 8t)$ Set *y* to 0 $\rightarrow A(4t, 0)$ $\rightarrow$ Area = $16t^2$	M1 M1 A1 [3]	Unsimplified or equivalent forms Attempt at both *A* and *B*, then using cao
(ii)	$m=\frac{1}{2}$ $\rightarrow y-2t=\frac{1}{2}(x-3t)(2y=x+t)$ Set *y* to 0 $\rightarrow C(-t, 0)$ Midpoint of *CP* is (t, t) This lies on the line $y=x$.	B1 M1 A1 A1 [4]	cao Unsimplified or equivalent forms co correctly shown.

124. 9709_s15_ms_12 Q: 6

	Answer	Mark	Partial Marks
	$h=60(1-\cos kt)$		
(i)	Max *h* when cos = −1 $\rightarrow$ 120	B1 [1]	Co
(ii)	$h=0$ and $t=30$, or $h=120$ and $t=15$ $\rightarrow \cos 30k=1$ or $\cos 15k=-1$ $\rightarrow 30k=2\pi$ or $15k=\pi$ $\rightarrow k=\frac{2\pi}{30}=\frac{\pi}{15}$	M1 A1 [2]	Substituting a correct pair of values into the equation. co ag
(iii)	$90=60(1-\cos kt)$ $\rightarrow \cos kt=\frac{-30}{60}=-0.5$ $\rightarrow kt=\frac{2\pi}{3}$ or $\rightarrow kt=\frac{4\pi}{3}$ $\rightarrow$ Either $t=10$ or 20 or both $\rightarrow t=10$ minutes	B1 B1 B1 [3]	co – but there must be evidence of correct subtraction.

125. 9709_s15_ms_13 Q: 7

	Answer	Mark	Partial Marks
(i)	$(9-p)^2+(3p)^2=169$	**M1**	Or $\sqrt{\ }=13$
	$10p^2-18p-88\,(=0)$ oe	**A1**	3-term quad
	$p=4$ ***or*** $-11/5$ oe	**A1** **[3]**	
(ii)	Gradient of given line $=-\frac{2}{3}$	**B1**	
	Hence gradient of AB $=\frac{3}{2}$	**M1**	Attempt using $m_1m_2=-1$
	$\frac{3}{2}=\frac{3p}{9-p}$ oe eg $\left(\frac{-2}{3}\right)\left(\frac{3p}{9-p}\right)=1$ (includes previous M1)	**M1**	Or vectors $\begin{pmatrix}9-p\\3p\end{pmatrix}\cdot\begin{pmatrix}3\\-2\end{pmatrix}$
	$p=3$	**A1** **[4]**	

126. 9709_w15_ms_11 Q: 6

	Answer	Mark	Partial Marks
(i)	$x^2-x+3=3x+a\rightarrow x^2-4x+(3-a)=0$	**B1** [1]	**AG**
(ii)	$5+(3-a)=0\rightarrow a=8$	**B1**	Sub $x=-1$ into **(i)**
	$x^2-4x-5=0\rightarrow x=5$	**B1** [2]	**OR B2** for x = 5 www
(iii)	$16-4(3-a)=0$ (applying $b^2-4ac=0$)	**M1**	**OR** $dy/dx=2x-1\rightarrow 2x-1=3$
	$a=-1$	**A1**	$x=2$
	$(x-2)^2=0\rightarrow x=2$	**A1**	$y=2^2-2+3\rightarrow y=5$
	$y=5$	**A1** [4]	$5=6+a\rightarrow a=-1$

127. 9709_w15_ms_12 Q: 6

	Answer	Mark	Partial Marks
	$A(-3, 7)$, $B(5, 1)$ and $C(-1, k)$		
(i)	$AB=10$	**B1**	
	$6^2+(k-1)^2=10^2$	**M1**	Use of Pythagoras
	$k=-7$ and 9	**A1** [3]	
(ii)	m of $AB=-¾$ m perp $=\frac{4}{3}$	**B1 M1**	B1 M1 Use of $m_1m_2=-1$
	$M=(1, 4)$		
	Eqn $y-4=\frac{4}{3}(x-1)$	**B1**	
	Set y to 0, $\rightarrow$ $x=-2$	**M1 A1** [5]	Complete method leading to D.

128. 9709_w15_ms_13 Q: 1

	Answer	Mark	Partial Marks
	$x^2 - 4x + c = 2x - 7 \to x^2 - 6x + c + 7(=0)$ $36 - 4(c+7) < 0$ $c > 2$	M1 DM1 A1 [3]	All terms on one side Apply $b^2 - 4ac < 0$. Allow $\leqslant$.

129. 9709_m22_ms_12 Q: 8

Question	Answer	Marks	Guidance
(a)	$(-2)^2 + y^2 = 8$ leading to $y = 2$ leading to $A = (0, 2)$	B1	
	Substitute $y = their\,2$ into circle leading to $(x-2)^2 + 4 = 8$	M1	Expect $x = 4$.
	$B = (4, 2)$	A1	
		3	
(b)	Attempt to find $[\pi]\int\left(8-(x-2)^2\right)dx$	*M1	
	$[\pi]\left[8x - \frac{(x-2)^3}{3}\right]$ or $[\pi]\left[8x - \left(\frac{x^3}{3} - 2x^2 + 4x\right)\right]$	A1	
	$[\pi]\left(32 - \frac{16}{3}\right)$ or $[\pi]\left[32 - \left(\frac{64}{3} - 32 + 16\right)\right]$	DM1	Apply limits $0 \to$ *their* 4.
	Volume of cylinder $= \pi \times 2^2 \times 4 = 16\pi$	B1 FT	OR from $\pi\int 2^2 dx$ with *their* limits from **(a)**. FT on *their A* and *B*
	[Volume of revolution $= 26\frac{2}{3}\pi - 16\pi =$] $10\frac{2}{3}\pi$	A1	Accept 33.5
		5	

130. 9709_m22_ms_12 Q: 10

Question	Answer	Marks	Guidance
(a)	$\tan A = \frac{12}{5}$ or $\cos A = \frac{5}{13}$ or $\sin A = \frac{12}{13}$	M1	OR $\tan B = \frac{5}{12}$ or $\cos B = \frac{12}{13}$ or $\sin B = \frac{5}{13}$
	$A = 1.176$ $B = 0.3948$	A1	Allow 1.18 or 67.4°, Allow 0.395 or 22.6°. May be implied by $\frac{\pi}{2} - 1.176$
	$DE = 4$	B1	If trigonometry used accept AWRT 4.00
	Arcs = $5 \times their\,1.176$ *and* $8 \times their\,0.3948$	M1	Or corresponding calculations in degrees.
	[Perimeter = 5.880 + 3.158 + 4 =] 13.0	A1	Accept 13. If *DE* is outside the given range this mark cannot be awarded.
		5	
(b)	Area of triangle = $\frac{1}{2} \times 5 \times their\,12$ [= 30]	B1 FT	
	Area of sectors = $\frac{1}{2} \times 5^2 \times their\ 1.176 + \frac{1}{2} \times 8^2 \times their\ 0.3948$	M1	Or corresponding calculations in degrees
	[Area = 30 – 14.70 –12.63 =] 2.67	A1	Allow 2.66 to 2.67
		3	

131. 9709_m21_ms_12 Q: 10

Question	Answer	Marks	Guidance
(a)	$\Delta ADE = \frac{1}{2}(ka)^2 \sin\frac{\pi}{6}$	M1	Attempt to find the area of ΔADE.
	$\frac{1}{4}k^2a^2$	A1	OE.
	Sector $ABC = \frac{1}{2}a^2\frac{\pi}{6}$	B1	
	$2\times\frac{1}{4}k^2a^2 = \frac{1}{2}a^2\frac{\pi}{6}$	M1	OE. For $2\times\Delta ADE = \text{sector}ABC$ with at least one correct area.
	$k = \left(\sqrt{\frac{\pi}{6}}\right) = 0.7236$	A1	
		5	
(b)	$2\times\frac{1}{2}(ka)^2\sin\theta = \frac{1}{2}a^2\theta$	M1	Condone omission of '2' or '1/2' on LHS for M1 only.
	$k^2 = \frac{\theta}{2\sin\theta}$	A1	
	$k^2 > \frac{1}{2}$ leading to $\frac{1}{\sqrt{2}} < k < 1$	A1	OE. Accept $k > \frac{1}{\sqrt{2}}$ or $k > 0.707$ (AWRT) or $0.707\text{(AWRT)} < k < 1$ or $k > \sqrt{\frac{1}{2}}$ OE
		3	

132. 9709_s21_ms_11 Q: 8

Question	Answer	Marks	Guidance
(a)	**Either** Let midpoint of *PQ* be *H*: $\sin HCP = \frac{2}{4} \Rightarrow$ Angle $HCP = \frac{\pi}{6}$ **Or** $\sin PSQ = \frac{4}{8} \Rightarrow$ Angle $PSQ = \frac{\pi}{6}$ **Or** using cosine rule: angle $PCQ = \frac{\pi}{3}$ **Or** by inspection: triangle *PCQ* or *PCT* is equilateral so angle $PCQ = \frac{\pi}{3}$	M1	
	Angle $PCS = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2}{3}\pi$	A1	AG
		2	
(b)	Perimeter = $2\times4\times\frac{2\pi}{3}$ or $8\pi - \frac{8\pi}{3}$	M1	Length of two arcs *PS* and *QR*
	$+2\pi\times2$	M1	Adding circumference of two semicircles
	$\frac{28\pi}{3}$	A1	Must be a single term
		3	

Question	Answer	Marks	Guidance
(c)	Area sector $CPQ = \frac{1}{2}\times 4^2 \times \frac{\pi}{3} = \frac{8\pi}{3}$	M1	Uses correct formula for sector
	Area of segment of large circle beyond CPQ $= \frac{8\pi}{3} - \frac{1}{2}\times 4^2 \times \sin\left(\frac{\pi}{3}\right) = \frac{8\pi}{3} - 4\sqrt{3}$	M1	Attempts to find area of segment
	Area of small semicircle = $\pi \times 2$ or area of small circle = $\pi \times 2^2$	M1	
	Area of plate = Large circle – [2 ×] small semicircle – [2 ×] segment area	M1	
	$\pi\times 4^2 - \pi\times 2^2 - 2\times\left(\frac{8\pi}{3} - 4\sqrt{3}\right) = \frac{20\pi}{3} + 8\sqrt{3}$	A1	AG
	Alternative method for Question 8(c)		
	Area of sector $PCS = \frac{1}{2}\times 4^2 \times \frac{2\pi}{3} = \frac{16\pi}{3}$	M1	Uses correct formula for sector
	Area of triangle $PCQ = \frac{1}{2}\times 4^2 \times \sin\frac{\pi}{3} = 4\sqrt{3}$	M1	Uses correct formula for triangle
	Area of small semicircle = $\pi \times 2$ or area of circle = $\pi \times 2^2$	M1	
	Area of plate = [2 ×] large sector + [2 ×] triangle – [2 ×] small semicircle	M1	
	$2\left(\frac{16\pi}{3}\right) + 2\left(4\sqrt{3}\right) - \pi\times 2^2 = \frac{20\pi}{3} + 8\sqrt{3}$	A1	AG
		5	

133. 9709_s21_ms_12 Q: 12

Question	Answer	Marks	Guidance
(a)	[By symmetry] [6 × $P\hat{A}Q$ = 2π], [$P\hat{A}Q$ =] 2π÷6,	M1	
	Explaining that there are six sectors around the diagram that make up a complete circle.	A1	AG
	Alternative method for Question 12(a)		
	Using area or circumference of circle centre A ÷ 6	M1	$\frac{400\pi}{6}$ or $\frac{40\pi}{6}$
	Justification for dividing by 6 followed by comparison with the sector area or arc length.	A1	AG
	Alternative method for Question 12(a)		
	Explain why ΔPAQ is an equilateral triangle	M1	Assumption of this scores M0
	Using ΔPAQ is an equilateral triangle $\therefore$ $P\hat{A}Q = \frac{\pi}{3}$	A1	AG
	Alternative method for Question 12(a)		
	Using the internal angle of a regular hexagon = $\frac{2\pi}{3}$ Or $F\hat{A}O + O\hat{A}B = \frac{2\pi}{3}$, equilateral triangles	M1	
	$P\hat{A}Q = 2\pi - \left(\frac{\pi}{2} + \frac{2\pi}{3} + \frac{\pi}{2}\right) = \frac{\pi}{3}$	A1	AG

Question	Answer	Marks	Guidance
(a)	**Alternative method for Question 12(a)**		
	$Sin\theta = \frac{20}{40}$, with θ clearly identified	M1	
	$\theta = \frac{\pi}{6}, 2\theta = \frac{\pi}{3} = F\hat{A}O$ and by similar triangles = $P\hat{A}Q$	A1	AG
		2	
(b)	Each straight section of rope has length 40 cm	B1	SOI
	Each curved section round each pipe has length $r\theta = 20\times\frac{\pi}{3}$	*M1	Use of $r\theta$ with $r = 20$ and θ in radians
	Total length = $6\times((their\ 40)+k\pi)$	DM1	$6\times$(*their* straight section + *their* curved section). *Their* curved section must be from acceptable use of $r\theta$ – this could now be numeric.
	$240+40\pi$ or 366 (AWRT) (cm)	A1	Or directly: (6× diameter) + circumference
		4	

Question	Answer	Marks	Guidance
(c)	[Triangle area =] $\frac{1}{2}\times40\times40\times\sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2}\times40\times20\sqrt{3}$ or $400\sqrt{3}$ or 693(AWRT)	B1	
	[Total area of hexagon = 6 × $400\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	[Trapezium area =] $\frac{1}{2}\times(40+80)\times40\sin\left(\frac{\pi}{3}\right)$ or $1200\sqrt{3}$ or 2080 (AWRT)	B1	
	[Total area of hexagon = 2 × $1200\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	Area of triangle *ABC* = $400\sqrt{3}$ or 693 (AWRT) or 4 × Area of half of triangle *ABC* = $4\times200\sqrt{3}$ or 1390 (AWRT) or Area of rectangle *ABDE* = $1600\sqrt{3}$ or 2770 (AWRT)	B1	
	[Total area of hexagon = $2\times400\sqrt{3}+1600\sqrt{3}$ =] $2400\sqrt{3}$ Or [= $4\times200\sqrt{3}+1600$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
			If B0B0, **SC B1** can be scored for sight of 4160 (AWRT) as final answer.
		2	

Question	Answer	Marks	Guidance
(d)	Each rectangle area = 40 × 20 (= 800)	B1	SOI, e.g. by sight of 4800
	Each sector area = $\frac{1}{2}r^2\theta = \frac{1}{2}\times20^2\times\frac{\pi}{3}\left[=\frac{200\pi}{3}\right]$	B1	SOI.
	Total area = $2400\sqrt{3}+4800+400\pi$ or 10 200 (cm^2) (AWRT)	B1	Or directly: part **(c)** + 6800 + area circle radius 20.
		3	

134. 9709_s21_ms_13 Q: 5

Question	Answer	Marks	Guidance
(a)	$\frac{1}{2}\times 4^2 \times \text{angle BAD} = 10$	**M1**	Use of sector area formula
	Angle BAD $= 1.25$	**A1**	OE. Accept 0.398π, $71.6°$ for **SC B1** only
		2	
(b)	Arc $BD = 4 \times \textit{their}1.25$	**M1**	Use of arc length formula. Expect 5.
	$BC = 4\tan(\textit{their}1.25)$	**M1**	Expect 12.0(4). May use ACB=0.321 or 18.4°
	$CD = \frac{4}{\cos(\textit{their}1.25)} - 4$ or $\sqrt{4^2 + (\textit{their } BC)^2} - 4$	**M1**	Expect 12.69 – 4 = 8.69. May use ACB.
	Perimeter = 5 + 12.0(4) + 8.69 = 25.7 (cm)	**A1**	AWRT
		4	

135. 9709_w21_ms_11 Q: 6

Question	Answer	Marks	Guidance
(a)	Recognise that at least one of angles A, B, C is $\frac{\pi}{3}$	**B1**	SOI; allow 60°.
	One arc $6 \times \textit{their } \frac{\pi}{3}$ leading to two arcs $2 \times 6 \times \textit{their } \frac{\pi}{3}$	**M1**	SOI e.g. may see 2π or 4π. Use of correct formula for length of arc and multiply by 2.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
	Alternative method for question 6(a)		
	Calculate circumference of whole circle = 12π	**B1**	
	One arc $\frac{1}{6}\times 12\pi$ leading to two arcs $2\times\frac{1}{6}\times 12\pi$	**M1**	SOI e.g. may see 2π or 4π.
	Perimeter = $6 + 4\pi$	**A1**	Must be exact value.
		3	

Question	Answer	Marks	Guidance
(b)	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$\frac{1}{2}\times\left(6^2\right)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)+6\pi\ \left[=6\pi-9\sqrt{3}+6\pi\right]$	M1 A1	M1 for attempt at strategy with values substituted: **area of segment + area of sector** A1 if correct (unsimplified).
	Area = $12\pi-9\sqrt{3}$	A1	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times 6^2\times their\left(\frac{\pi}{3}\right)\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)$	M1 A1	M1 for attempt at strategy with values substituted: **2 × sector – triangle** A1 if correct (unsimplified).
	Area = $12\pi-9\sqrt{3}$	A1	Must be simplified exact value.
	Alternative method for question 6(b)		
	Sector = $\frac{1}{2}\times 6^2 \times their\left(\frac{\pi}{3}\right)$	M1	Use of correct formula for area of sector. SOI e.g. may see 6π or 12π.
	$2\times\left(\frac{1}{2}\times\left(6^2\right)\times their\left(\frac{\pi}{3}\right)-\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\right)+$ $\frac{1}{2}\times\left(6^2\right)\times\sin\left(their\left(\frac{\pi}{3}\right)\right)\ \left[=12\pi-18\sqrt{3}+9\sqrt{3}\right]$	M1 A1	M1 for attempt at strategy with values substituted: **2 × segment + triangle** A1 if correct (unsimplified).
	Area $\left[=6\pi-9\sqrt{3}+6\pi\right]$ = $12\pi-9\sqrt{3}$	A1	Must be simplified exact value.
		4	

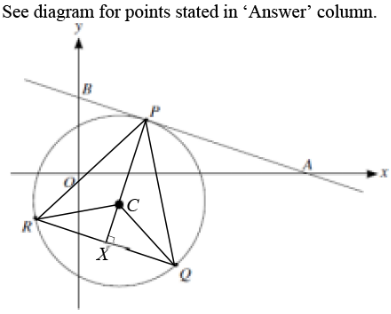
136. 9709_w21_ms_12 Q: 7

Question	Answer	Marks	Guidance
(a)	**EITHER** By using trigonometry: $B\hat{A}C=0.6435\ldots$ and $A\hat{B}C=\frac{\pi-0.6435}{2}$ **OR** By Pythagoras: $AP=12\Rightarrow BP=3$ so $\tan A\hat{B}C=\frac{9}{3}$ **OR** Using ΔPBC and either the sine or cosine rule $\sin A\hat{B}C=\frac{3}{\sqrt{10}}$ or $\cos A\hat{B}C=\frac{\sqrt{10}}{10}$	M1	$\frac{3}{\sqrt{10}}=0.9486\ldots$ $\frac{\sqrt{10}}{10}=0.3162\ldots$
	$A\hat{B}C=\frac{\pi-0.6435}{2}$ or $\tan^{-1}\frac{9}{3}$ or $\sin^{-1}\frac{3}{\sqrt{10}}$ or $\cos^{-1}\frac{\sqrt{10}}{10}$ or $1.249(04\ldots)$ or $71.56°$ = 1.25 radians (3 sf)	A1	AG. Final answer must be 1.25, more accurate value 1.24904… with no rounding to 3sf seen as the final answer gets M1A0. If decimals are used all values must be given to at least 4sf for A1.
		2	
(b)	$BC=\sqrt{(their\,3)^2+9^2}$ or $\frac{9}{\sin 1.25}$ [= $\sqrt{90}$, $3\sqrt{10}$ or 9.48697…]	M1	Using correct method(s) to find BC.
	Area of sector = $\frac{1}{2}\times(their\,BC)^2\times\tan^{-1}3$ [= 56.207 *or* 56.25]	M1	Using $\tan^{-1}3$ or 1.25 and *their BC*, but not 9 or 15, in correct area of sector formula.
	Area of triangle PBC = 13.4 to 13.6 or $\frac{1}{2}\times 9\times 3$	B1	
	[Area = (56.207 or 56.25) – *their* 13.5 =] 42.7 or 42.8	A1	AWRT
		4	

137. 9709_w21_ms_12 Q: 12

Question	Answer	Marks	Guidance
(a)	Centre is (3, – 2)	B1	
	Gradient of radius = $\frac{(their -2)-4}{(their\,3)-5}$ [= 3]	*M1	Finding gradient using *their* centre (not (0, 0)) and *P* (5,4).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using *P* and the negative reciprocal of *their* gradient to find the equation of *AB*.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	A1	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times 17=\right]\ \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.
	Alternative method for question 12(a)		
	$2x+2y\frac{dy}{dx}-6+4\frac{dy}{dx}=0$	B1	
	At *P*: $10+8\frac{dy}{dx}-6+4\frac{dy}{dx}=0\left[\Rightarrow\frac{dy}{dx}=-\frac{1}{3}\right]$	*M1	Find the gradient using *P* (5,4) in *their* implicit differential (with at least one correctly differentiated *y* term).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using *P* and *their* value for the gradient to find the equation of *AB*.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	A1	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times 17=\right]\ \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.

Question	Answer	Marks	Guidance
(a) cont'd	**Alternative method for question 12(a)**		
	$\left[y=-2\pm\left(40-(x-3)^2\right)^{\frac{1}{2}}\text{ OE leading to}\right]\frac{dy}{dx}=(3-x)\left(31+6x-x^2\right)^{-\frac{1}{2}}$	B1	OE. Correct differentiation of rearranged equation.
	$\frac{dy}{dx}=(3-5)\left(31+6(5)-(5)^2\right)^{-\frac{1}{2}}\left[\Rightarrow\frac{dy}{dx}=-\frac{1}{3}\right]$	*M1	Find the gradient using x = 5 in *their* differential (with clear use of chain rule).
	Equation of tangent $y-4=-\frac{1}{3}(x-5)$	DM1	Using *P* and *their* value for the gradient to find the equation of *AB*.
	Sight of [x =]17 and [y =] $\frac{17}{3}$	A1	
	$\left[\text{Area}=\frac{1}{2}\times\frac{17}{3}\times 17=\right]\ \frac{289}{6}$	A1	Or $48\frac{1}{6}$ or AWRT 48.2.
		5	

Question	Answer	Marks	Guidance
(b)	Radius of circle = $\sqrt{40}$,	B1	Or $2\sqrt{10}$ or 6.32 AWRT or $r^2 = 40$.
	Area of $\Delta CRQ = \frac{1}{2}\times(their\, r)^2 \sin 120\left[=\frac{1}{2}\times 40\times\frac{\sqrt{3}}{2}\right]$ OR Area of $\Delta CQX = \frac{1}{2} \times\sqrt{40}\cos 30\times\sqrt{40}\cos 60$ OE $\left[=\frac{1}{2}\times\sqrt{30}\times\sqrt{10}\right]$ OR Area of circle – 3× Area of segment = $40\pi - 3\times(40\frac{\pi}{3} - 10\sqrt{3})$ OR $QR = \sqrt{120}$ or $2\sqrt{30}$ and area = $\frac{1}{2}QR^2\sin 60$	M1	Using $\frac{1}{2}r^2\sin\theta$ with *their r* and 120 or 60 [×3] Using $\frac{1}{2}$ ×base×height in a correct right-angled triangle [×6]. Use of cosine rule and area of large triangle
	$30\sqrt{3}$	A1	AWRT 52[.0] implies B1M1A0.
		3	See diagram for points stated in 'Answer' column. 

138. 9709_w21_ms_13 Q: 5

Question	Answer	Marks	Guidance
(a)	Angle $XYC = \sin^{-1}\left(\frac{9}{11}\right) = 0.9582$ or $\sin XYC = \frac{9}{11}$ leading to $XYC = 0.9582$	B1	AG. OE using cosine rule.
		1	
(b)	$XY = \sqrt{11^2 - 9^2} = \sqrt{40}$ or using 0.9582 and trigonometry	*M1 A1	
	$AB = 9 + 11 - their XY$	B1 FT	OE e.g. $20 - 2\sqrt{10}$, $2 + 9 - 2\sqrt{10} + 11 - 2\sqrt{10}$
	Arc AC = 11 × 0.9582	M1	
	Arc $BC = 9\times\frac{\pi}{2}$	M1	
	Perimeter = [13.6(8) + 10.5(4) +14.1(4) =] 38.4	A1	AWRT. Answer must be evaluated as a single decimal.
		6	

139. 9709_m20_ms_12 Q: 7

	Answer	Mark	Partial Marks
	$OC = 6\cos 0.8 = 4.18(0)$	M1A1	SOI
	Area sector $OCD = \frac{1}{2}(their\, 4.18)^2 \times 0.8$	*M1	OE
	$\Delta OCA = \frac{1}{2}\times 6\times their\, 4.18\times\sin 0.8$	M1	OE
	Required area = *their* ΔOCA – *their* sectorOCD	DM1	SOI. If not seen *their* areas of sector and triangle must be seen
	2.01	A1	CWO. Allow or better e.g. 2.0064
		6	

140. 9709_s20_ms_11 Q: 8

	Angle $AOB = 15 \div 6 = 2.5$ radians	**B1**
	Angle $BOC = \pi - 2.5$ (**FT** on angle AOB)	**B1FT**
	$BC = 6(\pi - 2.5)$ $(BC = 3.850)$	**M1**
	$\sin(\pi - 2.5) = BX \div 6$ $(BX = 3.59)$	**M1**
	Either $OX = 6\cos(\pi - 2.5)$ **or** Pythagoras $(OX = 4.807)$	**M1**
	$XC = 6 - OX$ $(XC = 1.193) \rightarrow P = 8.63$	**A1**
		6

141. 9709_s20_ms_13 Q: 5

	$\cos POA = \frac{5}{13} \rightarrow POA = 1.17(6)$ Allow 67.4° **or** $\sin = \frac{12}{13}$ or $\tan = \frac{12}{5}$	**M1 A1**
	Reflex $AOB = 2\pi - 2\times$*their* $1.17(6)$ OE in degrees **or** minor arc AB = $5\times2\times$*their*$1.17(6)$	**M1**
	Major arc = $5 \times$ *their* 3.93(1) **or** $2\pi\times5$ - *their* 11.7(6)	**M1**
	AP (or BP) = $\sqrt{13^2 - 5^2} = 12$	**B1**
	Cord length = 43.7	**A1**
		6

142. 9709_s20_ms_13 Q: 10

(a)	Mid-point is (–1, 7)	**B1**
	Gradient, m, of AB is 8/12 OE	**B1**
	$y - 7 = -\frac{12}{8}(x+1)$	**M1**
	$3x + 2y = 11$ **AG**	**A1**
		4
(b)	Solve simultaneously $12x - 5y = 70$ and *their* $3x + 2y = 11$	**M1**
	$x = 5,\ y = -2$	**A1**
	Attempt to find distance between *their* (5, –2) and either (–7,3) or (5, 11)	**M1**
	$(r) = \sqrt{12^2 + 5^2}$ or $\sqrt{13^2 + 0} = 13$	**A1**
	Equation of circle is $(x-5)^2 + (y+2)^2 = 169$	**A1**
		5

143. 9709_w20_ms_11 Q: 10

	Answer	Mark	Partial Marks
(a)	$\left(\sin\theta = \frac{r}{OC} \rightarrow\right) OC = \frac{r}{\sin\theta}$	M1 A1	
	$CD = r + \frac{r}{\sin\theta}$	A1	
		3	
(b)	Radius of arc $AB = 4 + \frac{4}{\sin\frac{\pi}{6}} = 4 + 8 = 12$	B1	SOI
	(Arc $AB =$) *their* $12\times\frac{2\pi}{6}$ or $\left(\frac{1}{2}AB=\right)\left(\textit{their } 12\times\frac{\pi}{6}\right)$	M1	Expect 4π, must use *their* CD, not 4
	Perimeter = $24 + 4\pi$	A1	
		3	
(c)	Area $FOC = \frac{1}{2}\times 4\times \textit{their } OC \times \sin\frac{\pi}{3}$	M1	
	$8\sqrt{3}$	A1	
	Area sector $FOE = \frac{1}{2}\times\frac{2\pi}{3}\times 4^2 = \frac{16\pi}{3}$	B1	
	Shaded area = $16\sqrt{3} - \frac{16\pi}{3}$	A1	
	Alternative method for question 10(c)		
	$FC = \sqrt{(\textit{their } OC)^2 - 4^2}$	M1	$\sqrt{48}$ or $4\sqrt{3}$
	Area $FOC = \frac{1}{2}\times 4\times 4\sqrt{3} = 8\sqrt{3}$	A1	
	Area of half sector $FOE = \frac{1}{2}\times\frac{\pi}{3}\times 4^2 = \frac{8\pi}{3}$	B1	
	Shaded area = $16\sqrt{3} - \frac{16\pi}{3}$	A1	
		4	

144. 9709_w20_ms_12 Q: 8

	Answer	Mark	Partial Marks
(a)	Use of correct formula for the area of triangle *ABC*	M1	Use of 180–2θ scores M0. Condone $2\pi - 2\theta$
	$\frac{1}{2}r^2\sin(\pi-2\theta)$ **or** $\frac{1}{2}r^2\sin 2\theta$ **or** $2\times\frac{1}{2}r\times r\cos\theta\times\sin\theta$ **or** $2\times\frac{1}{2}r\cos\theta\times r\sin\theta$	A1	OE
	[Shaded area = triangle – sector] = *their* triangle area $-\frac{1}{2}r^2\theta$	B1 FT	FT for *their* triangle area $-\frac{1}{2}r^2\theta$ (Condone use of 180 degrees for triangle area for B1)
		3	
(b)	Arc $BD = r\theta = 6$ cm	B1	SOI
	$AC = 2r\cos\theta = (2\times 10\cos 0.6 = 20\cos 0.6 = 16.506)$ or $\sqrt{(2r^2 - 2r^2\cos(\pi-2\theta))}$ or $\frac{r\times\sin(\pi-2\theta)}{\sin\theta}$	*M1	Finding AC or $\frac{1}{2}AC$ (= 8.25)
	$DC = 2r\cos\theta - r$ or $\sqrt{(2r^2-2r^2\cos(\pi-2\theta))} - r$ (= 6.506)	DM1	Subtracting r from *their* AC or $r - r\cos\theta$ from *their* half AC (8.25-1.75)
	(Perimeter = 10 + 6 + 6.506 =) 22.5	A1	AWRT
		4	

145. 9709_w20_ms_13 Q: 9

	Answer	Mark	Partial Marks
(a)	$\cos BAO = \frac{6}{8}$ or $\frac{8^2+12^2-8^2}{2\times8\times12}$	M1	Or other correct method
	$BAO = 0.723$	A1	
		2	
(b)	Sector $ABC = ½\times12^2\times$ *their* 0.7227	*M1	Accept 52.1
	Triangle $AOB = ½\times8\times12\sin($*their* $0.7227)$ or $½\times12\times\sqrt{28}$	*M1	or $½\times8\times8\sin(\pi-2\times$ *their* $0.7227)$. Expect 31.7 or 31.8
	Shaded area = *their* 52.0 − *their* 31.7 = 20.3	DM1 A1	M1 dependent on both previous M marks
		4	
(c)	Arc $BC = 12\times$ *their* 0.7227	*M1	Expect 8.67
	Perimeter = 8 + 4 + *their* 8.67 = 20.7	DM1 A1	
		3	

146. 9709_w20_ms_13 Q: 11

	Answer	Mark	Partial Marks
(a)	$(-6-8)^2+(6-4)^2$	M1	OE
	$= 200$	A1	
	$\sqrt{200} > 10$, hence outside circle	A1	AG ('Shown' not sufficient). Accept equivalents of $\sqrt{200} > 10$
	Alternative method for question 11(a)		
	Radius = 10 and $C = (8, 4)$	B1	
	Min(x) on circle = 8 – 10 = –2	M1	
	Hence outside circle	A1	AG
		3	
(b)	angle $=\sin^{-1}\left(\frac{their\,10}{their\,10\sqrt{2}}\right)$	M1	Allow decimals for $10\sqrt{2}$ at this stage. If cosine used, angle ACT or BCT must be identified, or implied by use of 90°– 45°.
	angle = $\sin^{-1}\left(\frac{1}{\sqrt{2}} \text{ or } \frac{\sqrt{2}}{2} \text{ or } \frac{10}{10\sqrt{2}} \text{ or } \frac{10}{\sqrt{200}}\right) = 45°$	A1	AG Do not allow decimals
	Alternative method for question 11(b)		
	$(10\sqrt{2})^2 = 10^2 + TA^2$	M1	
	$TA = 10 \rightarrow 45°$	A1	AG
		2	
(c)	Gradient, m, of $CT = -\frac{1}{7}$	B1	OE
	Attempt to find mid-point (M) of CT	*M1	Expect (1, 5)
	Equation of AB is $y-5=7(x-1)$	DM1	Through *their* (1, 5) with gradient $-\frac{1}{m}$
	$y = 7x-2$	A1	
		4	
(d)	$(x-8)^2+(7x-2-4)^2=100$ or equivalent in terms of y	M1	Substitute *their* equation of AB into equation of circle.
	$50x^2-100x\ (=0)$	A1	
	$x = 0$ and 2	A1	WWW
	Alternative method for question 11(d)		
	MC = $\begin{pmatrix}7\\-1\end{pmatrix}$	M1	
	$\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}-1\\-7\end{pmatrix}=\begin{pmatrix}0\\-2\end{pmatrix}$, $\begin{pmatrix}1\\5\end{pmatrix}+\begin{pmatrix}1\\7\end{pmatrix}=\begin{pmatrix}2\\12\end{pmatrix}$	A1	
	$x = 0$ and 2	A1	
		3	

147. 9709_m19_ms_12 Q: 3

	Answer	Mark	Partial Marks
	Angle $CBA = \sin^{-1}\left(\frac{7}{8}\right) = 1.0654$ or $CBD = \cos^{-1}\left(\frac{-17}{32}\right) = 2.13$	**B1**	Accept 61.0°, 66° or 122°
	Sector $BCYD = ½ \times 8^2 \times 2 \times their 1.0654(\text{rad})$ soi or sector $CBY = ½ \times 8^2 \times their 1.0654(\text{rad})$	**M1**	Expect 68.1(9). Angle must be in radians (or *their* 61/360 × 2 × 8^2) Or sector DBY
	$\Delta BCD = 7 \times \sqrt{8^2 - 7^2}$ or $½ \times 8^2 \times \sin(2 \times their 1.0654)$ soi	**M1**	Expect 27.1(1). Award M1 for ABC or ABD
	Semi-circle $CXD = ½\pi \times 7^2 = 76.9(7)$	**M1**	M1M1 for segment area formula used correctly
	Total area = *their*68.19 – *their*27.11 + *their*76.97 = 118.0–118.1	**M1A1**	Cannot gain M1 without attempt to find angle CBA or CBD
		6	

148. 9709_s19_ms_11 Q: 3

	Answer	Mark	Partial Marks
	Uses $A = ½r^2\theta$	**M1**	Uses area formula.
	$\theta = \frac{2A}{r^2}$	**A1**	
	$P = r + r + r\theta$	**B1**	
	$P = 2r + \frac{2A}{r}$	**A1**	Correct simplified expression for *P*.
		4	

149. 9709_s19_ms_12 Q: 5

	Answer	Mark	Partial Marks
	Perimeter of $AOC = 2r + r\theta$	**B1**	
	Angle $COB = \pi - \theta$	**B1**	Could be on the diagram. Condone $180 - \theta$.
	Perimeter of $BOC = 2r + r(\pi - \theta)$	**B1**	**FT** on angle *COB* if of form $(k\pi - \theta)$, $k > 0$.
	$(2r +)\ \pi r - r\theta = 2((2r) + r\theta)$ $(2 + \pi - \theta = 4 + 2\theta \rightarrow \theta = \frac{\pi - 2}{3}$)	**M1**	Sets up equation using $r(k\pi - \theta)$ and ×2 on correct side. Condone any omissions of OA, OB and/or OC.
	$\theta = 0.38$	**A1**	Equivalent answer in degrees scores A0.
		5	

150. 9709_s19_ms_13 Q: 3

	Answer	Mark	Partial Marks
(i)	Angle EAD = Angle $ACD = \frac{3\pi}{10}$ or 54° or 0.942 soi **or** Angle $DAC = \frac{\pi}{5}$ or 36° or 0.628 soi	B1	
	$AD = 8\sin(\frac{3\pi}{10})$ **or** $8\cos(\frac{\pi}{5})$	M1	Angles used must be correct
	(AD =) 6.47	A1	
	Alternative method for question 3(i)		
	$AB = \frac{8}{\tan\left(\frac{\pi}{5}\right)}$ or $AB = \frac{8\sin\left(\frac{3\pi}{10}\right)}{\sin\left(\frac{\pi}{5}\right)}$ or $11.(01)$	B1	Angles used must be correct
	$AD = 11.0(1)\sin\frac{\pi}{5}$ **oe**	M1	
	(AD =) 6.47	A1	
		3	
(ii)	Area sector $= \frac{1}{2}(theirAD)^2 \times their\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$	M1	19.7(4)
	Area $\Delta ADC = \frac{1}{2} \times 8 \times theirAD \times \sin\frac{\pi}{5}$ or $\frac{1}{2} \times 8\cos\left(\frac{3\pi}{10}\right) \times 8\sin\left(\frac{3\pi}{10}\right)$	M1	Or e.g. $½\, theirAD \times \sqrt{8^2 - theirAD^2}$. 15.2(2)
	(Shaded area =) 35.0 or 34.9	A1	
		3	

151. 9709_w19_ms_11 Q: 8

	Answer	Mark	Partial Marks
(i)	$OA \times \frac{3}{8}\pi = 6$	M1	
	$OA = \frac{16}{\pi} = 5.093(0)$	A1	
(ii)	$AB = their5.0930 \times \tan\frac{3}{16}\pi$	M1	
	Perimeter $= 2 \times 3.4030 + 6 = 12.8$	A1	
(iii)	Area $OABC = (2 \times ½) \times their5.0930 \times their3.4030$	M1	
	Area sector $= ½ \times (their5.0930)^2 \times \frac{3}{8}\pi$	M1	
	Shaded area $= their17.331 - their15.279 = 2.05$	M1A1	

152. 9709_w19_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	Arc length $AB = 2r\theta$	B1	
	$\tan\theta = \frac{AT}{r}$ or $\frac{BT}{r} \to AT$ or $BT = r\tan\theta$	B1	Accept or $\sqrt{\left(\left(\frac{r}{\cos\theta}\right)^2 - r^2\right)}$ or $\frac{r\sin\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$ NOT $(90-\theta)$
	$P = 2r\theta + 2r\tan\theta$	B1FT	OE, FT for *their* arc length + 2 × *their AT*
		3	
(ii)	Area $\Delta AOT = ½ \times 5 \times 5\tan 1.2$ **or** Area $AOBT = 2 \times ½ \times 5 \times 5\tan 1.2$	B1	
	Sector area = ½ × 25 × 2.4 (or 1.2)	*M1	Use of $½r^2\theta$ with $\theta = 1.2$ or 2.4.
	Shaded area = 2 triangles – sector	DM1	Subtraction of sector, using 2.4 where appropriate, from 2 triangles
	Area = 34.3 (cm^2)	A1	AWRT
	Alternative method for question 4(ii)		
	Area of $\Delta ABT = ½ \times (5\times\tan 1.2)^2 \times \sin(\pi - 2.4)$ (= 55.86)	B1	
	Segment area = ½ × 25 × (2.4 – sin 2.4) (= 21.56)	*M1	Use of $½r^2(\theta - \sin\theta)$ with $\theta = 1.2$ or 2.4
	Shaded area = triangle – segment	DM1	Subtraction of segment from ΔABT, using 2.4 where appropriate.
	Area = 34.3 (cm^2)	A1	AWRT
		4	

153. 9709_w19_ms_13 Q: 4

	Answer	Mark	Partial Marks
(i)	Angle $CAO = \frac{\pi}{3}$	B1	
		1	
(ii)	(Sector AOC) = $\frac{1}{2}r^2 \times their\,\frac{\pi}{3}$	M1	SOI
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(their\,\frac{\pi}{3}\right)$ **or** $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ **or** $\frac{1}{2}(r)(r)\sqrt{3}$	M1	For M1M1, *their* $\frac{\pi}{3}$ must be of the form $k\pi$ where $0 < k < ½$
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(\frac{\pi}{3}\right)$ **or** $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ **or** $\frac{1}{2}(r)(r)\sqrt{3}$	A1	All correct
	$r^2\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$	A1	
		4	

154. 9709_m18_ms_12 Q: 6

	Answer	Mark	Partial Marks
(i)	$\frac{PQ}{2} = 10 \times \sin 1.1$	**M1**	Correct use of sin/cos rule
	(PQ =) 17.8 (17.82…implies **M1**, **A1**) AG	**A1**	OR $PQ = \frac{10\sin 2.2}{\sin\left(\frac{\pi}{2} - 1.1\right)}$ *or* $\frac{10\sin 2.2}{\sin 0.4708}$ *or* $\sqrt{200 - 200\cos 2.2}$ = 17.8
		2	
(ii)	Angle OPQ = (π/2 – 1.1) [accept 27°]	**B1**	OE Expect 0.4708 or 0.471. Can be scored in part **(i)**
	Arc QR =17.8 × *their* (π/2 – 1.1)	**M1**	Expect 8.39. (or 8.38).
	Perimeter $= 17.8 - 10 + 10 + \textit{their}\ \text{arc}\ QR$	**M1**	
	26.2	**A1**	For both parts allow correct methods in degrees
		4	

155. 9709_s18_ms_11 Q: 6

	Answer	Mark	Partial Marks
(i)	($\tan\theta = \frac{AT}{r}$) → $AT = r\tan\theta$ or $OT = \frac{r}{\cos\theta}$ SOI	**B1**	CAO
	→ $A = \frac{1}{2}r^2\tan\theta \quad - \frac{1}{2}r^2\theta$	**B1 B1**	B1 for $\frac{1}{2}r^2\tan\theta$. B1 for "$-\frac{1}{2}r^2\theta$" If Pythagoras used may see area of triangle as $\frac{1}{2}r\sqrt{r^2 + r^2 tan^2\theta}$ or $\frac{1}{2}r\left(\frac{r}{\cos\theta}\right)sin\theta$
		3	

	Answer	Mark	Partial Marks
(ii)	$\tan\theta = \frac{AT}{3}$ → $AT = 7.716$	**M1**	Correct use of trigonometry and radians in rt angle triangle
	Arc length = $r\theta$ = 3.6	**B1**	Accept 3×1.2
	OT by Pythagoras or $\cos 1.2 = \frac{3}{OT}$ (= 8.279)	**M1**	Correct method for OT
	Perimeter = AT + arc + OT – radius = 16.6	**A1**	CAO, www
		4	

156. 9709_s18_ms_12 Q: 6

	Answer	Mark	Partial Marks
(i)	AT or $BT = r\tan\theta$ or $OT = \frac{r}{cos\vartheta}$	**B1**	May be seen on diagram.
	$\frac{1}{2}r^2 2\theta$, & $\frac{1}{2} \times r \times (r\tan\theta$ or $AT)$ or $\frac{1}{2} \times r \times (\frac{r}{cos\vartheta}$ or $OT) \sin\theta$	**M1**	Both formulae, ($\frac{1}{2}r^2\theta$, $\frac{1}{2}bh$ *or* $\frac{1}{2}absin\theta$), seen with 2θ used when needed.
	$\frac{1}{2}r^2 2\theta = 2 \times \frac{1}{2} \times r \times r\tan\theta - \frac{1}{2}r^2 2\theta$ oe → $2\theta = \tan\theta$ **AG**	**A1**	Fully correct working from a correct statement. Note: $\frac{1}{2}r^2 2\theta = \frac{1}{2}r^2\tan\theta$ is a valid statement.
		3	

	Answer	Mark	Partial Marks
(ii)	$\theta = 1.2$ or sector area = 76.8	B1	
	Area of kite = 165 awrt	B1	
	164.6 – 76.8 = 87.8 awrt	B1	awrt 87.8 with little or no working can be awarded 3/3. SC Final answers that round to 88 with little or no working can be awarded 2/3.
		3	

157. 9709_s18_ms_13 Q: 5

	Answer	Mark	Partial Marks
	Angle $AOC = \frac{6}{5}$ or 1.2	M1	Allow 68.8°. Allow $\frac{5}{6}$
	AB = 5 × tan(*their* 1.2) OR by e.g. Sine Rule Expect 12.86	DM1	OR $OB = \frac{5}{\cos their 1.2}$. Expect 13.80
	Area $\Delta OAB = \frac{1}{2} \times 5 \times their\,12.86$ Expect 32.15	DM1	OR $\frac{1}{2}$ × 5 × *their OB* × sin *their* 1.2
	Area sector $\frac{1}{2} \times 5^2 \times their\,1.2$ Expect 15	DM1	All DM marks are dependent on the first M1
	Shaded region = 32.15 – 15 = 17.2	A1	Allow degrees used appropriately throughout. 17.25 scores A0
		5	

158. 9709_w18_ms_11 Q: 9

	Answer	Mark	Partial Marks
	Angle $OAB = \pi/2 - \pi/5 = 3\pi/10$ soi	B1	Allow 54° *or* 0.9425 *rads*
	Sector $CAB = \frac{1}{2} \times \left(their \frac{3\pi}{10}\right) \times 5^2$	M1	Expect 11.78
	$OA = \frac{5}{\sin\frac{\pi}{5}} = 8.507$	M1A1	May be implied by $OC = 3.507$
	Sector $COD = \frac{1}{2} \times (their\,3.507)^2 \times \frac{\pi}{5}$	M1	Expect 3.86
	$\Delta OAB = \frac{1}{2} \times 5 \times (their\,8.507) \sin\frac{3\pi}{10}$	M1	Or $\frac{1}{2} \times 5 \times \frac{5}{\tan\frac{\pi}{5}}$ or 2.5 × $\sqrt{(their\,8.507)^2 - 25}$
	= 17.20 or 17.21	A1	
	Shaded area $17.20 (or\,17.21) - 11.78 - 3.86 = 1.56$ or 1.57	A1	
		8	

159. 9709_w18_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	$A\hat{B}C$ using cosine rule giving $\cos^{-1}(\frac{-1}{8})$ or $2\sin^{-1}(¾)$ or $2\cos^{-1}\left(\frac{\sqrt{7}}{2}\right)$ **or** $B\hat{A}C = \cos^{-1}(¾)$ or $B\hat{A}C = \sin^{-1}\frac{\sqrt{7}}{4}$ or $B\hat{A}C = \tan^{-1}\frac{\sqrt{7}}{3}$	**M1**	Correct method for $A\hat{B}C$, expect 1.696^cawrt Or for $B\hat{A}C$, expect 0.723^cawrt
	$C\hat{B}Y = \pi - A\hat{B}C$ or $2\times C\hat{A}B$	**M1**	For attempt at $C\hat{B}Y = \pi - A\hat{B}C$ or $C\hat{B}Y = 2\times C\hat{A}B$
	OR		
	Find CY from ΔACY using Pythagoras or similar Δs	**M1**	Expect $4\sqrt{7}$
	$C\hat{B}Y = \cos^{-1}\left(\frac{8^2+8^2-(their\ CY)^2}{2\times8\times8}\right)$	**M1**	Correct use of cosine rule
	$C\hat{B}Y = 1.445^c$ AG	**A1**	Numerical values for angles in radians, if given, need to be correct to 3 decimal places. Method marks can be awarded for working in degrees. Need 82.8° awrt converted to radians for A1. Identification of angles must be consistent for A1.
		3	
(ii)	Arc $CY = 8\times1.445$	**B1**	Use of $s=8\theta$ for arc CY, Expect 11.56
	$B\hat{A}C = ½(\pi - A\hat{B}C)$ or $\cos^{-1}(¾)$	***M1**	For a valid attempt at $B\hat{A}C$, may be from **(i)**. Expect 0.7227^c
	Arc $XC = 12\times$ (their $B\hat{A}C$)	**DM1**	Expect 8.673
	Perimeter = 11.56 + 8.673 + 4 = **24.2 cm awrt www**	**A1**	Omission of '+4' only penalised here.
		4	

160. 9709_w18_ms_13 Q: 3

	Answer	Mark	Partial Marks
(i)	0.8 oe	**B1**	
		1	
(ii)	$BD = 5\sin their\ 0.8$	**M1**	Expect 3.58(7). Methods using degrees are acceptable
	$DC = 5 - 5\cos their\ 0.8$	**M1**	Expect 1.51(6)
	Sector = $½\times5^2\times their0.8$ OR Seg = $½\times5^2\times[their\ 0.8 - \sin their\ 0.8]$	**M1**	Expect 10 for sector. Expect 1.03(3) for segment
	Trap = $½(5 + theirDC)\times theirBD$ oe OR $\Delta BDC = ½theirBD\times theirCD$	**M1**	OR (for last 2 marks) if X is on AB and XC is parallel to BD:
	Shaded area = 11.69 – 10 OR 2.71(9) – 1.03(3) = 1.69 cao	**A1**	$BDCX$ –(sector – ΔAXC) = 5.43(8) – [10 – 6.24(9)] = 1.69 cao M1A1
		5	

161. 9709_m17_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$ABC = \pi/2 - \pi/7 = 5\pi/14$. $CBD = \pi - 5\pi/14 = 9\pi/14$	B1	**AG** Or other valid exact method.
	Total:	1	
(ii)	$\sin\frac{\pi}{7} = \frac{½BC}{8}$ **or** $\frac{BC}{\sin\frac{2\pi}{7}} = \frac{8}{\sin\frac{5\pi}{14}}$ **or** $BC^2 = 8^2 + 8^2 - 2(8)(8)\cos\frac{2\pi}{7}$	M1	
	$BC = 6.94(2)$	A1	
	arc CD = *their* $6.94 \times 9\pi/14$	M1	Expect 14.02(0)
	arc $CB = 8 \times 2\pi/7$	M1	Expect 7.18(1)
	perimeter = 6.94 + 14.02 + 7.18 = 28.1	A1	
	Total:	5	

162. 9709_s17_ms_11 Q: 8

	Answer	Mark	Partial Marks
(i)	Letting M be midpoint of AB		
	$OM = 8$ (Pythagoras) → $XM = 2$	B1	(could find √40 and use $\sin^{-1}$or $\cos^{-1}$)
	$\tan AXM = \frac{6}{2}$ $AXB = 2\tan^{-1}3 = 2.498$	M1 A1	AG Needs × 2 and correct trig for **M1**
	(Alternative 1: $\sin AOM = \frac{6}{10}, AOM = 0.6435, AXB = \pi - 0.6435$)		(Alternative 1: Use of isosceles triangles, **B1** for AOM, **M1**,**A1** for completion) (Alternative 2: Use of circle theorem, **B1** for AOB, **M1**,**A1**for completion)
	Total:	3	
(ii)	$AX = \sqrt{(6^2 + 2^2)} = \sqrt{40}$	B1	CAO, could be gained in part (i) or part (iii)
	Arc $AYB = r\theta = \sqrt{40} \times 2.498$	M1	Allow for incorrect √40 (not $r = 6 or 12 or 10$)
	Perimeter = 12 + arc = 27.8 cm	A1	
	Total:	3	
(iii)	area of sector $AXBY = ½ \times (\sqrt{40})^2 \times 2.498$	M1	Use of $½r^2\theta$ with their r, (not $r = 6 or r = 10$)
	Area of triangle $AXB = ½ \times 12 \times 2$, Subtract these → 38.0 cm²	M1 A1	Use of ½bh and subtraction. Could gain **M1** with $r = 10$.
	Total:	3	

163. 9709_s17_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$(AB) = 2r\sin\theta$ (or $r\sqrt{2-2\cos2\theta}$ or $\frac{r\sin2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	B1	Allow unsimplifed throughout eg r + r, $\frac{2\theta}{2}$ etc
	(Arc AB) = $2r\theta$	B1	
	$(P =)\ 2r + 2r\theta + 2r\sin\theta$ (or $r\sqrt{2-2\cos2\theta}$ or $\frac{r\sin2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)}$)	B1	
	Total:	3	

	Answer	Mark	Partial Marks
(ii)	Area sector *AOB* = (½ r^2 2*θ*) $\frac{25\pi}{6}$ or 13.1	B1	Use of segment formula gives 2.26 **B1B1**
	Area triangle *AOB* = (½×2*r*sin*θ*×*r*cos*θ* or ½ × r^2 sin2*θ*) $\frac{25\sqrt{3}}{4}$ or 10.8	B1	
	Area rectangle *ABCD* = (*r* × 2*r*sin*θ*) 25	B1	
	(Area =) Either 25 – (25π/6 – 25√3/4) or 22.7	B1	Correct final answer gets **B4**.
	Total:	4	

164. 9709_s17_ms_13 Q: 7

	Answer	Mark	Partial Marks
(i)	$\sin ABC = 8/10 \rightarrow ABC = 0.927(3)$	B1	Or cos = 6/10 or tan = 8/6. Accept 0.295π.
	Total:	1	
(ii)	$AB = 6(\text{Pythagoras}) \rightarrow \Delta BCD = 8\times 6 = 48.0$	M1A1	**OR** 8×10sin0.6435 or ½×10×10sin((2)×0.927)=48. 24or 40or80 gets **M1A0**
	Area sector $BCD = ½\times 10^2 \times (2)\times \textit{their}\, 0.9273$	*M1	Expect 92.7(3). 46.4 gets **M1**
	Area segment = 92.7(3) – 48	*A1	Expect 44.7(3). Might not appear until final calculation.
	Area semi-circle – segment = $½\times\pi\times 8^2 - \textit{their}(92.7-48)$	DM1	Dep. on previous **M1A1** OR $\pi\times 8^2 - (½\times\pi\times 8^2 + \textit{their}\, 44.7)$.
	Shaded area = 55.8 – 56.0	A1	
	Total:	6	

165. 9709_w17_ms_11 Q: 5

	Answer	Mark	Partial Marks
(i)	$\cos A = 8/10 \rightarrow A = 0.6435$	B1	AG Allow other valid methods e.g. $\sin A = 6/10$
		1	
(ii)	*EITHER:* Area $\Delta ABC = ½\times 16\times 6$ or $½\times 10\times 16\sin 0.6435 = 48$	(M1A1	
	Area 1 sector $½\times 10^2 \times 0.6435$	M1	
	Shaded area $= 2\times \textit{their}$ sector $- \textit{their}\, \Delta ABC$	M1)	
	OR: $\Delta BDE = 12$, $\Delta BDC = 30$	(B1 B1	
	Sector = 32.18	M1	
	2×segment + ΔBDE	M1)	
	=16.4	A1	
		5	

166. 9709_w17_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	Pythagoras → $r = \sqrt{72}$ OE or $\cos 45 = \frac{6}{r} \to r = \frac{6}{\cos 45} = 6\sqrt{2}$	M1	Correct method leading to $r =$
	Arc $DC = \sqrt{72} \times \frac{1}{4}\pi = \frac{3\sqrt{2}}{2}\pi$, 2.12π, 6.66	M1 A1	Use of $s=r\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
		3	
(ii)	Area of sector-BDC is $\frac{1}{2} \times 72 \times \frac{1}{4}\pi$ (= 9π or 28.274…)	*M1	Use of $\frac{1}{2}r^2\theta$ with their r (NOT 6) and $\frac{1}{4}\pi$
	Area $Q = 9\pi - 18$ (10.274…)	DM1	Subtracts their $\frac{1}{2} \times 6 \times 6$ from their $\frac{1}{2}r^2\theta$
	Area P is ($\frac{1}{4}\pi 6^2$ – area Q) = 18	M1	Uses {$\frac{1}{4}\pi 6^2$ – (their area Q using $\sqrt{72}$)}
	Ratio is $\frac{18}{9\pi - 18}$ $\left(\frac{18}{10.274}\right) \to 1.75$	A1	
		4	

167. 9709_w17_ms_13 Q: 7

	Answer	Mark	Partial Marks
(i)	$\sin^{-1}\left(\frac{3}{5}\right) = 0.6435$ AG	M1	OR $(PBC =)\cos^{-1}\left(\frac{3}{5}\right) = 0.9273 \Rightarrow (ABP =)\frac{\pi}{2} - 0.9273 = 0.6435$ Or other valid method. Check working and diagram for evidence of incorrect method
(ii)	Use (once) of sector area $= \frac{1}{2}r^2\theta$	M1	
	Area sector $BAP = \frac{1}{2} \times 5^2 \times 0.6435 = 8.04$	A1	
	Area sector $DAQ = \frac{1}{2} \times \frac{1}{2}\pi \times 3^2 = 7.07$, Allow $\frac{9\pi}{4}$	A1	
		3	

	Answer	Mark	Partial Marks
(iii)	*EITHER:* Region = sect + sect – (rect – Δ) or sect – [rect – (sect + Δ)]	(M1	Use of correct strategy
	(Area Δ BPC =) ½ × 3 × 4 = 6 Seen	A1	
	8.04 + 7.07 – (15 – 6) = 6.11	A1)	
	OR1: Region = sector ADQ – (trap $ABPD$ – sector ABP).	(M1	Use of correct strategy
	(Area trap $ABPD$ =) ½ (5 + 1) × 3 = 9 Seen	A1	
	7.07 – (9 – 8.04) = 7.07 – 0.96 = 6.11	A1)	
	OR2: Area segment AP= 2.5686 Area segment AQ = 0.5438 Region = segment AP + segment AQ + ΔAPQ.	(M1	Use of correct strategy
	(Area ΔAPQ =) ½ × 2 × 3 = 3 Seen	A1	
	2.57 + 0.54 + 3 = 6.11	A1)	
		3	

168. 9709_m16_ms_12 Q: 9

	Answer	Mark	Partial Marks
(a) (i)	$BAO = OBA = \frac{\pi}{2} - \alpha$ $AOB = \pi - \left(\frac{\pi}{2} - \alpha\right) - \left(\frac{\pi}{2} - \alpha\right) = 2\alpha$ **AG**	**M1A1** **[2]**	Allow use of 90º or 180º Or other valid reasoning
(ii)	$\frac{1}{2}r^2(2\alpha) - \frac{1}{2}r^2 \sin 2\alpha$ oe	**B2,1,0** **[2]**	SCB1 for reversed subtraction
(b)	Use of $\alpha = \frac{\pi}{6}$, $r = 4$ 1 segment $S = \left(\frac{1}{2}\right)4^2\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}$ $= \left(\frac{8\pi}{3} - 4\sqrt{3}\right)$ Area ABC $T = \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}$ $\left(= 4\sqrt{3}\right)$ $T - 3S = \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3} - 3$ $\left[\left(\frac{1}{2}\right)4^2\left(\frac{\pi}{3}\right) - \left(\frac{1}{2}\right)4^2 \sin\frac{\pi}{3}\right]$ $16\sqrt{3} - 8\pi$ cao	**B1B1** **M1** **B1** **M1** **A1** **[6]**	 Ft *their* **(ii)**, α, r OR $AXB = \frac{T}{3} = 4\tan\frac{\pi}{6}$ or $\frac{1}{2}\left(\frac{4}{\sqrt{3}}\right)^2 \sin\frac{2\pi}{3}\left(= \frac{4\sqrt{3}}{3}\right)$ OR $3\left[\frac{T}{3} - S\right] = 3\left[\frac{4\sqrt{3}}{3} - \left(\frac{8\pi}{3} - 4\sqrt{3}\right)\right]$

169. 9709_s16_ms_11 Q: 7

	Answer	Mark	Partial Marks
(i)	$CD = r\cos\theta$, $BD = r - r\sin\theta$ oe Arc $CB = r\left(\frac{1}{2}\pi - \theta\right)$ oe $\rightarrow P = r\cos\theta + r - r\sin\theta + r\left(\frac{1}{2}\pi - \theta\right)$ oe	**B1 B1** **B1** **B1**√ [4]	allow degrees but not for last B1 √ sum – assuming trig used
(ii)	Sector $= \frac{1}{2}.5^2.\left(\frac{1}{2}\pi - 0.6\right)$ (12.135) Triangle $= \frac{1}{2}.5\cos0.6.5\sin0.6$ (5.825) $\rightarrow$ Area = 6.31 (or $\frac{1}{4}$ circle − triangle – sector)	**M1** **M1** **A1** [3]	Uses $\frac{1}{2}r^2\theta$ Uses $\frac{1}{2}bh$ with some use of trig.

170. 9709_s16_ms_12 Q: 6

	Answer	Mark	Partial Marks
(i)	$PT = r\tan\alpha$ $QT = OT - OQ = \dfrac{r}{\cos\alpha} - r$ or $\sqrt{r^2 + r^2\tan^2\alpha} - r$ Perimeter = sum of the 3 parts including $r\alpha$	**B1** **B1** **B1** [3]	
(ii)	Area of triangle = $½\times10\times10\tan\dfrac{\pi}{3}$ Area of sector = $½ \times 10^2 \times ⅓\pi$ Shaded region has area 34 (2sf)	**M1** **M1** **A1** [3]	Correct formula used, $50\sqrt{3}$,86.6 Correct formula used, $\dfrac{50\pi}{3}$, 52.36

171. 9709_w16_ms_11 Q: 3

	Answer	Mark		Partial Marks
(i)	$2r\alpha + r\alpha + 2r = 4.4r$ $\alpha = 0.8$	**M1** **A1**	[2]	At least 3 of the 4 terms required
(ii)	$½(2r)^2\, 0.8 - ½(r^2)0.8 = 30$ $(3/2)r^2 \times 0.8 = 30 \;\rightarrow\; r = 5$	**M1A1**√ **A1**	[3]	Ft through on *their* α

172. 9709_w16_ms_12 Q: 6

	Answer	Mark		Partial Marks
(i)	$\dfrac{r}{10}$ = sin 0.6 or $\dfrac{r}{10}$ = cos 0.97 or $BD = \sqrt{200 - 200\cos 1.2}\,(=11.3)$ r = 10 × 0.5646, r = 10 × sin 0.6, r = 10 × cos 0.971 or r = ½ BD → r = 5.646 AG	**M1** **A1**	[2]	Or other valid alternative.
(ii)	Major arc = 10(θ) (= 50.832) θ = 2π – 1.2 (= 5.083) or C = 2π × 10, Minor arc = 1.2 × 10 Semicircle = 5.646π (= 17.737) Major arc + semicircle = 68.6	**M1** **B1** **A1**	[3]	θ = 2π – 1.2 or π – 1.2 Implied by 5.1
(iii)	Area of major sector $= ½10^2(\theta)$ (= 254.159) Area of triangle *OBD* = ½10²sin1.2 (= 46.602) Area = semicircle + sector + triangle (= 50.1 + 254.2 + 46.6) = 351	 **M1** **M1** **A1**	[3]	 θ = 2π – 1.2 or π – 1.2 Use of ½*ab*sin*C* or other complete method

173. 9709_w16_ms_13 Q: 5

	Answer	Mark		Partial Marks
(i)	$\cos 0.9 = OE/6$ or $= \sin\left(\frac{\pi}{2} - 0.9\right)$ oe $OE = 6\cos 0.9 = 3.73$ oe AG	**M1** **A1**	[2]	Other methods possible
(ii)	Use of $(2\pi - 1.8)$ or equivalent method Area of large sector $= \frac{1}{2} \times 6^2 \times (2\pi - 1.8)$ oe Area of small sector $\frac{1}{2} \times 3.73^2 \times 1.8$ Total area = 80.7(0) + 12.5(2) = 93.2	**M1** **M1** **M1** **A1**	[4]	Expect 4.48 Or $\pi 6^2 - \frac{1}{2}6^2 1.8$. Expect 80.70 Expect 12.52 Other methods possible

174. 9709_s15_ms_11 Q: 5

	Answer	Mark	Partial Marks
(i)	$24 = r + r + r\theta$ $\rightarrow \theta = \frac{24 - 2r}{r}$ $A = \frac{1}{2} r^2\theta = \frac{24r}{2} - r^2 = 12r - r^2$. aef, ag	M1 M1A1 [3]	$($May not use $\theta)$ Attempt at $s = r\theta$ linked with 24 and r Uses A formula with θ as f(r). cao
(ii)	$(A =)36 - (r - 6)^2$	B1 B1 [2]	cao
(iii)	Greatest value of $A = 36$ $(r = 6) \rightarrow \theta = 2$	B1√ B1 [2]	Ft on **(ii).** cao, may use calculus or the discriminant on $12r - r^2$

175. 9709_s15_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$. Divides top and bottom by cos θ $\rightarrow \frac{t-1}{t+1}$	B1 [1]	Answer given.
(ii)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{1}{6}\tan\theta$ $\rightarrow \frac{t-1}{t+1} = \frac{t}{6}$ $\rightarrow t^2 - 5t + 6 = 0$ $\rightarrow t = 2$ or $t = 3$ $\rightarrow \theta = 63.4°$ or $71.6°$	B1 M1 A1 A1 [4]	Using the identity. Forms a 3 term quadratic with terms all on same side. co co

176. 9709_s15_ms_13 Q: 11

	Answer	Mark	Partial Marks
(i)	$OC = r\cos\alpha$ or $AC = r\sin\alpha$ or oe soi (Area $\Delta OAC =$) $\frac{1}{2}r^2 \sin\alpha \cos\alpha$ $\frac{1}{2}r^2 \sin\alpha\cos\alpha = \frac{1}{2}\times\frac{1}{2}r^2\alpha$ oe $\boldsymbol{\sin\alpha \cos\alpha = \frac{1}{2}\alpha}$	**M1** **A1** **M1** **A1** **[4]**	 Or e.g. $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\cos\alpha\sin\alpha = \frac{1}{4}r^2\alpha$ $\frac{1}{2}r^2\alpha - \frac{1}{2}r^2\cos\alpha\sin\alpha = \frac{1}{2}r^2\cos\alpha\sin\alpha$ AG
(ii)	Perimeter $\Delta OAC = r + r\sin\alpha + r\cos\alpha = 2.4(0)r$ Perim. $ACB = r\alpha + r\sin\alpha + r - r\cos\alpha = 2.18r$ or $2.17r$ Ratio $= \dfrac{2.4(0)}{2.18 \text{ or } 2.17} : 1 = 1.1 : 1$	**M1A1** **M1A1** **A1** **[5]**	Allow with r a number. 2.0164 gets M1A0 Allow with r a number. 0.9644 gets M1A0 Allow 2.2 www. Use of cos = 0.6, sin = 0.8, $\alpha = 0.9$ is PA 1
(iii)	54.3° cao	**B1** **[1]**	

177. 9709_w15_ms_11 Q: 7

	Answer	Mark	Partial Marks
(i)	$BC^2 = r^2 + r^2 = 2r^2 \rightarrow BC = r\sqrt{2}$	**B1** [1]	**AG**
(ii)	Area sector $BCFD = \frac{1}{4}\pi(r\sqrt{2})^2$ soi	**M1**	Expect $\frac{1}{2}\pi r^2$
	Area $\Delta\, BCAD = \frac{1}{2}(2r)r$	**M1**	Expect r^2 (could be embedded)
	Area segment $CFDA = \frac{1}{2}\pi r^2 - r^2$ .oe	**A1**	
	Area semi-circle $CADE = \frac{1}{2}\pi r^2$	**B1**	
	Shaded area $\frac{1}{2}\pi r^2 - \left(\frac{1}{2}\pi r^2 - r^2\right)$		
	or $\pi r^2 - \left(\frac{1}{2}\pi r^2 + \left(\frac{1}{2}\pi r^2 - r^2\right)\right)$	**DM1**	Depends on the area $\Delta\, BCD$
	$= r^2$	**A1** [6]	

178. 9709_w15_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	Length of $OB = \dfrac{6}{\cos 0.6} = 7.270$	**M1** [1]	ag Any valid method
(ii)	AB = 6tan0.6 or 4.1 Arc length = 7.27 × (½π – 0.6) = (7.06) Perimeter = 6 + 7.27 + 7.06 + 6tan0.6 = 24.4	**B1** **M1** **A1** [3]	Sight of in **(ii)** Use of $s = r\theta$ with sector angle
(iii)	Area of AOB = ½ × 6 × 7.27 × sin0.6 Area of OBC = ½ × 7.27² × (½π – 0.6) → area = 12.31 + 25.65 = 38.0	**M1** **M1** **A1** [3]	Use of any correct area method Use of $\frac{1}{2}r^2\theta$.

179. 9709_w15_ms_13 Q: 4

	Answer	Mark	Partial Marks
(i)	Sector $OCD = \frac{1}{2}(2r)^2\theta \ (= 2r^2\theta)$ Sector(s) $OAB/OEF = (2)\frac{1}{2}r^2(\pi - \theta)$ Total $= r^2(\pi + \theta)$	**B1** **B1** **B1** [3]	$2r^2\theta$ seen somewhere Accept with/without factor (2) **AG www**
(ii)	Arc $CD = 2r\theta$ Arc(s) AB/EF $(2)r(\pi - \theta)$ Straight edges $= 4r$ Total $2\pi r + 4r$ (which is independent of θ)	**B1** **B1** **B1** **B1** [4]	Accept with/without factor (2) Must be simplified

180. 9709_m22_ms_12 Q: 7

Question	Answer	Marks	Guidance
(a)	$\dfrac{(\sin\theta + 2\cos\theta)(\cos\theta + 2\sin\theta) - (\sin\theta - 2\cos\theta)(\cos\theta - 2\sin\theta)}{(\cos\theta - 2\sin\theta)(\cos\theta + 2\sin\theta)}$	*M1	Obtain an expression with a common denominator
	$\dfrac{5\sin\theta\cos\theta + 2\cos^2\theta + 2\sin^2\theta - (5\sin\theta\cos\theta - 2\sin^2\theta - 2\cos^2\theta)}{\cos^2\theta - 4\sin^2\theta}$ $= \dfrac{4(\cos^2\theta + \sin^2\theta)}{\cos^2\theta - 4\sin^2\theta}$	A1	
	$\dfrac{4}{\cos^2\theta - 4(1 - \cos^2\theta)}$	DM1	Use $\cos^2\theta + \sin^2\theta = 1$ twice
	$\dfrac{4}{5\cos^2\theta - 4}$	A1	AG
		4	
(b)	$\dfrac{4}{5\cos^2\theta - 4} = 5$ leading to $25\cos^2\theta = 24$ leading to $\cos\theta = \sqrt{\dfrac{24}{25}} \left[= (\pm)0.9798\right]$	M1	Make $\cos\theta$ the subject
	$\theta = 11.5°$ or $168.5°$	A1 A1 FT	 FT on 180° – 1st solution
		3	

181. 9709_m21_ms_12 Q: 3

Question	Answer	Marks	Guidance
	$\tan\theta + 2\sin\theta = 3\tan\theta - 6\sin\theta$ leading to $2\tan\theta - 8\sin\theta\ [=0]$	M1	OE
	$2\sin\theta - 8\sin\theta\cos\theta\ (=0)$ leading to $[2]\sin\theta(1-4\cos\theta)\ [=0]$	M1	
	$\cos\theta = \frac{1}{4}$	A1	Ignore $\sin\theta = 0$
	$\theta = 75.5°$ **only**	A1	
		4	

182. 9709_s21_ms_11 Q: 4

Question	Answer	Marks	Guidance
	$a = 2$	B1	
	$b = \frac{\pi}{4}$	B1	or $\frac{2\pi}{8}$
	$c = 1$	B1	
		3	

183. 9709_s21_ms_11 Q: 7

Question	Answer	Marks	Guidance
(a)	Reach $\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$ **or** $\frac{1-\sin^2\theta}{1-\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$ **or** $\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} - 2\tan^2\theta$ **or** $\sec^2\theta - \frac{2\sin^2\theta}{\cos^2\theta}$ **or** $2 - \sec^2\theta$ **or** $\frac{\cos 2\theta}{\cos^2\theta}$	M1	May start with $1-\tan^2\theta$
	$1-\tan^2\theta$	A1	AG, must show sufficient stages
		2	
(b)	$1-\tan^2\theta = 2\tan^4\theta \Rightarrow 2\tan^4\theta + \tan^2\theta - 1\ [=0]$	M1	Forming a 3-term quadratic in $\tan^2\theta$ or e.g. u
	$\tan^2\theta = 0.5$ or -1 leading to $\tan\theta = [\pm]\sqrt{0.5}$	M1	
	$\theta = 35.3°$ and $144.7°$ (AWRT)	A1	Both correct. Radians 0.615, 2.53 scores A0.
		3	

184. 9709_s21_ms_12 Q: 10

Question	Answer	Marks	Guidance
(a)	$\frac{1+\sin x}{1-\sin x}-\frac{1-\sin x}{1+\sin x}\equiv\frac{(1+\sin x)^2-(1-\sin x)^2}{(1-\sin x)(1+\sin x)}$	*M1	For using a common denominator of $(1-\sin x)(1+\sin x)$ and reasonable attempt at the numerator(s).
	$\equiv\frac{1+2\sin x+\sin^2 x-(1-2\sin x+\sin^2 x)}{(1-\sin x)(1+\sin x)}$	DM1	For multiplying out the numerators correctly. Condone sign errors for this mark.
	$\equiv\frac{4\sin x}{1-\sin^2 x}\equiv\frac{4\sin x}{\cos^2 x}$	DM1	For simplifying denominator to $\cos^2 x$.
	$\equiv\frac{4\sin x}{\cos x\cos x}\equiv\frac{4\tan x}{\cos x}$	A1	AG. Do not award A1 if undefined notation such as s, c, t or missing x's used throughout or brackets are missing.
	Alternative method for Question 10(a)		
	$\frac{4\tan x}{\cos x}\equiv\frac{4\sin x}{\cos^2 x}\equiv\frac{4\sin x}{1-\sin^2 x}$	*M1	Using $\tan x=\frac{\sin x}{\cos x}$ and $\cos^2 x=1-\sin^2 x$
	$\equiv\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}$	DM1	Separating into partial fractions.
	$\equiv 1+\frac{-2}{1+\sin x}+\frac{2}{1-\sin x}-1$	DM1	Use of 1-1 or similar
	$\equiv-\frac{1-\sin x}{1+\sin x}+\frac{1+\sin x}{1-\sin x}$	A1	
		4	

Question	Answer	Marks	Guidance
(b)	$\cos x=\frac{1}{2}$	*B1	OE. WWW.
	$x=\frac{\pi}{3}$	DB1	Or AWRT 1.05
	$x=0$ from $\tan x=0$ or $\sin x=0$	B1	WWW. Condone extra solutions outside the domain 0 to $\frac{\pi}{2}$ but B0 if any inside.
		3	

185. 9709_s21_ms_13 Q: 4

Question	Answer	Marks	Guidance
(a)	$\dfrac{\tan x+\sin x}{\tan x-\sin x}[=k]$ leading to $\dfrac{\sin x+\sin x\cos x}{\sin x-\sin x\cos x}[=k]$ or $\dfrac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}$ [=k] or $\dfrac{\tan x+\tan x\cos x}{\tan x-\tan x\cos x}$ [=k]	M1	Multiply numerator and denominator by $\cos x$, or divide numerator and denominator by tan x or sin x
	$\dfrac{\sin x(1+\cos x)}{\sin x(1-\cos x)}$ or $\dfrac{\frac{1}{\cos x}+1}{\frac{1}{\cos x}-1}\cdot\dfrac{\cos x}{\cos x}$ or $\dfrac{\tan x(1+\cos x)}{\tan x(1-\cos x)}$ leading to $\dfrac{1+\cos x}{1-\cos x}[=k]$	A1	AG, WWW
		2	
(b)	$k-k\cos x=1+\cos x$ leading to $k-1=k\cos x+\cos x$	M1	Gather like terms on LHS and RHS
	$k-1=(k+1)\cos x$ leading to $\cos x=\dfrac{k-1}{k+1}$	A1	WWW, OE
		2	
(c)	Obtaining $\cos x$ from *their* **(b)** or **(a)**	M1	Expect $\cos x=\dfrac{3}{5}$
	±0.927 (only solutions in the given range)	A1	AWRT. Accept ±0.295π
		2	

186. 9709_w21_ms_11 Q: 3

Question	Answer	Marks	Guidance
	$3\cos\theta(2\tan\theta-1)+2(2\tan\theta-1)\ [=0]$	M1	Or similar partial factorisation; condone sign errors.
	$(2\tan\theta-1)\ (3\cos\theta+2)\ [=0]$ [leading to $\tan\theta=\dfrac{1}{2}$, $\cos\theta=-\dfrac{2}{3}$]	M1	OE. At least 2 out of 4 products correct.
	26.6°, 131.8°	A1 A1	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°.
	Alternative method for question 3		
	$6\cos\theta\left(\dfrac{\sin\theta}{\cos\theta}\right)-3\cos\theta+4\left(\dfrac{\sin\theta}{\cos\theta}\right)-2\ [=0]$ $6\cos\theta\sin\theta-3\cos^2\theta+4\sin\theta-2\cos\theta\ [=0]$ $2\sin\theta(3\cos\theta+2)-\cos\theta(3\cos\theta+2)\ \ [=0]$	M1	Using $\tan\theta=\dfrac{\sin\theta}{\cos\theta}$ and reaching a partial factorisation; condone sign errors.
	$(2\sin\theta-\cos\theta)(3\cos\theta+2)\ \ [=0]$ [leading to $\tan\theta=\dfrac{1}{2}$, $\cos\theta=-\dfrac{2}{3}$]	M1	At least 2 out of 4 products correct.
	26.6°, 131.8°	A1 A1	WWW. Must be 1 d.p. or better. Final A0 if extra solution within the interval. **SC B1** No factorisation: Division by $2\tan\theta-1$ leading to 131.8° or division by $3\cos\theta+2$ or similar leading to 26.6°.
		4	

187. 9709_w21_ms_11 Q: 5

Question	Answer	Marks	Guidance
(a)	$a = 5$	**B1**	
	$b = 2$	**B1**	
	$c = 3$	**B1**	
		3	

Question	Answer	Marks	Guidance
(b)(i)	3	**B1**	
		1	
(b)(ii)	2	**B1**	
		1	

188. 9709_w21_ms_12 Q: 1

Question	Answer	Marks	Guidance
	$2\cos^2\theta - 7\cos\theta + 3[=0]$	**M1**	Forming a 3-term quadratic expression with all terms on the same side or correctly set up prior to completing the square. Allow ± sign errors.
	$(2\cos\theta - 1)(\cos\theta - 3) = 0$	**DM1**	Solving *their* 3-term quadratic using factorisation, formula or completing the square.
	$[\cos\theta = \frac{1}{2}$ or $\cos\theta = 3$ leading to] $\theta = -60°$ or $\theta = 60°$	**A1**	
	$\theta = -60°$ **and** $\theta = 60°$	**A1 FT**	FT for ± same answer between 0° and 90° or 0 and $\frac{\pi}{2}$. $\pm\frac{\pi}{3}$ or ± 1.05 AWRT scores maximum M1M1A0A1FT. **Special case:** If M1 DM0 scored then SC B1 for $\theta = -60°$ **or** $\theta = 60°$, and SC B1 FT can be awarded for $\pm$(*their* 60°) .
		4	

189. 9709_w21_ms_13 Q: 7

Question	Answer	Marks	Guidance
(a)	$\tan x + \cos x = k(\tan x - \cos x)$ leading to $\sin x + \cos^2 x = k(\sin x - \cos^2 x)$	**M1**	Use $\tan x = \frac{\sin x}{\cos x}$ and clear fraction.
	$\sin x + 1 - \sin^2 x = k\sin x - k + k\sin^2 x$	***M1**	Use $\cos^2 x = 1 - \sin^2 x$ twice to obtain an equation in sine.
	$k\sin^2 x + \sin^2 x + k\sin x - \sin x - k - 1 = 0$	**DM1**	Gather like terms on one side of the equation.
	$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0$	**A1**	AG. Factorise to obtain answer.
		4	
(b)	$5\sin^2 x + 3\sin x - 5 = 0$	**B1**	
	$\sin x = \frac{-3 \pm \sqrt{9+100}}{10}$	**M1**	Use formula or complete the square.
	$x = 48.1°, 131.9°$	**A1** **A1 FT**	AWRT. Maximum A1 if extra solutions in range. FT for 180 – *their* answer or 540 – *their* answer if sin*x* is negative If M0 given and correct answers only **SCB1B1** available. If answers in radians; 0.839, 2.30 can score **SCB1** for both.
		4	

190. 9709_m20_ms_12 Q: 5

	Answer	Mark	Partial Marks
	$2\tan\theta - 6\sin\theta + 2 = \tan\theta + 3\sin\theta + 2 \rightarrow \tan\theta - 9\sin\theta\ (=0)$	M1	Multiply by denominator and simplify
	$\sin\theta - 9\sin\theta\cos\theta\ (=0)$	M1	Multiply by $\cos\theta$
	$\sin\theta(1-9\cos\theta)\ (=0) \rightarrow \sin\theta = 0, \quad \cos\theta = \frac{1}{9}$	M1	Factorise and attempt to solve at least one of the factors = 0
	$\theta = 0$ or $83.6°$ (only answers in the given range)	A1A1	
		5	

191. 9709_m20_ms_12 Q: 11

	Answer	Mark	Partial Marks
(a)	$(\tan x - 2)(3\tan x + 1)\ (=0)$. **or** formula **or** completing square	M1	Allow reversal of signs in the factors. Must see a method
	$\tan x = 2$ or $-\frac{1}{3}$	A1	
	$x = 63.4°$(only value in range) or $161.6°$(only value in range)	B1FT B1FT	
		4	
(b)	Apply $b^2 - 4ac < 0$	M1	SOI. Expect $25 - 4(3)(k) < 0$, $\tan x$ must not be in coefficients
	$k > \frac{25}{12}$	A1	Allow $b^2 - 4ac = 0$ leading to correct $k > \frac{25}{12}$ for M1A1
		2	
(c)	$k = 0$	M1	SOI
	$\tan x = 0$ or $\frac{5}{3}$	A1	
	$x = 0°$ or $180°$ or $59.0°$	A1	All three required
		3	

192. 9709_s20_ms_11 Q: 4

(a)	$-1 \leqslant f(x) \leqslant 2$	B1 B1
		2
(b)	$k = 1$	B1
	Translation by 1 unit upwards parallel to the y-axis	B1
		2
(c)	$y = -\frac{3}{2}\cos 2x - \frac{1}{2}$	B1
		1

193. 9709_s20_ms_11 Q: 7

(a)	$\dfrac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}$	M1
	Use of $\sin^2\theta+\cos^2\theta=1 \rightarrow \dfrac{2+2\sin\theta}{\cos\theta(1+\sin\theta)} \rightarrow \dfrac{2}{\cos\theta}$.	M1A1
		3
(b)	$\dfrac{2}{\cos\theta}=\dfrac{3}{\sin\theta} \rightarrow \tan\theta=1.5$	M1
	$\theta = 0.983$ or 4.12 (**FT** on second value for 1st value + π)	A1 A1FT
		3

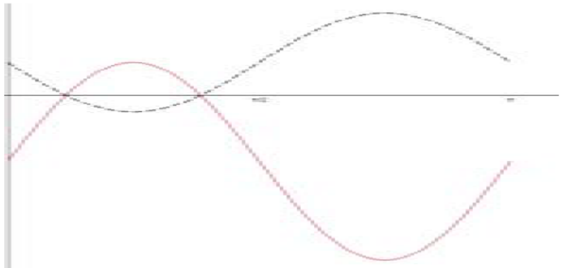
194. 9709_s20_ms_12 Q: 2

(a)	$3\cos\theta=8\tan\theta \rightarrow 3\cos\theta=\dfrac{8\sin\theta}{\cos\theta}$	M1
	$3(1-\sin^2\theta)=8\sin\theta$	M1
	$3\sin^2\theta+8\sin\theta-3=0$	A1
		3
(b)	$(3\sin\theta-1)(\sin\theta+3)=0 \rightarrow \sin\theta=⅓$	M1
	$\theta=19.5°$	A1
		2

195. 9709_s20_ms_12 Q: 7

(a)	$BC^2=r^2+4r^2-2r.2r\times\cos\left(\dfrac{\pi}{6}\right)=5r^2-2r^2\sqrt{3}$	M1
	$BC=r\sqrt{(5-2\sqrt{3})}$	A1
		2
(b)	Perimeter $=\dfrac{2\pi r}{6}+r+r\sqrt{(5-2\sqrt{3})}$	M1 A1
		2
(c)	Area = sector – triangle	
	Sector area $=\dfrac{1}{2}4r^2\dfrac{\pi}{6}$	M1
	Triangle area $=½\,r.\,2r\sin\dfrac{\pi}{6}$	M1
	Shaded area $=r^2\left(\dfrac{\pi}{3}-\dfrac{1}{2}\right)$	A1
		3

196. 9709_s20_ms_12 Q: 9

(a)	f(x) from −1 to 5	B1B1
	g(x) from −10 to 2 (**FT** from part (**a**))	B1FT
		3
(b)		B2, 1
		2
(c)	Reflect in x-axis	B1
	Stretch by factor 2 in the y direction	B1
	Translation by $-\pi$ in the x direction OR translation by $\begin{pmatrix} 0 \\ -\pi \end{pmatrix}$.	B1
		3

197. 9709_s20_ms_13 Q: 7

(a)	$\frac{\tan\theta}{1+\cos\theta}+\frac{\tan\theta}{1-\cos\theta}=\frac{\tan\theta(1-\cos\theta)+\tan\theta(1+\cos\theta)}{1-\cos^2\theta}$	M1
	$=\frac{2\tan\theta}{\sin^2\theta}$	M1
	$=\frac{2\sin\theta}{\cos\theta\sin^2\theta}$	M1
	$=\frac{2}{\sin\theta\cos\theta}$ **AG**	A1
		4
(b)	$\frac{2}{\sin\theta\cos\theta}=\frac{6\cos\theta}{\sin\theta}$	M1
	$\cos^2\theta=\frac{1}{3}\to\quad\cos\theta=(\pm)0.5774$	A1
	54.7°, 125.3° (**FT** for 180° – 1st solution)	A1 A1FT
		4

198. 9709_w20_ms_11 Q: 4

	Answer	Mark	Partial Marks
	$(y=)[3]+[2]\left[\cos\frac{1}{2}\theta\right]$	B1 B1 B1	
		3	

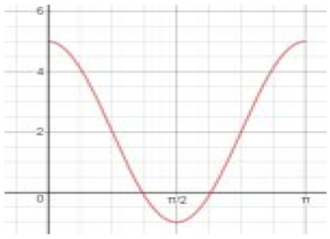
199. 9709_w20_ms_11 Q: 7

	Answer	Mark	Partial Marks
(a)	$\left(\frac{\sin\theta}{1-\sin\theta}-\frac{\sin\theta}{1+\sin\theta}=\right)\frac{\sin\theta(1+\sin\theta)-\sin\theta(1-\sin\theta)}{1-\sin^2\theta}$	*M1	Put over a single common denominator
	$\frac{2\sin^2\theta}{\cos^2\theta}$	DM1	Replace $1-\sin^2\theta$ by $\cos^2\theta$ and simplify numerator
	$2\tan^2\theta$	A1	AG
		3	
(b)	$2\tan^2\theta=8 \rightarrow \tan\theta=(\pm)2$	B1	SOI
	$(\theta =)$ 63.4°, 116.6°	B1 B1 FT	FT on 180 – 1st solution (with justification)
		3	

200. 9709_w20_ms_12 Q: 6

	Answer	Mark	Partial Marks
(a)	$\left(\frac{1}{\cos x}-\frac{\sin x}{\cos x}\right)\left(\frac{1}{\sin x}+1\right)$	B1	Uses "$\tan x = \sin x \div \cos x$" throughout
	$\left(\frac{1-\sin x}{\cos x}\right)\left(\frac{1+\sin x}{\sin x}\right)$ or $\left(\frac{1-\sin^2 x}{\cos x\sin x}\right)$	M1	Correct algebra leading to two or four terms
	$\left(\frac{\cos^2 x}{\cos x\sin x}\right)$	A1	OE. A correct expression which can be cancelled directly to $\frac{\cos x}{\sin x}$ e.g. $\frac{\cos x\left(1-\sin^2 x\right)}{\sin x\left(1-\sin^2 x\right)}$
	$\left(\frac{\cos^2 x}{\cos x\sin x}\right)=\left(\frac{\cos x}{\sin x}\right)=\frac{1}{\tan x}$	A1	AG. Must show cancelling. If x is missing throughout their working withhold this mark.
		4	
(b)	Uses **(a)** $\rightarrow \frac{1}{\tan x}=2\tan^2 x$ $\tan^3 x=\frac{1}{2}$	M1	Reducing to $\tan^3 x = k$.
	$(x =)$ 38.4°	A1	AWRT. Ignore extra answers outside the range 0 to 180° but A0 if within.
		2	

201. 9709_w20_ms_12 Q: 11

	Answer	Mark	Partial Marks
(a)	5, –1	B1 B1	Sight of each value
		2	
(b)		*B1	Needs to be a curve, not straight lines. One complete cycle starting and finishing at *their* largest value.
		DB1	One complete cycle starting and finishing at $y = 5$ and going down to $y = -1$ and starting to level off at least one end.
		2	
(c)(i)	0 solution	B1	
		1	
(c)(ii)	2 solutions	B1	
		1	
(c)(iii)	1 solution	B1	
		1	
(d)	Stretch by (scale factor) $\frac{1}{2}$, parallel to x-axis or in x direction (or horizontally)	B1	
	Translation of $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$	B1	Accept translation/shift Accept translation 4 units in positive y-direction.
		2	
(e)	Translation of $\begin{pmatrix} -\frac{\pi}{2} \\ 0 \end{pmatrix}$	B1	Accept translation/shift Accept translation $-\frac{\pi}{2}$ units in x-direction.
	Stretch by (scale factor) 2 parallel to y-axis (or vertically).	B1	
		2	

202. 9709_w20_ms_13 Q: 3

	Answer	Mark	Partial Marks
	$3\tan^4\theta + \tan^2\theta - 2\ (=0)$	M1	SOI 3-term quartic, condone sign errors for this mark only
	$(3\tan^2\theta - 2)(\tan^2\theta + 1)\ (=0)$	M1	Attempt to factorise or solve 3-term quadratic in $\tan^2\theta$.
	$\tan\theta = (\pm)\sqrt{\frac{2}{3}}$ or $(\pm)0.816$ or $(\pm)0.817$	A1	SOI Implied by final answer = 39.2° after 1st M1 scored
	39.2°, 140.8°	A1 A1 FT	FT for 2nd solution =180° – 1st solution
		5	

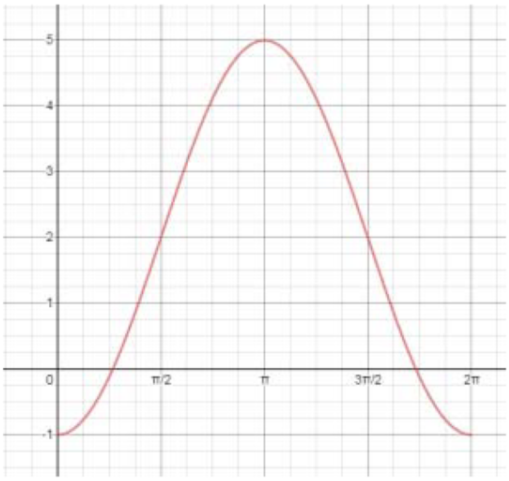
203. 9709_m19_ms_12 Q: 7

	Answer	Mark	Partial Marks
(a)	$3\left(1-\cos^2 2\theta\right)+8\cos 2\theta=0 \;\rightarrow\; 3\cos^2 2\theta-8\cos 2\theta-3\;(=0)$	M1	Use $s^2=1-c^2$ and simplify to 3-term quadratic in 2θ
	$\cos 2\theta=-\frac{1}{3}$ soi	A1	Ignore other solution
	$2\theta = 109.(47)°$ or $250.(53)°$	A1	One solution is sufficient, may be implied by either of the next solns
	$\theta = 54.7°$ or $125.3°$	A1A1ft	Ft for 180° – other solution Use of double angles leads to $3c^4-7c^2+2=0 \Rightarrow c=\pm 1/\sqrt{3}$ for M1A1A1 then A1A1 for each angle Similar marking if $3\sin^2 2\theta=-8\cos 2\theta$ is squared leading to $9\sin^4 2\theta+64\sin^2 2\theta-64=0$
		5	
(b)	$\sqrt{3}=a+\tan 0 \rightarrow a=\sqrt{3}$	B1	$b = 8$ or -4 (or -10, 14 etc) scores M1A0
	$0=\tan(-b\pi/6)+\sqrt{3}$ taken as far as $\tan^{-1}$, angle units consistent	M1	A0 if $\tan^{-1}\left(-\sqrt{3}\right)$ is not exact; (b=2 no working scores B2)
	$b=2$	A1	
		3	

204. 9709_s19_ms_11 Q: 6

	Answer	Mark	Partial Marks
(i)	LHS $=\left(\frac{1}{c}-\frac{s}{c}\right)^2=\frac{(1-s)(1-s)}{c^2}=\frac{(1-s)(1-s)}{1-s^2}$	B1	Expresses tan in terms of sin and cos
		B1	correctly $1-s^2$ as the denominator
	$=\frac{(1-s)(1-s)}{(1-s)(1+s)}$	M1	Factors and correct cancelling www
	$\frac{1-\sin x}{1+\sin x}$ AG	A1	
		4	
(ii)	Uses part **(i)** to obtain $\frac{1-\sin 2x}{1+\sin 2x}=\frac{1}{3} \rightarrow \sin 2x = ½$	M1	Realises use of $2x$ and makes $\sin 2x$ the subject
	$x=\frac{\pi}{12}$	A1	Allow decimal (0.262)
	(or) $x=\frac{5\pi}{12}$	A1	**FT** for $½\pi$ – 1st answer. Allow decimal (1.31) $\frac{\pi}{12}$ and $\frac{5\pi}{12}$ only, and no others in range. SC $\sin x = ½ \rightarrow \frac{\pi}{6}\;\frac{5\pi}{6}$ B1
		3	

205. 9709_s19_ms_11 Q: 9

	Answer	Mark	Partial Marks
(i)	$-1 \leqslant f(x) \leqslant 5$ or $[-1, 5]$ (may use y or f instead of f(x))	B1 B1	$-1 < f(x) \leqslant 5$ or $-1 \leqslant x \leqslant 5$ or $(-1,5)$ or $[5,-1]$ B1 only
		2	
(ii)		*B1	Start and end at –ve y, symmetrical, centre +ve.
	$g(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$	DB1	Shape all ok. Curves not lines. One cycle $[0,2\pi]$ Flattens at each end.
		2	
(iii)	(greatest value of $p =$) π	B1	
		1	
(iv)	$x = 2 - 3\cos x \rightarrow \cos x = ⅓(2 - x)$	M1	Attempt at $\cos x$ the subject. Use of cos^{-1}
	$g^{-1}(x) = \cos^{-1}\frac{2-x}{3}$ (may use 'y =')	A1	Must be a function of x,
		2	

206. 9709_s19_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$a^2 + 2ab + b^2, a^2 - 2ab + b^2$	B1	Correct expansions.
	$\sin^2 x + \cos^2 x = 1$ used $\rightarrow (a+b)^2 + (a-b)^2 = 1$	M1	Appropriate use of $\sin^2 x + \cos^2 x = 1$ with $(a+b)^2$ and $(a-b)^2$
	$a^2 + b^2 = ½$	A1	No evidence of $\pm 2ab$, scores 2/3
	Alternative method for question 4(i)		
	$2a = (s+c)$ & $2b = (s-c)$ or $a = ½(s+c)$ & $b = ½(s-c)$	B1	
	$a^2+b^2 = \frac{1}{4}(s+c)^2 + \frac{1}{4}(s-c)^2 = ½(s^2+c^2)$	M1	Appropriate use of $\sin^2 x + \cos^2 x = 1$
	$a^2 + b^2 = ½$	A1	Method using only $(\sin x - b)^2$ and $(a - \cos x)^2$ scores 0/3.
		3	SC B1 for assuming θ is acute giving $a = \frac{1}{\sqrt{5}} + b$ or $2\sqrt{5} - b$
(ii)	$\tan x = \frac{\sin x}{\cos x} \rightarrow \frac{a+b}{a-b} = 2$	M1	Use of $\tan x = \frac{\sin x}{\cos x}$ to form an equation in a and b only
	$a = 3b$	A1	
		2	

207. 9709_s19_ms_12 Q: 6

	Answer	Mark	Partial Marks
(i)	3, –3	**B1**	Accept ± **3**
	–½	**B1**	
	2½	**B1**	
		3	Condone misuse of inequality signs.
(ii)			Only mark the curve from $0 \rightarrow 2\pi$. If the x axis is not labelled assume that $0 \rightarrow 2\pi$ is the range shown. Labels on axes are not required.
	2 complete oscillations of a cosine curve starting with a maximum at (0,a), a>0	**B1**	
	Fully correct curve which must appear to level off at 0 and/or 2π.	**B1**	
	Line starting on positive y axis and finishing below the x axis at 2π. Must be straight.	**B1**	
		3	
(iii)	4	**B1**	
		1	

208. 9709_s19_ms_13 Q: 9

	Answer	Mark	Partial Marks
(i)	$q \leqslant f(x) \leqslant p+q$	**B1B1**	B1 each inequality – allow two separate statements Accept < , $(q, p+q)$, $[q, p+q]$ Condone y or x or f in place of f(x)
		2	
(ii)	**(a)** 2	**B1**	Allow $\frac{\pi}{4}, \frac{3\pi}{4}$
	(b) 3	**B1**	Allow $0, \frac{\pi}{2}, \pi$
	(c) 4	**B1**	Allow $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$
		3	
(iii)	$3\sin^2 2x + 2 = 4 \rightarrow \sin^2 2x = \frac{2}{3}$ soi	**M1**	
	Sin2x = (±)0.816(5). Allow $\sin 2x = (\pm)\sqrt{\frac{2}{3}}$ or $2x = \sin^{-1}(\pm)\sqrt{\frac{2}{3}}$	**A1**	OR Implied by at least one correct value for x. Allow $\sin^{-1}$ form
	($2x$ =) **at least two** of 0.955(3), 2.18(6), 4.09(7) , 5.32(8)	**A1**	Can be implied by corresponding values of x below Allow for at least two of 0.304π, 0.696π, 1.30(4)π, 1.69(6)π **OR** at least <u>two</u> of 54.7(4)°, 125.2(6)°, 234.7(4)°, 305.2(6)°
	(x =) 0.478, 1.09, 2.05, 2.66.	**A1A1**	Allow 0.152π, 0.348π, 0.652π, 0.848π SC A1 for 2 or 3 correct. SC A1 for all of 27.4º, 62.6º, 117.4º, 152.6º $\text{Sin}2x = \pm\frac{2}{3} \rightarrow x = 0.365, 1.21, 1.94, 2.78$ scores SC M1A0A0A1
		5	

209. 9709_w19_ms_11 Q: 5

	Answer	Mark	Partial Marks
(i)	$4\tan x + 3\cos x + \frac{1}{\cos x} = 0 \to 4\sin x + 3\cos^2 x + 1 = 0$	M1	Multiply by $\cos x$ or common denominator of $\cos x$
	$4\sin x + 3\left(1-\sin^2 x\right) + 1 = 0 \to 3\sin^2 x - 4\sin x - 4 = 0$	M1	Use $\cos^2 x = 1 - \sin^2 x$ and simplify to 3-term quadratic in $\sin x$
	$\sin x = -\frac{2}{3}$	A1	AG
		3	
(ii)	$2x - 20° = 221.8°, 318.2°$	M1A1	Attempt to solve $\sin(2x-20) = -2/3$(M1). At least 1 correct (A1)
	$x = 120.9°, 169.1°$	A1 A1FT	FT for 290° – other solution. **SC** A1 both answers in radians
		4	

210. 9709_w19_ms_12 Q: 6

	Answer	Mark	Partial Marks
(a)	$(2x + 1) = \tan^{-1}(⅓)$ (= 0.322 or 18.4 OR −0.339 rad or 8.7°)	*M1	Correct order of operations. Allow degrees.
	Either *their* 0.322 + π or 2π Or *their* −0.339 + $\frac{\pi}{2}$ or π	DM1	Must be in radians
	$x = 1.23$ **or** $x = 2.80$	A1	AWRT for either correct answer, accept 0.39π or 0.89π
		A1	For the second answer with no other answers between 0 and 2.8 **SC1** For both 1.2 and 2.8
		4	
(b)(i)	$5\cos^2 x - 2$	B1	Allow $a = 5$, $b = -2$
		1	
(b)(ii)	−2	B1FT	FT for sight of *their b*
	3	B1FT	FT for sight of *their a + b*
		2	

211. 9709_w19_ms_13 Q: 7

	Answer	Mark	Partial Marks
(i)	$3\cos^4\theta + 4\left(1-\cos^2\theta\right) - 3(=0)$	M1	Use $s^2 = 1 - c^2$
	$3x^2 + 4(1-x) - 3(=0) \to 3x^2 - 4x + 1(=0)$	A1	AG
		2	
(ii)	Attempt to solve for x	M1	Expect $x = 1, 1/3$
	$\cos\theta = (\pm)1,\ (\pm)0.5774$	A1	Accept $(\pm)\left(\frac{1}{\sqrt{3}}\right)$ SOI
	$(\theta =)$ 0°, 180°, 54.7°, 125.3°	A3,2,1,0	A2,1,0 if more than 4 solutions in range
		5	

212. 9709_m18_ms_12 Q: 5

	Answer	Mark	Partial Marks
(a)	$2\tan x+5=2\tan^2 x+5\tan x+3 \rightarrow 2\tan^2 x+3\tan x-2(=0)$	**M1A1**	Multiply by denom., collect like terms to produce 3-term quad. in tan*x*
	0.464 (accept 0.148π), 2.03 (accept 0.648π)	**A1A1**	**SCA1** for both in degrees 26.6°, 116.6° only
		4	
(b)	$\alpha=30°\quad k=4$	**B1B1**	Accept $\alpha=\pi/6$
		2	

213. 9709_s18_ms_11 Q: 4

	Answer	Mark	Partial Marks
(i)	$(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)\equiv\sin^3\theta+\cos^3\theta$.		Accept abbreviations s and c
	LHS $=\sin\theta+\cos\theta-\sin^2\theta\cos\theta-\sin\theta\cos^2\theta$	**M1**	Expansion
	$=\sin\theta(1-\cos^2\theta)+\cos\theta(1-\sin^2\theta)$ or $(s+c-c(1-c^2)-s(1-s^2))$	**M1A1**	Uses identity twice. Everything correct. AG
	Uses $\sin^2\theta+\cos^2\theta=1 \rightarrow \sin^3\theta+\cos^3\theta$ (RHS)		or from RHS: M1 for use of trig ID twice
	Or		
	LHS $=(\sin\theta+\cos\theta)(\sin^2\theta+\cos^2\theta-\sin\theta\cos\theta)$	**M1**	M1 for factorisation
	$=\sin^3\theta+\sin\theta\cos^2\theta-\sin^2\theta\cos\theta+\cos\theta\sin^2\theta+\cos^3\theta-\sin\theta\cos^2\theta=\sin^3\theta+\cos^3\theta$	**M1A1**	
		3	

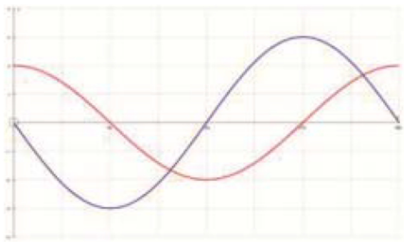
	Answer	Mark	Partial Marks
(ii)	$(\sin\theta+\cos\theta)(1-\sin\theta\cos\theta)=3\cos^3\theta \rightarrow \sin^3\theta=2\cos^3\theta$	**M1**	
	$\rightarrow \tan^3\theta=2 \rightarrow \theta=51.6°$ or $231.6°$ (only)	**A1A1FT**	Uses $\tan^3=\sin^3\div\cos^3$. A1 CAO. A1FT, 180 + their acute angle. $\tan^3\theta=0$ gets M0
		3	

214. 9709_s18_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$a+\frac{1}{2}b=5$	**B1**	Alternatively these marks can be awarded when ½ and –1 appear after *a* or *b* has been eliminated.
	$a-b=11$	**B1**	
	$\rightarrow a=7$ and $b=-4$	**B1**	
		[3]	
(ii)	$a+b$ or *their a* + *their b* (3)	**B1**	Not enough to be seen in a table of values – must be selected. Graph from their values can get both marks. **Note: Use of b^2-4ac scores 0/3**
	$a-b$ or *their a* – *their b* (11).	**B1**	
	$\rightarrow k<3$, $k>11$	**B1**	Both inequalities correct. Allow combined statement as long as correct inequalities if taken separately. Both answers correct from T & I or guesswork 3/3 otherwise 0/3
		3	

215. 9709_s18_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$2\cos x = -3\sin x \to \tan x = -\frac{2}{3}$	M1	Use of tan=sin/cos to get tan =, or other valid method to find sin or cos =. M0 for $\tan x = +/-\frac{3}{2}$
	$\to x = 146.3°$ or $326.3°$awrt	A1 A1FT	FT for 180 added to an incorrect first answer in the given range. The second A1 is withheld if any further values in the range $0° \leqslant x \leqslant 360°$ are given. Answers in radians score A0, A0.
		3	

	Answer	Mark	Partial Marks
(ii)	No labels required on either axis. Assume that the diagram is 0° to 360° unless labelled otherwise. Ignore any part of the diagram outside this range.		
		B1	Sketch of $y = 2\cos x$. One complete cycle; start and finish at top of curve at roughly the same positive y value and go below the x axis by roughly the same distance. (Can be a poor curve but not straight lines.)
		B1	Sketch of $y = -3\sin x$ One complete cycle; start and finish on the x axis, must be inverted and go below and then above the x axis by roughly the same distance. (Can be a poor curve but not straight lines.)
		B1	Fully correct answer including the sine curve with clearly larger amplitude than cosine curve. Must now be reasonable curves.
			Note: Separate diagrams can score 2/3
		3	
(iii)	$x < 146.3°$, $x > 326.3°$	B1FT B1FT	Does not need to include 0°, 360°. √ from their answers in (i) Allow combined statement as long as correct inequalities if taken separately. SC For two correct values including ft but with $\leqslant$ and $\geqslant$ B1
		2	

216. 9709_s18_ms_13 Q: 7

	Answer	Mark	Partial Marks
(a)(i)	$\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{\frac{\sin\theta^2}{\cos\theta^2} - 1}{\frac{\sin\theta^2}{\cos\theta^2} + 1}$	M1	
	$= \frac{\sin\theta^2 - \cos\theta^2}{\sin\theta^2 + \cos\theta^2}$	A1	multiplying by $\cos\theta^2$ Intermediate stage can be omitted by multiplying directly by $\cos\theta^2$
	$= \sin\theta^2 - \cos\theta^2 = \sin\theta^2 - (1 - \sin\theta^2) = 2\sin^2\theta - 1$	A1	Using $\sin\theta^2 + \cos\theta^2 = 1$ twice. Accept $a = 2,\ b = -1$
	ALT 1 $\frac{\sec^2\theta - 2}{\sec^2\theta}$	M1	ALT 2 $\frac{\tan^2\theta - 1}{\sec^2\theta}$
	$1 - \frac{2}{\sec^2\theta} = 1 - 2\cos^2\theta$	A1	$(\tan^2\theta - 1)\cos^2\theta$
	$1 - 2(1 - \sin^2\theta) = 2\sin^2\theta - 1$	A1	$\sin^2\theta - \cos^2\theta = \sin^2\theta - (1 - \sin^2\theta) = 2\sin^2\theta - 1$
		3	
(a)(ii)	$2\sin^2\theta - 1 = \frac{1}{4} \to \sin\theta = (\pm)\sqrt{\frac{5}{8}}$ or $(\pm)0.7906$	M1	OR $\frac{t^2 - 1}{t^2 + 1} = \frac{1}{4} \to 3t^2 = 5 \to t = (\pm)\sqrt{\frac{5}{3}}$ or $t = (\pm)1.2910$
	$\theta = -52.2$	A1	
		2	

	Answer	Mark	Partial Marks
(b)(i)	$\sin x = 2\cos x \rightarrow \tan x = 2$	M1	Or $\sin x = \sqrt{\frac{4}{5}}$ or $\cos x = \sqrt{\frac{1}{5}}$
	$x = 1.11$ with no additional solutions	A1	Accept 0.352π or 0.353π. Accept in co-ord form ignoring *y* co-ord
		2	
(b)(ii)	Negative answer in range $-1 < y < -0.8$	B1	
	-0.894 or -0.895 or -0.896	B1	
		2	

217. 9709_w18_ms_11 Q: 5

	Answer	Mark	Partial Marks
(i)	$\frac{(\cos\theta - 4)(5\cos\theta - 2) - 4\sin^2\theta}{\sin\theta(5\cos\theta - 2)} \quad (=0)$	M1	Accept numerator only
	$\frac{5\cos^2\theta - 22\cos\theta + 8 - 4(1-\cos^2\theta)}{\sin\theta(5\cos\theta - 2)} \quad (=0)$	M1	Simplify numerator and use $s^2 = 1 - c^2$. Accept numerator only
	$9\cos^2\theta - 22\cos\theta + 4 = 0$ www **AG**	A1	
		3	
(ii)	Attempt to solve for $\cos\theta$, (formula, completing square expected)	M1	Expect $\cos\theta = 0.1978$. Allow 2.247 in addition
	$\theta = 78.6°, 281.4°$ (only, second solution in the range)	A1A1FT	Ft for (360° – 1st solution)
		3	

218. 9709_w18_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$fg(x) = 2 - 3\cos(\frac{1}{2}x)$	B1	Correct *fg*
	$2 - 3\cos(\frac{1}{2}x) = 1 \rightarrow \cos(\frac{1}{2}x) = \frac{1}{3} \rightarrow \left(\frac{1}{2}x\right) = \cos^{-1}\left(their\frac{1}{3}\right)$	M1	M1 for correct order of operations to solve their *fg*(*x*) = 1 as far as using inverse cos expect 1.23, (or 70.5°) condone *x* =.
	$x = \mathbf{2.46}$ **awrt or** $\frac{4.7\pi}{6}$ $(0.784\pi$ **awrt**$)$	A1	One solution only in the given range, ignore answers outside the range. Answer in degrees A0.
			Alternative: Solve $f(y) = 1 \rightarrow y = 1.23 \rightarrow \frac{1}{2}x = 1.23$ **B1M1** $\rightarrow x = 2.46$ **A1**
		3	

	Answer	Mark	Partial Marks
(ii)		B1	One cycle of ± cos curve, evidence of turning at the ends not required at this stage. Can be a poor curve but not an inverted "V". If horizontal axis is not labelled mark everything to the right of the vertical axis. If axis is clearly labelled mark 0 → 2π.
		B1	Start and finish at roughly the same negative *y* value. Significantly more above the *x* axis than below or correct range implied by labels .
		B1	Fully correct. Curves not lines. Must be a reasonable curve clearly turning at both ends. Labels not required but must be appropriate if present.
		3	

219. 9709_w18_ms_12 Q: 6

	Answer	Mark	Partial Marks
(i)	In Δ ABD, $\tan\theta = \frac{9}{BD} \to BD = \frac{9}{\tan\theta}$ or $9\tan(90-\theta)$ or $9\cot\theta$ or $\sqrt{\left[(20\tan\theta)^2 - 9^2\right]}$ (Pythag) or $\frac{9\sin(90-\theta)}{\sin\theta}$ (Sine rule)	B1	Both marks can be gained for correct equated expressions.
	In Δ DBC, $\sin\theta = \frac{BD}{20} \to BD = 20\sin\theta$	B1	
	$20\sin\theta = \frac{9}{\tan\theta}$	M1	Equates their expressions for BD and uses $\sin\theta/\cos\theta = \tan\theta$ or $\cos\theta/\sin\theta = \cot\theta$ if necessary.
	$\to 20\sin^2\theta = 9\cos\theta$ AG	A1	Correct manipulation of their expression to arrive at given answer.
			SC: In Δ DBC, $\sin\theta = \frac{BD}{20} \to BD = 20\sin\theta$ B1 In Δ ABD, $BA = \frac{9}{\sin\theta}$ and $\cos\theta = \frac{BD}{BA}$ $\cos\theta = \frac{20\sin\theta}{9/\sin\theta} \to \cos\theta = \frac{20\sin^2\theta}{9}$ M1 $\to$ **$20\sin^2\theta = 9\cos\theta$** A1 Scores 3/4
		4	
(ii)	Uses $s^2 + c^2 = 1 \to 20\cos^2\theta + 9\cos\theta - 20 \ (= 0)$	M1	Uses $s^2 + c^2 = 1$ to form a three term quadratic in $\cos\theta$
	$\to \cos\theta = 0.8$	A1	www
	$\to \theta = 36.9°$ awrt	A1	www. Allow 0.644^c awrt. Ignore 323.1° or 2.50^c. Note: correct answer without working scores 0/3.
		3	

220. 9709_w18_ms_13 Q: 7

	Answer	Mark	Partial Marks
(i)	$\frac{(\tan\theta+1)(1-\cos\theta)+(\tan\theta-1)(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$ soi	M1	
	$\frac{\tan\theta - \tan\theta\cos\theta + 1 - \cos\theta + \tan\theta - 1 + \tan\theta\cos\theta - \cos\theta}{1-\cos^2\theta}$ www	A1	
	$\frac{2(\tan\theta - \cos\theta)}{\sin^2\theta}$ www **AG**	A1	
		3	

	Answer	Mark	Partial Marks
(ii)	$(2)(\tan\theta - \cos\theta)\ (=0) \to (2)\left(\frac{\sin\theta}{\cos\theta} - \cos\theta\right)\ (=0)$ soi	M1	Equate numerator to zero and replace $\tan\theta$ by $\sin\theta/\cos\theta$
	$(2)\left(\sin\theta - \left(1-\sin^2\theta\right)\right)\ (=0)$	DM1	Multiply by $\cos\theta$ and replace $\cos^2\theta$ by $1-\sin^2\theta$
	$\sin\theta = 0.618(0)$ soi	A1	Allow (√5–1)/2
	$\theta = 38.2°$	A1	Apply penalty –1 for extra solutions in range
		4	

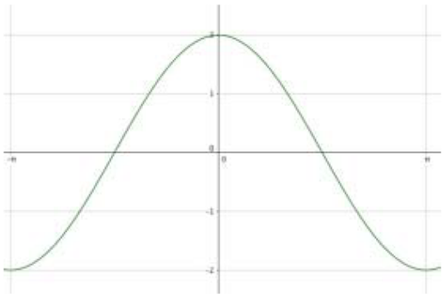
221. 9709_m17_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	$\tan x = \cos x \to \sin x = \cos^2 x$	**M1**	Use tan = sin/cos and multiply by cos
	$\sin x = 1 - \sin^2 x$	**M1**	Use $\cos^2 x = 1 - \sin^2 x$
	$\sin x = 0.6180$. Allow $(-1 + \sqrt{5})/2$	**M1**	Attempt soln of quadratic in $\sin x$. Ignore solution –1.618. Allow x = 0.618
	x-coord of A = $\sin^{-1} 0.618 = 0.666$ cao	**A1**	Must be radians. Accept 0.212π
	Total:	**4**	
(ii)	***EITHER*** x-coord of B is $\pi - \textit{their}\,0.666$	**(M1**	Expect 2.475(3). Must be radians throughout
	y-coord of B is $\tan(\textit{their}\,2.475)$ or $\cos(\textit{their}\,2.475)$	**M1**	
	$x = 2.48$, $y = -0.786$ or -0.787 cao	**A1)**	Accept $x = 0.788\pi$
	OR y-coord of B is – (cos or tan (*their* 0.666))	**(M1**	
	x-coord of B is $\cos^{-1}(\textit{their } y)$ or $\pi + \tan^{-1}(\textit{their } y)$	**M1**	
	$x = 2.48$, $y = -0.786$ or -0.787	**A1)**	Accept $x = 0.788\pi$
	Total:	**3**	

222. 9709_s17_ms_11 Q: 3

	Answer	Mark	Partial Marks
(i)	$\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} \equiv \frac{2}{\sin\theta}$.		
	$\frac{(1+c)^2 + s^2}{s(1+c)} = \frac{1+2c+c^2+s^2}{s(1+c)}$	**M1**	Correct use of fractions
	$= \frac{2+2c}{s(1+c)} = \frac{2(1+c)}{s(1+c)} \to \frac{2}{s}$	**M1 A1**	Use of trig identity, **A1** needs evidence of cancelling
	Total:	**3**	
(ii)	$\frac{2}{s} = \frac{3}{c} \to t = \frac{2}{3}$	**M1**	Use part **(i)** and $t = s \div c$, may restart from given equation
	$\to \theta = 33.7°$ or $213.7°$	**A1 A1FT**	FT for 180° + 1st answer. 2nd **A1** lost for extra solns in range
	Total:	**3**	

223. 9709_s17_ms_11 Q: 5

	Answer	Mark	Partial Marks
	$y = 2\cos x$		
(i)		**B1**	One whole cycle – starts and finishes at –ve value
		**DB1**	Smooth curve, flattens at ends and middle. Shows (0, 2).
	Total:	**2**	
(ii)	$P(\frac{\pi}{3}, 1)$ $Q(\pi, -2)$		
	$\to PQ^2 = \left(\frac{2\pi}{3}\right)^2 + 3^2 \to PQ = 3.7$	**M1 A1**	Pythagoras (on their coordinates) must be correct, OE.
	Total:	**2**	

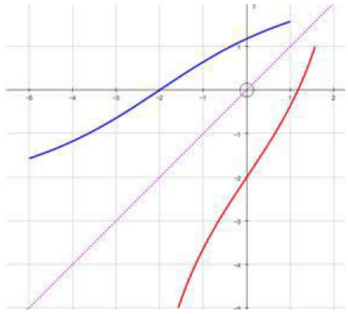
	Answer	Mark	Partial Marks
(iii)	Eqn of PQ $y - 1 = -\frac{9}{2\pi}\left(x - \frac{\pi}{3}\right)$	**M1**	Correct form of line equation or sim equations from their P & Q
	If $y = 0 \to h = \frac{5\pi}{9}$	**A1**	AG, condone $x = \frac{5\pi}{9}$
	If $x = 0 \to k = \frac{5}{2}$,	**A1**	SR: non-exact solutions **A1** for both
	Total:	**3**	

224. 9709_s17_ms_12 Q: 3

	Answer	Mark	Partial Marks
(i)	LHS = $\left(\frac{1}{c} - \frac{s}{c}\right)^2$	**M1**	Eliminates tan by replacing with $\frac{\sin}{\cos}$ leading to a function of sin and/or cos only.
	$= \frac{(1-s)^2}{1-s^2}$	**M1**	Uses $s^2 + c^2 = 1$ leading to a function of sin only.
	$= \frac{(1-s)(1-s)}{(1-s)(1+s)} = \frac{1-\sin\theta}{1+\sin\theta}$	**A1**	AG. Must show use of factors for **A1**.
	Total:	**3**	
(ii)	Uses part (i) $\to 2 - 2s = 1 + s$		
	$\to s = ⅓$	**M1**	Uses part (i) to obtain $s = k$
	$\theta = 19.5°$ or $160.5°$	**A1A1 FT**	FT from error in 19.5° Allow 0.340^c (0.3398^c) & 2.80(2) or $0.108\pi^c$ & $0.892\pi^c$ for **A1** only. Extra answers in the range lose the second **A1** if gained for 160.5°.
	Total:	**3**	

225. 9709_s17_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$3\tan\left(\frac{1}{2}x\right)=-2 \rightarrow \tan\left(\frac{1}{2}x\right)=-⅔$	M1	Attempt to obtain $\tan\left(\frac{1}{2}x\right)=k$ from $3\tan\left(\frac{1}{2}x\right)+2=0$
	$½x=-0.6\ (-0.588) \rightarrow x=-1.2$	M1 A1	$\tan^{-1}k$. Seeing $½x=-33.69°$ or $x=-67.4°$ implies **M1M1**.
			Extra answers between -1.57 &1.57 lose the **A1**. Multiples of π are acceptable (eg -0.374π)
	Total:	**3**	
(ii)	$\frac{y+2}{3}=\tan\left(\frac{1}{2}x\right)$	M1	Attempt at isolating tan(½x)
	$\rightarrow f^{-1}(x)=2\tan^{-1}\left(\frac{x+2}{3}\right)$	M1 A1	Inverse tan followed by × 2. Must be function of x for **A1**.
	$-5,1$	B1 B1	Values stated **B1** for -5, **B1** for 1.
	Total:	**5**	

	Answer	Mark	Partial Marks
(iii)		B1 B1 B1	A tan graph through the first, third and fourth quadrants. (**B1**) An invtan graph through the first, second and third quadrants.(**B1**) Two curves clearly symmetrical about $y=x$ either by sight or by exact end points. Line not required. Approximately in correct domain and range. (Not intersecting.) (**B1**) Labels on axes not required.
	Total:	**3**	

226. 9709_s17_ms_13 Q: 5

	Answer	Mark	Partial Marks
(i)	$\frac{2\sin\theta+\cos\theta}{\sin\theta+\cos\theta}=\frac{2\sin\theta}{\cos\theta}$	M1	Replace $\tan\theta$ by $\sin\theta/\cos\theta$
	$2\sin\theta\cos\theta+\cos^2\theta=2\sin^2\theta+2\sin\theta\cos\theta \Rightarrow c^2=2s^2$	M1 A1	Mult by c(s + c) or making this a common denom.. For **A1** simplification to **AG** without error or omission must be seen.
	Total:	**3**	
(ii)	$\tan^2\theta=1/2$ or $\cos^2\theta=2/3$ or $\sin^2\theta=1/3$	B1	Use $\tan\theta=s/c$ or $c^2+s^2=1$ and simplify to one of these results
	$\theta=35.3°$ or $144.7°$	B1 B1 FT	FT for 180 – other solution. SR **B1** for radians 0.615, 2.53 (0.196π, 0.804π) Extra solutions in range amongst solutions of which 2 are correct gets **B1B0**
	Total:	**3**	

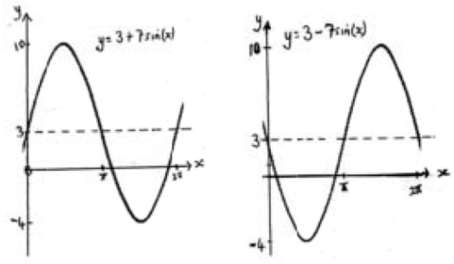
227. 9709_w17_ms_11 Q: 7

	Answer	Mark	Partial Marks
(a)	$a=-2, \quad b=3$	B1B1	
		2	
(b)(i)	$s+s^2-sc+2c+2sc-2c^2=s+sc \to s^2-2c^2+2c=0$	B1	Expansion of brackets must be correct
	$1-\cos^2\theta-2\cos^2\theta+2\cos\theta=0$	M1	Uses $s^2=1-c^2$
	$3\cos^2\theta-2\cos\theta-1=0$	A1	AG
		3	
(b)(ii)	$\cos\theta=1$ or $-\frac{1}{3}$	B1	
	$\theta=0°$ or $109.5°$ or $-109.5°$	B1B1B1 FT	FT for – *their* 109.5°
		4	

228. 9709_w17_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	*EITHER:* Uses $\tan^2 2x=\frac{\sin^2 2x}{\cos^2 2x}$	(M1	Replaces $\tan^2 2x$ by $\frac{\sin^2 2x}{\cos^2 2x}$ not $\frac{\sin^2}{\cos^2}2x$
	Uses $\sin^2 2x=(1-\cos^2 2x)$	M1	Replaces $\sin^2 2x$ by $(1-\cos^2 2x)$
	$\to 2\cos^2 2x+3\cos 2x+1=0$	A1)	AG. All correct
	OR: $\tan^2 2x=\sec^2 2x-1$	(M1	Replaces $\tan^2 2x$ by $\sec^2 2x-1$
	$\sec^2 2x=\frac{1}{\cos^2 2x}$ Multiply through by $\cos^2 2x$ and rearrange	M1	Replaces $\sec^2 2x$ by $\frac{1}{\cos^2 2x}$
	$\to 2\cos^2 2x+3\cos 2x+1=0$	A1)	AG. All correct
		3	
(ii)	$\cos 2x=-½, -1$	M1	Uses **(i)** to get values for $\cos 2x$. Allow incorrect sign(s).
	$2x=120°, 240°$ or $2x=180°1$ $x=60°$ or $120°$	A1 A1 FT	**A1** for 60° or 120° FT for 180−1st answer
	or $x=90°$	A1	Any extra answer(s) in given range only penalise fourth mark so max 3/4.
		4	

229. 9709_w17_ms_12 Q: 6

	Answer	Mark	Partial Marks
(a)(i)	$4 = a + \frac{1}{2}b$ $3 = a + b$	M1	Forming simultaneous equations and eliminating one of the variables – probably a. May still include $\sin\frac{\pi}{2}$ and / or $\sin\frac{\pi}{6}$
	$\rightarrow a = 5,\ b = -2$	A1 A1	
		3	
(a)(ii)	$ff(x) = a + b\sin(a + b\sin x)$	M1	Valid method for ff. Could be f(0) = N followed by f(N) = M.
	$ff(0) = 5 - 2\sin 5 = 6.92$	A1	
(b)	*EITHER:* $10 = c + d$ *and* $-4 = c - d$ $10 = c - d$ *and* $-4 = c + d$	(M1	Either pair of equations stated.
	$c = 3$, $d = 7$, -7 or ±7	A1 A1)	Either pair solved ISW **Alternately** c=3 **B1**, range = 14 **M1**→ d = 7, –7 or ±7 **A1**
	OR: 	(M1 A1 A1)	Either of these diagrams can be awarded M1.Correct values of c and/or d can be awarded the A1, A1
		3	

230. 9709_w17_ms_13 Q: 5

	Answer	Mark	Partial Marks
(i)	$\cos\theta + 4 + 5\sin^2\theta + 5\sin\theta - 5\sin\theta - 5\ (=0)$	M1	Multiply throughout by $\sin\theta + 1$. Accept if $5\sin\theta - 5\sin\theta$ is not seen
	$5(1-\cos^2\theta) + \cos\theta - 1\ (=0)$	M1	Use $s^2 = 1 - c^2$
	$5\cos^2\theta - \cos\theta - 4 = 0$ AG	A1	Rearrange to AG
		3	
(ii)	$\cos\theta = 1$ and -0.8	B1	Both required
	$\theta = [0°, 360°]$, $[143.1°]$, $[216.9°]$	B1 B1 B1 FT	Both solutions required for 1st mark. For 3rd mark FT for $(360° - \textit{their}\,143.1°)$ Extra solution(s) in range (e.g. 180º) among 4 correct solutions scores $\frac{3}{4}$
		4	

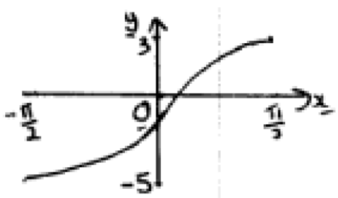
231. 9709_m16_ms_12 Q: 4

	Answer	Mark	Partial Marks
(a)	$3x = -\sqrt{3}/2$ $x = \dfrac{-\sqrt{3}}{6}$ oe	M1 A1 **[2]**	Accept −0.866 at this stage Or $\dfrac{-3}{6\sqrt{3}}$ or $\dfrac{-1}{2\sqrt{3}}$
(b)	$(2\cos\theta - 1)(\sin\theta - 1) = 0$ $\cos\theta = 1/2$ or $\sin\theta = 1$ $\theta = \pi/3$ *or* $\pi/2$	M1 A1 A1A1 **[4]**	Reasonable attempt to factorise and solve Award B1B1 www Allow 1.05, 1.57. SCA1for both 60°, 90°

232. 9709_s16_ms_11 Q: 2

	Answer	Mark	Partial Marks
	$3\sin^2\theta = 4\cos\theta - 1$ Uses $s^2 + c^2 = 1$ $\rightarrow 3c^2 + 4c - 4\ (= 0)$ $(\rightarrow c = \frac{2}{3}$ or $-2)$ $\rightarrow \theta = 48.2°$ or $311.8°$ 0.841, 5.44 rads, **A1** only (0.268π, 1.73π)	 **M1 A1** **A1 A1√** [4]	 Equation in $\cos\theta$ only. All terms on one side of (=) For 360° − 1st answer.

233. 9709_s16_ms_11 Q: 11

	Answer	Mark	Partial Marks
(i)	$f : x \rightarrow 4\sin x - 1$ for $-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}$ Range $-5 \leqslant f(x) \leqslant 3$	**B1** **B1** [2]	–5 and 3 Correct range
(ii)	$4s - 1 = 0 \rightarrow s = \frac{1}{4} \rightarrow x = 0.253$ $x = 0 \rightarrow y = -1$	**M1 A1** **B1** [3]	Makes $\sin x$ subject. Degrees **M1 A0**, (14.5°)
(iii)		**B1√** **B1** [2]	Shape from their range in (i) Flattens, curve.
(iv)	range $-\frac{1}{2}\pi \leqslant f^{-1}(x) \leqslant \frac{1}{2}\pi$ domain $-5 \leqslant x \leqslant 3$ Inverse $f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{4}\right)$	**B1** **B1√** **M1 A1** [4]	 √ on part (i) (only for 2 numerical values) Correct order of operations

234. 9709_s16_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	$\tan\left(\frac{\pi}{3}\right) = \frac{AC}{2x}$ or $\cos\left(\frac{\pi}{3}\right)\left(=\sin\frac{\pi}{6}\right)=\frac{2x}{AB}$ $\to AC = 2\sqrt{3}x$ or $AB = 4x$	**B1**	Either trig ratio
	$AM = \sqrt{13\boldsymbol{x}^2}, \sqrt{13}\boldsymbol{x}, 3.61\boldsymbol{x}$	**M1A1** [3]	Complete method.
(ii)	$\tan(M\hat{A}C) = \frac{x}{\text{Their } AC}$	**M1**	"Their AC" must be f(x), $\left(M\hat{A}C\right) \neq \theta$.
	$\theta = \frac{1}{6}\pi - \tan^{-1}\frac{1}{2\sqrt{3}}$ **AG**	**A1** [2]	Justifies $\frac{\pi}{6}$ and links MAC & θ

235. 9709_s16_ms_12 Q: 7

	Answer	Mark	Partial Marks
(i)	$\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} \equiv \frac{4}{\sin\theta\tan\theta}$		
	LHS $= \frac{1+2c+c^2-\left(1-2c+c^2\right)}{(1-c)(1+c)}$	**M1**	Attempt at combining fractions.
	$= \frac{4c}{1-c^2}$	**A1 A1**	A1 for numerator. A1 denominator
	$= \frac{4c}{s^2}$		Essential step for award of A1
	$= \frac{4}{ts}$ **AG**	**A1** [4]	
(ii)	$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$		Uses part (i) to eliminate "s" correctly.
	$\to s \times \frac{4}{ts} = 3$ $\left(\to t = \frac{4}{3}\right)$	**M1**	
	$\theta = 53.1°$ and $233.1°$	**A1 A1√** [3]	√ for 180° + 1st answer.

236. 9709_s16_ms_13 Q: 6

	Answer	Mark	Partial Marks
	$BAC = \sin^{-1}(3/5)$ or $\cos^{-1}(4/5)$ or $\tan^{-1}(3/4)$	**B1**	Accept 36.8(7)°
	$ABC = \sin^{-1}(4/5)$ or $\cos^{-1}(3/5)$ or $\tan^{-1}(4/3)$	**B1**	Accept 53.1(3)°
	$ACB = \pi/2$ (Allow 90°)	**B1**	
	Shaded area = ΔABC – sectors ($AEF + BEG + CFG$)	**M1**	
	$\Delta ABC = \frac{1}{2} \times 4 \times 3$ oe	**B1**	
	Sum sectors $= \frac{1}{2}\left[3^2 0.6435\right)$ + $2^2 0.9273 + 1^2 1.5708]$ **OR** $\frac{\pi}{360}\left[3^2 36.8(7) + 2^2 53.1(3) + 1^2 90\right]$	**M1**	
	6 – 5.536 = 0.464	**A1** [7]	

237. 9709_s16_ms_13 Q: 8

	Answer	Mark	Partial Marks
(i)	$3\sin^2 x - \cos^2 x + \cos x = 0$	**M1**	Multiply by $\cos x$
	Use $s^2 = 1 - c^2$ and simplify to 3-term quad	**M1**	Expect $4c^2 - c - 3 = 0$
	$\cos x = -3/4$ and 1	**A1**	
	$x = 2.42$ (allow 0.77π) or 0 (extra in range max 1)	**A1A1** [5]	SC1 for 0.723 (or 0.23π), π following $4c^2 + c - 3 = 0$
(ii)	$2x = 2\pi - their 2.42$ or 360 – 138.6	**B1**√	Expect $2x = 3.86$
	x = 1.21 (0.385π), 1.93 (0.614/5π), 0, π (3.14) (extra max 1)	**B1B1** [3]	Any 2 correct B1. Remaining 2 correct B1. SCB1for all 69.3, 110.7, 0, 180 (degrees) SCB1 for .361, π/2, 2.78 after $4c^2 + c - 3 = 0$

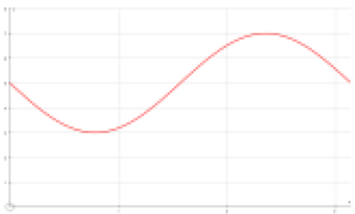
238. 9709_w16_ms_11 Q: 6

	Answer	Mark		Partial Marks
(i)	$\cos^4 x = \left(1 - \sin^2 x\right)^2 \quad = 1 - 2\sin^2 x + \sin^4 x$ AG	**B1**	[1]	Could be LHS to RHS or vice versa
(ii)	$8\sin^4 x + 1 - 2\sin^2 x + \sin^4 x = 2\left(1 - \sin^2 x\right)$	**M1**		Substitute for $\cos^4 x$ and $\cos^2 x$ or
	$9\sin^4 x = 1$	**A1**		OR sub for $\sin^4 x \to 3\cos^2 x = 2$ $\to \cos x = (\pm)\sqrt{2/3}$
	$x = 35.3^o$ (or any correct solution)	**A1**		
	Any correct second solution from 144.7°, 215.3°, 324.7°	**A1**√		Allow the first 2 **A1** marks for radians
	The remaining 2 solutions	**A1**	[5]	(0.616, 2.53, 3.76, 5.67)

239. 9709_w16_ms_12 Q: 2

	Answer	Mark		Partial Marks
(i)	$2\sin 2x = 6\cos 2x$ $\tan 2x = k$ $\rightarrow \tan 2x = 3$ or $k = 3$	**M1** **A1**	[2]	Expand and collect as far as $\tan 2x$ = a constant from sin ÷ cos soi cwo
(ii)	$x = (\tan^{-1}(their\ k)) \div 2$ $(71.6^\circ$ or $-108.4^\circ) \div 2$ $x = 35.8^\circ, -54.2^\circ$ $x = 0.624^c, -0.946^c$ $x = 0.198\pi^c, -0.301\pi^c$	**M1** **A1 A1√**	[3]	Inverse then ÷2. soi. √ on 1st answer +/ − 90° if in given range but no extra solutions in the given range. Both SR A1A0

240. 9709_w16_ms_12 Q: 10

	Answer	Mark		Partial Marks
(i)	$3 \leqslant f(x) \leqslant 7$	**B1** **B1**	[2]	Identifying both 3 and 7 or correctly stating one inequality. Completely correct statement. **NB** $3 \leqslant x \leqslant 7$ scores B1B0
(ii)		**B1*** **DB1**	[2]	One complete oscillation of a sinusoidal curve between 0 and π. All correct, initially going downwards, all above $f(x)=0$
(iii)	$5\text{-}2\sin 2x = 6 \rightarrow \sin 2x = -\frac{1}{2}$ $\rightarrow 2x = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ $\rightarrow x = \frac{7\pi}{12}$ or $\frac{11\pi}{12}$ 0.583π or 0.917π $\frac{\pi + 0.524}{2}$ or $\frac{2\pi - 0.524}{2}$ 1.83^c or 2.88^c	**M1** **A1 A1√**	[3]	Make $\sin 2x$ the subject. √ for $\frac{3\pi}{2}$ − 1[st] answer from $\sin 2x = -\frac{1}{2}$ only, if in given range SR A1A0 for both.
(iv)	$k = \frac{\pi}{4}$	**B1**	[1]	
(v)	$2\sin 2x = 5 - y \rightarrow \sin 2x = \frac{1}{2}(5 - y)$ $(g^{-1}(x)) = \frac{1}{2}\sin^{-1}\frac{(5-x)}{2}$	**M1** **M1** **A1**	[3]	Makes $\pm\sin 2x$ the subject soi by final answer. Correct order of operations including correctly dealing with "–". Must be a function of x

241. 9709_w16_ms_13 Q: 3

	Answer	Mark		Partial Marks
	$4\sin^2 x = 6\cos^2 x \Rightarrow \tan^2 x = \frac{6}{4}$ or $4\sin^2 x = 6(1-\sin^2 x)$ [tan x = (±)1.225 or sin x = (±)0.7746 or cos x = (±)0.6325] x= 50.8 (Allow 0.886 (rad)) Another angle correct $x = 50.8°, 129.2°, 230.8°, 309.2°$ [0.886, 2.25/6, 4.03, 5.40 (rad)]	**M1** **A1** **A1**√ **A1**	 [4]	Or $4(1-\cos^2 x) = 6\cos^2 x$ Or any other angle correct Ft from 1st angle (Allow radians) All 4 angles correct in degrees

242. 9709_s15_ms_11 Q: 1

	Answer	Mark	Partial Marks
	θ is obtuse , $\sin\theta = k$		
(i)	$\cos\theta = -\sqrt{(1-k^2)}$	B1 [1]	cao
(ii)	$\tan\theta = \frac{\sin\theta}{\cos\theta}$ used	M1	Used, attempt at cosine seen in (i)
	$\rightarrow \tan\theta = -\frac{k}{\sqrt{(1-k^2)}}$ aef	A1√ [2]	Ft for their cosine as a function of k only, from part (i)
(iii)	$\sin(\theta+\pi) = -k$	B1 [1]	cao

243. 9709_s15_ms_11 Q: 8

	Answer	Mark	Partial Marks
	$f : x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ for $0 \leqslant x \leqslant 2\pi$.		
(i)	$5 + 3\cos\left(\frac{1}{2}x\right) = 7$ $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$ $\frac{1}{2}x = 0.84$ $x = 1.68$ only, aef (in given range)	B1 M1A1 [3]	Makes $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$ Looks up $\cos^{-1}$ first, then ×2
(ii)		B1 B1 [2]	y always +ve, m always –ve. from (0, 8) to (2π, 2) (may be implied)
(iii)	No turning point on graph or 1:1	B1 [1]	cao, independent of graph in (ii)
(iv)	$y = 5 + 3\cos\left(\frac{1}{2}x\right)$ Order; −5, ÷3, $\cos^{-1}$, ×2 $x = 2\cos^{-1}\left(\frac{x-5}{3}\right)$	M1 M1 A1 [3]	Tries to make x subject. Correct order of operations cao

244. 9709_s15_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$f'(x) = 5 - 2x^2$ and (3, 5) $f(x) = 5x - \frac{2x^3}{3}$ $(+c)$ Uses (3, 5) $\rightarrow c = 8$	B1 M1 A1 [3]	For integral Uses the point in an integral co

245. 9709_s15_ms_12 Q: 8

	Answer	Mark	Partial Marks
(a)	1st, 2nd, nth are 56, 53 and −22 $a = 56,\ d = -3$ $-22 = 56 + (n-1)(-3)$ $\rightarrow\ n = 27$ $S_{27} = \frac{27}{2}(112 + 26(-3))$ $\rightarrow$ 459	 M1 A1 M1 A1 [4]	 Uses correct u_n formula. co Needs positive integer n Co
(b)	1[st], 2[nd], 3[rd] are $2k + 6$, $2k$ and $k + 2$.		
(i)	Either $\frac{2k}{2k+6} = \frac{k+2}{2k}$ or uses a, r and eliminates $\rightarrow 2k^2 - 10k - 12 = 0$ $\rightarrow k = 6$	 M1 DM1 A1 [3]	 Correct method for equation in k. Forms quad. or cubic equation with no brackets or fractions. Co
(ii)	$S_\infty = \frac{a}{1-r}$ with $r = \frac{2k}{2k+6}$ or $\frac{k+2}{2k}$ $(= \frac{2}{3})$ $\rightarrow$ 54	M1 A1 [2]	Needs attempt at a and r and S_∞ Co

246. 9709_s15_ms_13 Q: 4

	Answer	Mark	Partial Marks
(i)	$\tan\theta = 1/3$ $\theta = 18.4°$ only	**M1** **A1** **[2]**	 Ignore solns. outside range 0→180
(ii)	$\mathbf{tan}\,2x = (\pm)1/\sqrt{3}$ Must be sq. root soi $(x) = 15$ $(x) =$ any correct second value (75, 105, 165) $(x) =$ cao	**M1** **A1** **A1**√ **A1** **[4]**	$\sin 2x = (\pm)1/2$ **or** $\cos 2x = (\pm)\sqrt{\mathbf{3/2}}$ using $c^2 + s^2 = 1$. Not $\tan x = (\pm)\frac{1}{\sqrt{3}}$ etc. ft for (90 ± their 15) or (180 – their 15) All four correct. Extra solns in range 1

247. 9709_w15_ms_11 Q: 3

	Answer	Mark	Partial Marks
	$4x^2 + x^2 = 1/2$ soi Solve as quadratic in x^2 $x^2 = 1/4$ $x = \pm 1/2$	**B1** **M1** **A1** **A1** [4]	 E.g. $(4x^2 - 1)(2x^2 + 1)$ or $x^2 =$ formula Ignore other solution

248. 9709_w15_ms_11 Q: 4

	Answer	Mark	Partial Marks
(i)	$4\cos^2\theta + 15\sin\theta = 0$	**M1**	Replace $\tan\theta$ by $\frac{\sin\theta}{\cos\theta}$ and multiply by $\sin\theta$ or equivalent
	$4(1-s^2)+15s = 0 \rightarrow 4\sin^2\theta - 15\sin\theta - 4 = 0$	**M1A1** **[3]**	Use $c^2 = 1 - s^2$ and rearrange to **AG** (www)
(ii)	$\sin\theta = -1/4$ $\theta = 194.5$ or 345.5	**B1** **B1B1**√ [3]	Ignore other solution Ft from 1st solution, SC B1 both angles in rads (3.39 and 6.03)

249. 9709_w15_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \left(\frac{1}{s} - \frac{c}{s}\right)^2$	**M1**	Use of tan = sin/cos
	$\frac{(1-c)^2}{s^2} = \frac{(1-c)^2}{1-c^2}$	**M1**	Use of $s^2 = 1 - c^2$
	$= \frac{(1-c)(1-c)}{(1-c)(1+c)}$ or $\frac{(1-c)^2}{(1-c)(1+c)}$	**A1**	
	$\equiv \frac{1-\cos x}{1+\cos x}$	**A1** [4]	ag
(ii)	$\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$		
	$\frac{1-\cos x}{1+\cos x} = \frac{2}{5} \rightarrow \cos x \frac{3}{7}$	**M1**	Making cos*x* the subject
	$\rightarrow$ $x = 1.13$ or 5.16	**A1 A1**√ [3]	2π – 1st answer.

250. 9709_w15_ms_13 Q: 7

	Answer	Mark	Partial Marks
(a)	$1 + 3\sin^2\theta + 4\cos\theta = 0$	**M1**	Attempt to multiply by $\cos\theta$
	$1 + 3(1-\cos^2\theta) + 4\cos\theta + 0$	**M1**	Use $c^2 + s^2 = 1$
	$3\cos^2\theta - 4\cos\theta - 4 = 0$ **AG**	**A1**	
	$\cos\theta = -2/3$	**B1**	Ignore other solution
	$\theta = 131.8$ *or* 228.2	**B1B1**√ [6]	Ft for 360 – 1st soln. –1 extra solns in range
(b)	$c = b/a$ cao	**B1**	Radians 2.30 & 3.98 scores SCB1
	$d = a - b$	**B1** [2]	Allow $D = (0, a - b)$

251. 9709_m22_ms_12 Q: 3

Question	Answer	Marks	Guidance
(a)	${}^6C_2 \times (3x)^4\left(\frac{2}{x^2}\right)^2$	**B1**	Can be seen within an expansion.
	$15\times 3^4 \times 2^2$	**B1**	Identified. Powers must be correct.
	4860	**B1**	Without any power of *x*
		3	
(b)	*Their* 4860 and one other relevant term	**M1**	Using *their* 4860 and an attempt to find a term in x^{-3}
	Other term = $6C3(3x)^3\left(\frac{2}{x^2}\right)^3$ or $6C3\times 3^3\times 2^3$ or 4320	**A1**	Must be identified. If M0 scored then **SC B1** for 4320 as the only answer.
	[4860 – 4320 =] 540	**A1**	
		3	

252. 9709_m22_ms_12 Q: 4

Question	Answer	Marks	Guidance
	$ar^2 = a + d$	**B1**	
	$ar^4 = a + 5d$	**B1**	
	$a^2r^4 = a(a+5d)$ leading to $a^2 + 5ad = (a+d)^2$	***M1**	Eliminating *r* or complete elimination of *a* and *d*.
	$\left[3ad - d^2 = 0 \text{ leading to}\right]$ $d = 3a$ OR $\left[r = 2 \text{ leading to}\right]$ $d = 3a$	**A1**	
	$S_{20} = \frac{20}{2}[2a + 19\times 3a]$	**DM1**	Use of formula with *their d* in terms of *a*.
	$590a$	**A1**	
		6	

253. 9709_m21_ms_12 Q: 1

Question	Answer	Marks	Guidance
(a)	$1 + 5x + 10x^2$	**B1**	
		1	
(b)	$1 - 12x + 60x^2$	**B2, 1, 0**	B2 all correct, B1 for two correct components.
		2	
(c)	$\left(1+5x+10x^2\right)\left(1-12x+60x^2\right)$ leading to 60 – 60 + 10	**M1**	3 products required
	10	**A1**	Allow $10x^2$
		2	

254. 9709_m21_ms_12 Q: 9

Question	Answer	Marks	Guidance
(a)(i)	$\frac{\cos\theta}{1-r}=\frac{1}{\cos\theta}$	B1	
	$1-r=\cos^2\theta$ leading to $r=1-\cos^2\theta$	M1	Eliminate fractions
	$r=\sin^2\theta$ leading to 2nd term = $\cos\theta\sin^2\theta$	A1	AG
		3	
(a)(ii)	$S_{12}=\frac{\cos\left(\frac{\pi}{3}\right)\left[1-\left(\sin^2\left(\frac{\pi}{3}\right)\right)^{12}\right]}{1-\sin^2\left(\frac{\pi}{3}\right)}=\frac{0.5\left[1-(0.75)^{12}\right]}{1-0.75}$	M1	Evidence of correct substitution, use of S_n formula and attempt to evaluate
	1.937	A1	
		2	
(b)	$[d=]\cos\theta\sin^2\theta-\cos\theta$	M1	Use of $d=u_2-u_1$
	$-\frac{1}{8}$	A1	
	[85th term =] $\frac{1}{2}+84\times-\frac{1}{8}$	M1	Use of $a+84d$ with a calculated value of d
	−10	A1	
		4	

255. 9709_s21_ms_11 Q: 2

Question	Answer	Marks	Guidance
	$10(2a+19d)=405$	B1	
	$20(2a+39d)=1410$	B1	
	Solving simultaneously two equations obtained from using the correct sum formulae [$a=6$, $d=1.5$]	M1	Reach $a=$ or $d=$
	Using the correct formula for 60th term with their a and d	M1	
	60th term = 94.5	A1	OE, e.g. $\frac{189}{2}$
		5	

256. 9709_s21_ms_11 Q: 3

Question	Answer	Marks	Guidance
(a)	243	B1	
	$-810x$	B1	
	$+1080x^2$	B1	
		3	
(b)	$(4+x)^2=16+8x+x^2$	B1	
	Coefficient of x^2 is $16\times1080+8\times(-810)+243$	M1	Allow if at least 2 pairs used correctly
	11043	A1	Allow $11043x^2$
		3	

257. 9709_s21_ms_11 Q: 5

Question	Answer	Marks	Guidance
	$(-12)^2 = 8k \times 2k$	**M1**	Forming an equation in k
	$k = -3$	**A1**	
	Using correct formula for S_∞ [$r = 0.5$, $a = -384$]	**M1**	With $-1 < r < 1$
	$S_\infty = -768$	**A1**	
	Alternative method for Question 5		
	$r^2 = \frac{2k}{8k}$	**M1**	
	$r = [\pm]0.5$	**A1**	
	Using correct formula for S_∞ [$r = 0.5$, $a = -384$]	**M1**	$-1 < r < 1$
	$S_\infty = -768$	**A1**	
		4	

258. 9709_s21_ms_12 Q: 4

Question	Answer	Marks	Guidance
	[Coefficient of x or p =] 480	B1	SOI. Allow $480x$ even in an expansion.
	$\left[\text{Term in } \frac{1}{x} \text{ or } q = \right][10\times](2x)^3\left(\frac{k}{x^2}\right)^2$	M1	Appropriate term identified and selected.
	$[10 \times 2^3k^2 =]\ 80k^2$	A1	Allow $\frac{80k^2}{x}$
	$p = 6q$ used ($480 = 6 \times 80k^2$ or $80 = 80k^2$)	M1	Correct link used for *their* coefficient of x and $\frac{1}{x}$ (p and q) with no x's.
	$[k^2 = 1 \Rightarrow]\ k = \pm 1$	A1	A0 if a range of values given. Do not allow $\pm\sqrt{1}$.
		5	

259. 9709_s21_ms_12 Q: 8

Question	Answer	Marks	Guidance
(a)	$\left(a+b=2\times\frac{3}{2}a\right)\Rightarrow b=2a$	B1	SOI
	$18^2 = a(b+3)$ OE or 2 correct statements about r from the GP, e.g. $r=\frac{18}{a}$ and b + 3 = 18r or $r^2=\frac{b+3}{a}$	B1	SOI
	$324 = a(2a+3) \Rightarrow 2a^2+3a-324[=0]$ or $b^2+3b-648[=0]$ or $6r^2-r-12[=0]$ or $4d^2+3d-162[=0]$	M1	Using the correct connection between AP and GP to form a 3-term quadratic with all terms on one side.
	$(a-12)(2a+27)[=0]$ or $(b-24)(b+27)[=0]$ or $(2r-3)(3r+4)[=0]$ or $(d-6)(4d+27)[=0]$	M1	Solving *their* 3-term quadratic by factorisation, formula or completing the square to obtain answers for *a, b, r* or *d*.
	$a = 12$, $b = 24$	A1	WWW. Condone extra 'solution' $a=-13.5, b=-27$ only.
		5	

Question	Answer	Marks	Guidance
(b)	Common difference $d = 6$	B1 FT	SOI. FT *their* $\frac{a}{2}$
	$S_{20}=\frac{20}{2}(2\times12+19\times6)$	M1	Using correct sum formula with *their a, their* calculated *d* and 20.
	1380	A1	
		3	

260. 9709_s21_ms_13 Q: 7

Question	Answer	Marks	Guidance
(a)	$(a-x)^6=a^6-6a^5x+15a^4x^2-20a^3x^3+\ldots$	**B2, 1, 0**	Allow extra terms. Terms may be listed. Allow a^6x^0.
		2	
(b)	$\left(1+\frac{2}{ax}\right)(\ldots15a^4x^2-20a^3x^3+\ldots)$ leading to $\left[x^2\right](15a^4-40a^2)$	**M1**	Attempting to find 2 terms in x^2
	$15a^4-40a^2=-20$ leading to $15a^4-40a^2+20[=0]$	**A1**	Terms on one side of the equation
	$(5a^2-10)(3a^2-2)$ $[=0]$	**M1**	OE. M1 for attempted factorisation or solving for a^2 or u ($=a^2$) using e.g. formula or completing the square
	$a=\pm\sqrt{2},\ \pm\sqrt{\frac{2}{3}}$	**B1 B1**	OE exact form only If B0B0 scored then **SC B1** for $\sqrt{2},\ \sqrt{\frac{2}{3}}$ WWW or ±1.41,±0,816 WWW
		5	

261. 9709_s21_ms_13 Q: 9

Question	Answer	Marks	Guidance
(a)	$ar = \frac{24}{100} \times \frac{a}{1-r}$	M1	Form an equation using a numerical form of the percentage and correct formula for u_2 and S_∞
	$100r^2 - 100r + 24[=0]$	A1	OE. All 3 terms on one side of an equation.
	$(20r-8)(5r-3)[=0] \rightarrow r = \frac{2}{5}, \frac{3}{5}$	A1	Dependent on factors or formula seen from their quadratic.
		3	

Question	Answer	Marks	Guidance
(b)	$3 \times \{(a+4d)\} = \{(2(a+1)+11(d+1))\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $a+d=13$	A1	
	$\left[\frac{5}{2}\right] \times 3\{(2a+4d)\} = \left[\frac{5}{2}\right] \times 2\{(4(a+1)+4(d+1))\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $-a+2d=8$	A1	
	Solve 2 linear equations simultaneously	DM1	Elimination or substitution expected
	$d=7,\ a=6$	A1	**SC B1** for a=6, d=7 without complete working
		6	

262. 9709_w21_ms_11 Q: 1

Question	Answer	Marks	Guidance
(a)	$1 - \frac{1}{x} + \frac{1}{4x^2}$	B1	OE. Multiply or use binomial expansion. Allow unsimplified.
		1	
(b)	$1 + 12x + 60x^2 + 160x^3$	B2, 1, 0	Withhold 1 mark for each error; B2, 1, 0. ISW if more than 4 terms in the expansion.
		2	
(c)	*their* $(1 \times 12) +$ *their* $(-1 \times 60) +$ *their* $\left(\frac{1}{4} \times 160\right)$	M1	Attempts at least 2 products where each product contains one term from each expansion.
	$[12 - 60 + 40 =]\ -8$	A1	Allow $-8x$.
		2	

263. 9709_w21_ms_11 Q: 4

Question	Answer	Marks	Guidance
(a)	$\frac{5a}{1-\left(\pm\frac{1}{4}\right)}$	B1	Use of correct formula for sum to infinity.
	$\frac{8}{2}\left[2a + 7(-4)\right]$	*M1	Use of correct formula for sum of 8 terms and form equation; allow 1 error.
	$4a = 8a - 112$ leading to $a = [28]$	DM1	Solve equation to reach a value of a.
	$a = 28$	A1	Correct value.
		4	
(b)	*their* $28 + (k-1)(-4) = 0$	M1	Use of correct method with *their* a.
	$[k=]8$	A1	
		2	

264. 9709_w21_ms_12 Q: 5

Question	Answer	Marks	Guidance
(a)	[(3rd term – 1st term) = (5th term – 3rd term) leading to…] $-6\sqrt{3}\sin x - 2\cos x = 10\cos x + 6\sqrt{3}\sin x$ $\left[\text{leading to } -12\sqrt{3}\sin x = 12\cos x\right]$ OR [(1st term + 5th term) = 2 × 3rd term leading to…] $12\cos x = -12\sqrt{3}\sin x$	*M1	OE. From the given terms, obtain 2 expressions relating to the common difference of the arithmetic progression, attempt to solve them simultaneously and achieve an equation just involving sin*x* and cos*x*.
	Elimination of sin*x* and cos*x* to give an expression in tan*x* $\left[\tan x = -\frac{1}{\sqrt{3}}\right]$	DM1	For use of $\frac{\sin x}{\cos x} = \tan x$
	$[x =]\frac{5\pi}{6}$ only	A1	CAO. Must be exact.
		3	
(b)	$d = 2\cos x$ or $d = 2\cos(\textit{their } x)$	B1 FT	Or an equivalent expression involving sin*x* and cos*x* e.g. $-3\sqrt{3}\sin(\textit{their } x) - \cos(\textit{their } x)\ \left[=-\sqrt{3}\right]$ FT for *their x* from **(a)** only. If not $\pm\sqrt{3}$, must see unevaluated form.
	$S_{25} = \frac{25}{2}\left(2\times\left(2\cos(\textit{their } x)\right) + (25-1)\times(\textit{their } d)\right)$ $\left[=12.5\left(2\times\left(-\sqrt{3}\right) + 24\left(-\sqrt{3}\right)\right)\right]$	M1	Using the correct sum formula with $\frac{25}{2}$, (25 – 1) and with *a* replaced by either 2(cos(*their x*)) or $\pm\sqrt{3}$ and *d* replaced by either 2(cos(*their x*)) or $\pm\sqrt{3}$.
	$-325\sqrt{3}$	A1	Must be exact.
		3	

265. 9709_w21_ms_12 Q: 6

Question	Answer	Marks	Guidance
	$ar = 54$ and $\frac{a \text{ or } \textit{their } a}{1-r} = 243$	B1	SOI
	$\frac{54}{r} = 243(1-r)$ leading to $243r^2 - 243r + 54[=0]$ $[9r^2 - 9r + 2 = 0]$ OR $a^2 - 243a + 13122[=0]$	*M1	Forming a 3-term quadratic expression in *r* or *a* using *their* 2nd term and S_∞. Allow ± sign errors.
	$k(3r-2)(3r-1)[=0]$ OR $(a-81)(a-162)[=0]$	DM1	Solving *their* 3-term quadratic using factorisation, formula or completing the square. If factorising, factors must expand to give ±*their* coefficient of r^2.
	$54 \div \left(\textit{their } \frac{2}{3}\right) = a$ OR $54 \div (\textit{their } 81) = r$	DM1	May be implied by final answer.
	Tenth term $= \frac{512}{243}$ $\left[\text{OR } 81\times\left(\frac{2}{3}\right)^9 \text{ OR } 54\times\left(\frac{2}{3}\right)^8\right]$	A1	OE. Must be exact. **Special case:** If B1M1DM0DM1 scored then SC B1 can be awarded for the correct final answer.
		5	

266. 9709_w21_ms_12 Q: 8

Question	Answer	Marks	Guidance
(a)	Terms required for x^2: $-5\times 2^4\times ax+10\times 2^3\times a^2x^2\left[=-80ax+80a^2x^2\right]$	B1	Can be seen as part of an expansion or in correct products.
	2× (±*their* coefficient of x) + 4× (±*their* coefficient of x^2)	*M1	
	x^2 coefficient is $320a^2-160a=-15$ $\Rightarrow 64a^2-32a+3\Rightarrow(8a-3)(8a-1)$	DM1	Forming a 3-term quadratic in a, with all terms on the same side or correctly setting up prior to completing the square and solving using factorisation, formula or completing the square. If factorising, factors must expand to give *their* coefficient of a^2.
	$a=\frac{1}{8}$ or $a=\frac{3}{8}$	A1	OE. **Special case:** If DM0 for solving quadratic, SC B1 can be awarded for correct final answers.
		4	

Question	Answer	Marks	Guidance
(b)	$320a^2-160a=k\Rightarrow 320a^2-160a-k[=0]$	M1	Forming a 3-term quadratic in a with all terms on the same side. Allow ± sign errors.
	Their b^2-4ac [= 0], $[160^2-4\times320\times(-k)=0]$	M1	Any use of discriminant on a 3-term quadratic.
	$k=-20$	A1	
	$a=\frac{1}{4}$	B1	Condone $a=\frac{1}{4}$ from $k=20$.
	Alternative method for question 8(b)		
	$320a^2-160a=k$ and divide by 320 $\left[a^2-\frac{a}{2}=\frac{k}{320}\right]$	M1	Allow ± sign errors.
	Attempt to complete the square $\left[\left(a-\frac{1}{4}\right)^2-\frac{1}{16}=\frac{k}{320}\right]$	M1	Must have $\left(a-\frac{1}{4}\right)^2$
	$a=\frac{1}{4}$	A1	
	$k=-20$	B1	

Question	Answer	Marks	Guidance
(b) cont'd	**Alternative method for question 8(b)**		
	$320a^2-160a=k$ and attempt to differentiate LHS $[640a-160]$	M1	Allow ± sign errors.
	Setting *their* $(640a-160)=0$ and attempt to solve.	M1	
	$a=\frac{1}{4}$	A1	
	$k=-20$	B1	
		4	

267. 9709_w21_ms_13 Q: 2

Question	Answer	Marks	Guidance
(a)	$1+6ax+15a^2x^2$	B1	Terms must be evaluated.
		1	
(b)	*their* $15a^2 \pm (3\times$ *their* $6a)$	*M1	Expect $15a^2-18a$.
	$15a^2-18a=-3$	A1	
	$(3)(a-1)(5a-1)\ [=0]$	DM1	Dependent on 3-term quadratic. Or solve using formula or completing the square.
	$a=1,\ \frac{1}{5}$	A1	WWW. If DM0 awarded **SC B1** if both answers correct.
		4	

268. 9709_w21_ms_13 Q: 4

Question	Answer	Marks	Guidance
(a)	$84-3(n-1)=0$	M1	OE, SOI. Allow either = 0 or < 0 (to -3).
	Smallest n is 30	A1	**SC B2** for answer only n = 30 WWW.
		2	
(b)	$\left(\frac{2k}{2}\right)[168+(2k-1)(-3)]=\left(\frac{k}{2}\right)[168+(k-1)(-3)]$	M1 A1	M1 for forming an equation using correct formula. A1 for at least one side correct.
	$k=19$	A1	
		3	

269. 9709_m20_ms_12 Q: 6

	Answer	Mark	Partial Marks
(a)	$5C2\ [2(x)]^3\left[\frac{a}{(x^2)}\right]^2$	B1	SOI Can include correct x's
	$10\times8\times a^2\left(\frac{x^3}{x^4}\right)=720\left(\frac{1}{x}\right)$	B1	SOI Can include correct x's
	$a=\pm3$	B1	
		3	
(b)	$5C4\ [2(x)]\left[\frac{\textit{their } a}{(x^2)}\right]^4$	B1	SOI *Their a* can be just one of their values (e.g. just 3). Can gain mark from within an expansion but must use *their* value of a
	810 identified	B1	Allow with x^{-7}
		2	

270. 9709_m20_ms_12 Q: 8

	Answer	Mark	Partial Marks
(a)	2%	**B1**	
		1	
(b)	Bonus = 600 + 23× 100 = 2900	**B1**	
	Salary = 30000×1.03^{23}	**M1**	Allow 30000×1.03^{24} (60984)
	= 59207.60	**A1**	Allow answers of 3significant figure accuracy or better
	$\frac{\textit{their}\ 2900}{\textit{their}\ 59200}$	**M1**	SOI
	4.9(0)%	**A1**	
		5	

271. 9709_s20_ms_11 Q: 1

	$117 = \frac{9}{2}(2a+8d)$	**B1**
	Either $91 = S_4$ with 'a' as $a + 4d$ **or** $117 + 91 = S_{13}$ (**M1** for overall approach. **M1** for S_n)	**M1M1**
	Simultaneous Equations → $a = 7$, $d = 1.5$	**A1**
		4

272. 9709_s20_ms_11 Q: 2

	$\left(kx+\frac{1}{x}\right)^5 + \left(1-\frac{2}{x}\right)^8$ Coefficient in $\left(kx+\frac{1}{x}\right)^5 = 10 \times k^2$ (**B1** for 10. **B1** for k^2)	**B1B1**
	Coefficient in $\left(1-\frac{2}{x}\right)^8 = 8 \times -2$	**B2,1,0**
	$10k^2 - 16 = 74 \rightarrow k = 3$	**B1**
		5

273. 9709_s20_ms_11 Q: 3

(a)	$36 000 × $(1.05)^n$ (**B1** for $r = 1.05$. **M1** method for rth term)	**B1M1**
	$53 200 after 8 years.	**A1**
		3
(b)	$S_{10} = 36000\frac{(1.05^{10}-1)}{(1.05-1)}$	**M1**
	$453 000	**A1**
		2

274. 9709_s20_ms_12 Q: 1

(a)	$(2+3x)\left(x-\frac{2}{x}\right)^6$ Term in x^2 in $\left(x-\frac{2}{x}\right)^6 = 15x^4 \times \left(\frac{-2}{x}\right)^2$	B1
	Coefficient = 60	B1
		2
(b)	Constant term in $\left(x-\frac{2}{x}\right)^6 = 20x^3 \times \left(\frac{-2}{x}\right)^3$ (−160)	B2, 1
	Coefficient of x^2 in $(2+3x)\left(x-\frac{2}{x}\right)^6 = 120 - 480 = -360$	B1FT
		3

275. 9709_s20_ms_12 Q: 4

	1st term is −6, 2nd term is −4.5 (**M1** for using *k*th terms to find both *a* and *d*)	M1
	$\rightarrow a = -6, d = 1.5$	A1 A1
	$S_n = 84 \rightarrow 3n^2 - 27n - 336 = 0$	M1
	Solution $n = 16$	A1
		5

276. 9709_s20_ms_13 Q: 4

(a)	$1+5a+10a^2+10a^3+\ldots$	B1
		1
(b)	$1+5(x+x^2)+10(x+x^2)^2+10(x+x^2)^3+\ldots$ SOI	M1
	$1+5(x+x^2)+10(x^2+2x^3+\ldots)+10(x^3+\ldots)+\ldots$ SOI	A1
	$1+5x+15x^2+30x^3+\ldots$	A1
		3

277. 9709_s20_ms_13 Q: 8

(a)	$r = \cos^2\theta$ SOI	**M1**
	$S_\infty = \dfrac{\sin^2\theta}{1-\cos^2\theta}$	**M1**
	1	**A1**
		3
(b)(i)	$d = \sin^2\theta\cos^2\theta - \sin^2\theta$	**M1**
	$\sin^2\theta\left(\cos^2\theta - 1\right)$	**M1**
	$-\sin^4\theta$	**A1**
		3
(b)(ii)	Use of $S_{16} = \dfrac{16}{2}[2a+15d]$	**M1**
	With both $a = \dfrac{3}{4}$ and $d = -\dfrac{9}{16}$	**A1**
	$S_{16} = -55\dfrac{1}{2}$	**A1**
		3

278. 9709_w20_ms_11 Q: 5

	Answer	Mark	Partial Marks
(a)	$6C2\times\left[2\left(x^2\right)\right]^4\times\left[\dfrac{a}{(x)}\right]^2$, $6C3\times\left[2\left(x^2\right)\right]^3\times\left[\dfrac{a}{(x)}\right]^3$	**B1 B1**	SOI Can be seen in an expansion
	$15\times 2^4\times a^2 = 20\times 2^3\times a^3$	**M1**	SOI Terms must be from a correct series
	$a = \dfrac{15\times 2^4}{20\times 2^3} = \dfrac{3}{2}$	**A1**	OE
		4	
(b)	0	**B1**	
		1	

279. 9709_w20_ms_11 Q: 8

	Answer	Mark	Partial Marks
(a)	$S=\frac{a}{1-r}$, $2S=\frac{a}{1-R}$	B1	SOI at least one correct
	$\frac{2a}{1-r}=\frac{a}{1-R}$	M1	SOI
	$2-2R=1-r \rightarrow r=2R-1$	A1	AG
		3	
(b)	$ar^2=aR \rightarrow (a)(2R-1)^2=R(a)$	*M1	
	$4R^2-5R+1\ (=0) \rightarrow (4R-1)(R-1)\ (=0)$	DM1	Allow use of formula or completing square.
	$R=\frac{1}{4}$	A1	Allow R = 1 in addition
	$S=\frac{2a}{3}$	A1	
	Alternative method for question 8(b)		
	$ar^2=aR \rightarrow (a)r^2=½(r+1)(a)$	*M1	Eliminating 1 variable
	$2r^2-r-1\ (=0) \rightarrow (2r+1)(r-1)\ (=0)$	DM1	Allow use of formula or completing square. Must solve a quadratic.
	$r=-\frac{1}{2}$	A1	Allow r = 1 in addition
	$S=\frac{2a}{3}$	A1	
		4	

280. 9709_w20_ms_12 Q: 1

	Answer	Mark	Partial Marks
	Coefficient of x^3 in $(1-2x)^5$ is -80	B1	Can be seen in an expansion but must be simplified correctly.
	Coefficient of x^2 in $(1-2x)^5$ is 40	B1	
	Coefficient of x^3 in $(1+kx)(1-2x)^5$ is $40k-80=20$	M1	Uses the relevant two terms to form an equation = 20 and solves to find k. Condone x^3 appearing in some terms if recovered.
	$(k=)\frac{5}{2}$	A1	
		4	

281. 9709_w20_ms_12 Q: 2

	Answer	Mark	Partial Marks
	$(-2p)^2 = (2p+6) \times (p+2)$ or $\frac{-2p}{2p+6} = \frac{p+2}{-2p}$	M1	OE. Using "*a, b, c* then $b^2 = ac$" or $a = 2p+6$, $ar = -2p$ and $ar^2 = p+2$ to form a correct relationship in terms of *p* only
	$(2p^2 - 10p - 12 = 0)$ $p = 6$	A1	
	$a = 18$ and $r = -⅔$	A1	
	$(s_\infty) =$ *their a* ÷ (1 − *their r*) $\left(= 18 \div \frac{5}{3}\right)$	M1	Correct formula used with their values for *a* and *r* , $\|r\| < 1$ Both *a* & *r* from the same value of p.
	$(s_\infty =)10.8$	A1	OE. A0 if an extra solution given
			SC B2 for $s_\infty = \frac{2p+6}{1-\frac{-2p}{2p+6}}$ *or* $\frac{2p+6}{1-\frac{p+2}{-2p}}$ ignore any subsequent algebraic simplification.
		5	

282. 9709_w20_ms_12 Q: 4

	Answer	Mark	Partial Marks
	S_x *and* S_{x+1}	M1	Using two values of *n* in the given formula
	$a = 5$, $d = 2$	A1 A1	
	$a + (n-1)d > 200 \rightarrow 5 + 2(k-1) > 200$	M1	Correct formula used with their *a* and *d* to form an equation or inequality with 200, condone use of *n*
	$(k =)$ 99	A1	Condone ⩾ 99
	Alternative method for question 4		
	$\frac{n}{2}(2a + (n-1)d) \equiv n^2 + 4n \rightarrow \left(\frac{d}{2} = 1,\ a - \frac{1}{2}d = 4\right)$	M1	Equating two correct expressions of S_n and equating coefficients of *n* and n^2
	$d = 2$, $a = 5$	A1 A1	
	$a + (n-1)d > 200 \rightarrow 5 + 2(k-1) > 200$	M1	Correct formula used with their *a* and *d* to form an equation or inequality with 200, condone use of *n*
	$(k =)$ 99	A1	Condone ⩾ 99
	Alternative method for question 4		
	$sum_k - sum_{k-1} \rightarrow k^2 + 4k - (k-1)^2 - 4(k-1)$	M1 A1	Using given formula with consecutive expressions subtracted. Allow *k*+1 and *k*.
	$2k + 3 > 200$ or $= 200$	M1 A1	Simplifying to a linear equation or inequality
	$(k =)$ 99	A1	Condone ⩾ 99
		5	

283. 9709_w20_ms_13 Q: 5

	Answer	Mark	Partial Marks
	$\left[7C1a^6b(x)\right]$, $\left[7C2a^5b^2(x^2)\right]$, $[7C4a^3b^4(x^4)]$	B2, 1, 0	SOI, can be seen in an expansion.
	$\frac{7C2a^5b^2(x^2)}{7C1a^6b(x)} = \frac{7C4a^3b^4(x^4)}{7C2a^5b^2(x^2)} \rightarrow \frac{21a^5b^2}{7a^6b} = \frac{35a^3b^4}{21a^5b^2}$	M1 A1	M1 for a correct relationship OE (Ft from *their* 3 terms). For A1 binomial coefficients must be correct & evaluated.
	$\frac{a}{b} = \frac{5}{9}$	A1	OE
		5	

284. 9709_w20_ms_13 Q: 7

	Answer	Mark	Partial Marks
(a)	$(d=)-\frac{\tan^2\theta}{\cos^2\theta}-\frac{1}{\cos^2\theta}$	B1	Allow sign error(s). Award only at form (*d* =)... stage
	$-\frac{\sin^2\theta}{\cos^4\theta}-\frac{1}{\cos^2\theta}$ or $\frac{-\sec^2\theta}{\cos^2\theta}$	M1	Allow sign error(s). Can imply B1
	$\frac{-\sin^2\theta-\cos^2\theta}{\cos^4\theta}$ or $\frac{-\frac{1}{\cos^2\theta}}{\cos^2\theta}$	M1	
	$-\frac{1}{\cos^4\theta}$	A1	AG, WWW
		4	
(b)	$a=\frac{4}{3}$, $d=-\frac{16}{9}$	B1	SOI, both required. Allow $a=\frac{1}{\frac{3}{4}}$, $d=-\frac{1}{\frac{9}{16}}$
	$u_{13}=\frac{1}{\cos^2\theta}-\frac{12}{\cos^4\theta}$ = $\frac{4}{3}+12\left(\frac{-16}{9}\right)$	M1	Use of correct formula with *their a* and *their d*. The first 2 steps could be reversed
	–20	A1	WWW
		3	

285. 9709_m19_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$5C3\left[(-)(px)^3\right]$ soi	B1	Can be part of expansion. Condone omission of – sign
	$(-1)10p^3=-2160$ then ÷ and cube root	M1	Condone omission of – sign.
	$p=6$	A1	
		3	

286. 9709_m19_ms_12 Q: 6

	Answer	Mark	Partial Marks
(i)	$S_n=\frac{p(2^n-1)}{2-1}$ soi	M1	
	$p(2^n-1)>1000p \rightarrow 2^n>1001$ **AG**	A1	
		2	
(ii)	$p+(n-1)p=336$	B1	Expect $np=336$
	$\frac{n}{2}\left[2p+(n-1)p\right]=7224$	B1	Expect $\frac{n}{2}(p+np)=7224$
	Eliminate *n* or *p* to an equation in one variable	M1	Expect e.g. $168(1+n)=7224$ or $1+336/p=43$ etc
	$n=42$, $p=8$	A1A1	
		5	

287. 9709_s19_ms_11 Q: 1

	Answer	Mark	Partial Marks
(i)	Ind term $= (2x)^3 \times \left(\frac{k}{x}\right)^3 \times {}_6C_3$	B2,1,0	Term must be isolated
	$= 540 \rightarrow k = 1\frac{1}{2}$	B1	
		3	
(ii)	Term, in x^2 is $(2x)^4 \times \left(\frac{k}{x}\right)^2 \times {}_6C_2$	B1	All correct – even if k incorrect.
	$15 \times 16 \times k^2 = 540$ (or $540x^2$)	B1	**FT** For $240k^2$ or $240k^2x^2$
		2	

288. 9709_s19_ms_11 Q: 8

	Answer	Mark	Partial Marks
(a)	$ar^2 = 48$, $ar^3 = 32$, $r = \frac{2}{3}$ **or** $a = 108$	M1	Solution of the 2 eqns to give r (or a). A1 (both)
	$r = \frac{2}{3}$ **and** $a = 108$	A1	
	$S\infty = \frac{108}{\frac{1}{3}} = 324$	A1	**FT** Needs correct formula and r between −1 and 1.
		3	
(b)	Scheme A $a = 2.50$, $d = 0.16$ $S_n = 12(5 + 23\times0.16)$	M1	Correct use of either AP S_n formula.
	$S_n = 104$ tonnes.	A1	
	Scheme B $a = 2.50$, $r = 1.06$	B1	Correct value of r used in GP.
	$= \frac{2.5(1.06^{24} - 1)}{1.06 - 1}$	M1	Correct use of either S_n formula.
	$S_n = 127$ tonnes.	A1	
		5	

289. 9709_s19_ms_12 Q: 1

	Answer	Mark	Partial Marks
	For $\left(\frac{2}{x} - 3x\right)^5$ term in x is 10 or $5C_3$ or $5C2 \times \left(\frac{2}{x}\right)^2 \times (-3x)^3$ or $\left(\frac{2}{x}\right)^5 \frac{5.4.3}{3!}\left(-\frac{3}{2}x^2\right)^3$ or $(-3x)^5 \frac{5.4}{2!}\left(\frac{2}{3x^2}\right)^2$	B2,1	3 elements required. –1 for each error with or without x's. Can be seen in an expansion.
	−1080 identified	B1	Allow $-1080x$ Allow if expansion stops at this term. Allow from expanding brackets.
		3	

290. 9709_s19_ms_12 Q: 10

	Answer	Mark	Partial Marks
(a)(i)	$S_{10} = S_{15} - S_{10}$ or $S_{10} = S_{(11\text{ to }15)}$	M1	Either statement seen or implied.
	$5(2a + 9d)$ oe	B1	
	$7.5(2a + 14d) - 5(2a + 9d)$ or $\frac{5}{2}[(a + 10d) + (a+14d)]$ oe	A1	
	$d = \frac{a}{3}$ **AG**	A1	Correct answer from convincing working
		4	Condone starting with $d = \frac{a}{3}$ and evaluating both summations as 25a.
(a)(ii)	$(a+9d) = 36 + (a+3d)$	M1	Correct use of $a+(n-1)d$ twice and addition of ± 36
	$a = 18$	A1	
		2	Correct answer www scores 2/2
(b)	$S_\infty = 9 \times S_4$; $\frac{a}{1-r} = 9\frac{a(1-r^4)}{1-r}$ or $9(a + ar + ar^2 + ar^3)$	B1	May have 12 in place of a.
	$9(1 - r^n) = 1$ where $n = 3,4$ or 5	M1	Correctly deals with a and correctly eliminates '$1 - r$'
	$r^4 = \frac{8}{9}$ oe	A1	
	(5^{th} term =) $10\frac{2}{3}$ or 10.7	A1	
		4	Final answer of 10.6 suggests premature approximation – award 3/4 www.

291. 9709_s19_ms_13 Q: 2

	Answer	Mark	Partial Marks
(i)	$\frac{-5}{x} + \frac{5}{8x^3} - \frac{1}{32x^5}$ (or $-5x^{-1} + \frac{5}{8}x^{-3} - \frac{1}{32}x^{-5}$)	B1B1B1	B1 for each correct term SCB1 for both $\frac{+5}{x}$ & $\frac{+1}{32x^5}$
		3	
(ii)	$1 \times 20 + 4 \times their(-5) = 0$	M1A1	Must be from exactly 2 terms SCB1 for $20 + 20 = 40$
		2	

292. 9709_s19_ms_13 Q: 5

	Answer	Mark	Partial Marks
(i)	$\frac{x}{2}\left[2+(x-1)(-/+0.02)\right]$ or $1.01x-0.01x^2$ or $0.99x+0.01x^2$ **oe**	**B1**	Allow – or + 0.02. Allow n used
		1	
(ii)	Equate to 13 **then** *either* simplify to a 3-term quadratic equation **or** find at least 1 solution (need not be correct) to an unsimplified quadratic	**M1**	Expect $n^2-101n+1300$ (=0) or $0.99x+0.01x^2=13$. Allow x used
	16	**A1**	Ignore 85.8 or 86
		2	
(iii)	Use of $\frac{a(1-r^n)}{1-r}$ with $a=1$, $r=0.92$, $n=20$ soi	**M1**	
	(=) 10.1	**A1**	
	Use of $\left(S_\infty=\right)\frac{a}{1-r}$ with $a=1$, $r=0.92$	**M1**	OR $\frac{(1)(1-0.92^n)}{1-0.92}=13 \rightarrow 0.92^n=-0.04$ oe
	$S_\infty=12.5$ so never reaches target or < 13	**A1**	Conclusion required – 'Shown' is insufficient No solution so never reaches target or < 13
		4	

293. 9709_w19_ms_11 Q: 1

	Answer	Mark	Partial Marks
	$6C2\times(2x)^4\times\frac{1}{(4x^2)^2}$	**B1**	SOI **SC:** Condone errors in $(4^{-1})^2$ evaluation or interpretation for B1 only
	$15\times2^4\times\frac{1}{4^2}$	**B1**	Identified as required term.
	15	**B1**	
		3	

294. 9709_w19_ms_11 Q: 4

	Answer	Mark	Partial Marks
(i)	Identifies common ratio as 1.1	**B1**	
	Use of $x(1.1)^{20}=20$	**M1**	SOI
	$x\left(=\frac{20}{(1.1)^{20}}\right)=3.0$	**A1**	Accept 2.97
		3	
(ii)	$their3.0\times\frac{\left[(1.1)^{21}-1\right]}{1.1-1}\rightarrow 192$	**M1** **A1**	Correct formula used for M mark. Allow 2.97 used from **(i)** Accept 190 from $x = 2.97\ldots$
		2	

295. 9709_w19_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$\frac{6x}{2}$, $15\times\frac{x^2}{4}$	**B1 B1**	OE In or from a correct expansion. Can be implied by correct equation.
	$\times(4+ax)\rightarrow 3a+15=3$	**M1**	2 terms in x^2 equated to 3 or $3x^2$. Condone x^2 on one side only.
	$a=-4$	**A1**	CAO
		4	

296. 9709_w19_ms_12 Q: 8

	Answer	Mark	Partial Marks
(a)(i)	21st term = 13 + 20 × 1.2 = 37 (km)	B1	
		1	
(a)(ii)	S_{21}= ½×21 × (26 + 20 × 1.2) or ½ × 21 × (13 + *their* 37)	M1	A correct sum formula used with correct values for a, d and n.
	525 (km)	A1	
		2	
(b)(i)	$\frac{x-3}{x}=\frac{x-5}{x-3}$ oe (or use of a, ar and ar^2)	M1	Any valid method to obtain an equation in one variable.
	(a = *or* x =) 9	A1	
		2	
(b)(ii)	$r=\left(\frac{x-3}{x}\right)$ or $\left(\frac{x-5}{x-3}\right)$ or $\sqrt{\frac{x-5}{x}}$ = ⅔. Fourth term = 9 × (⅔)^3	M1	Any valid method to find r and the fourth term with *their* a & r.
	2⅔ **or** 2.67	A1	OE, AWRT
		2	
(b)(iii)	$S\infty=\frac{a}{1-r}=\frac{9}{1-\frac{2}{3}}$	M1	Correct formula and using *their* 'r' and 'a', with $\lvert r\rvert<1$, to obtain a numerical answer.
	27 **or** 27.0	A1	AWRT
		2	

297. 9709_w19_ms_13 Q: 1

	Answer	Mark	Partial Marks
(i)	$1 + 6y + 15y^2$	B1	CAO
		1	
(ii)	$1+6\left(px-2x^2\right)+15\left(px-2x^2\right)^2$	M1	SOI. Allow $6C1\times1^5\left(px-2x^2\right)$, $6C2\times1^4\left(px-2x^2\right)^2$
	$\left(15p^2-12\right)\left(x^2\right)=48\left(x^2\right)$	A1	1 term from each bracket and equate to 48
	$p=2$	A1	**SC:** A1 p = 4 from $15p - 12 = 48$
		3	

298. 9709_w19_ms_13 Q: 9

	Answer	Mark	Partial Marks
(i)	$\frac{5k-6}{3k}=\frac{6k-4}{5k-6} \rightarrow (5k-6)^2=3k(6k-4)$	M1	OR any valid relationship
	$25k^2-60k+36=18k^2-12k \rightarrow 7k^2-48k+36$	A1	AG
		2	
(ii)	$k=\frac{6}{7}$, 6	B1B1	Allow 0.857(1) for $\frac{6}{7}$
	When $k=\frac{6}{7}$, $r=-\frac{2}{3}$	B1	Must be exact
	When $k=6$, $r=\frac{4}{3}$	B1	
		4	
(iii)	Use of $S_\infty=\frac{a}{1-r}$ with $r=their-\frac{2}{3}$ and $a=3\times their\frac{6}{7}$	M1	Provided 0 < |*their* –2/3| < 1
	$\frac{18}{7}\div\left(1+\frac{2}{3}\right)=\frac{54}{35}$ or 1.54	A1	FT if 0.857(1) has been used in part (ii).
		2	

299. 9709_m18_ms_12 Q: 2

	Answer	Mark	Partial Marks
(i)	${}^7C_2(+/-2x)^2$ or ${}^7C_3(-2x)^3$	M1	SOI, Allow for either term correct. Allow + or – inside first bracket.
	$84(x^2)$, $-280(x^3)$	A1A1	
		3	
(ii)	$2\times(their-280)+5\times(their\,84)$ only	M1	
	–140	A1	
		2	

300. 9709_m18_ms_12 Q: 3

	Answer	Mark	Partial Marks
(i)	$40+60\times1.2 = 112$	M1A1	Allow 1.12 m. Allow **M1** for 40 + 59 × 1.2 OE
		2	

	Answer	Mark	Partial Marks
(ii)	Find rate of growth e.g. 41.2/40 or 1.2/40	*M1	SOI, Also implied by 3% , 0.03 or 1.03 seen
	$40\times(1+their\,0.03)^{60\ or\ 59}$	DM1	
	236	A1	Allow 2.36 m
		3	

301. 9709_s18_ms_11 Q: 1

	Answer	Mark	Partial Marks
(i)	$(1-2x)^5 = 1 - 10x + 40x^2$ (no penalty for extra terms)	B2,1	Loses a mark for each incorrect term. Treat $-32x^5 + 80x^4 - 80x^3$ as MR –1
		2	
(ii)	$\rightarrow (1 + ax + 2x^2)(1 - 10x + 40x^2)$		
	3 terms in x^2 $\rightarrow$ $40 - 10a + 2$	M1 A1FT	Selects 3 terms in x^2. FT from **(i)**
	Equate with 12 $\rightarrow$ a = 3	A1	CAO
		3	

302. 9709_s18_ms_11 Q: 8

	Answer	Mark	Partial Marks
(a)	$ar = 12$ and $\frac{a}{1-r} = 54$	B1 B1	CAO, OE CAO, OE
	Eliminates a or r $\rightarrow$ $9r^2 - 9r + 2 = 0$ or $a^2 - 54a + 648 = 0$	M1	Elimination leading to a 3-term quadratic in a or r
	$\rightarrow r = \frac{2}{3}$ or $\frac{1}{3}$ hence to a $\rightarrow$ $a = 18$ or 36	A1	Needs both values.
		4	
(b)	nth term of a progression is $p + qn$		
(b)(i)	first term = $p + q$. Difference = q or last term = $p + qn$	B1	Need first term and, last term or common difference
	$S_n = \frac{n}{2}(2(p+q)+(n-1)q)$ or $\frac{n}{2}(2p+q+nq)$	M1A1	Use of S_n formula with their a and d. ok unsimplified for A1.
		3	
(b)(ii)	Hence $2(2p+q+4q)=40$ and $3(2p+q+6q)=72$	DM1	Uses their S_n formula from **(i)**
	Solution $\rightarrow$ $p = 5$ and $q = 2$ [Could use S_n with a and d $\rightarrow$ $a = 7, d = 2$ $\rightarrow$ $p = 5, q = 2$.]	A1	Note: answers 7, 2 instead of 5, 2 gets M1A0 – must attempt to solve for M1
		2	

303. 9709_s18_ms_12 Q: 1

	Answer	Mark	Partial Marks
	Coefficient of x^2 in $\left(2+\frac{x}{2}\right)^6$ is ${}_6C_2 \times 2^4 \times (½)^2 (x^2)$ (= 60)	B2,1,0	3 things wanted –1 each incorrect component, must be multiplied together. Allow ${}_6C_4$, $\binom{6}{4}$ and factorial equivalents. Marks can be awarded for correct term in an expansion.
	Coefficient of x^2 in $(a+x)^5$ is ${}_5C_2 \times a^3 (x^2)$ (= $10a^3$)	B1	Marks can be awarded for correct term in an expansion.
	$\rightarrow 60 + 10a^3 = 330$	M1	Forms an equation '*their 60*' + '*their 10a³*' = 330, OK with x^2 in all three terms initially. This can be recovered by a correct answer.
	$a = 3$	A1	Condone ±3 as long as +3 is selected.
		5	

304. 9709_s18_ms_12 Q: 3

	Answer	Mark	Partial Marks
(i)	$r = 1.02$ or $\frac{102}{100}$ used in a GP in some way.	B1	Can be awarded here for use in S_n formula.
	Amount in 12th week = 8000 (*their r*)^{11} or (*their a from* $\frac{8000}{their\ r}$) (*their r*)^{12}	M1	Use of ar^{n-1} with a = 8000 & n = 12 or with a = $\frac{8000}{1.02}$ and n = 13.
	= 9950 (kg) awrt	A1	Note: Final answer of either 9943 or 9940 implies M1. Full marks can be awarded for a correct answer from a list of terms.
		3	

	Answer	Mark	Partial Marks
(ii)	In 12 weeks, total is $\frac{8000\left((their\ r)^{12}-1\right)}{\left((their\ r)-1\right)}$	M1	Use of S_n with a = 8000 and n = 12 or addition of 12 terms.
	= 107000 (kg) awrt	A1	Correct answer but no working 2/2
		2	

305. 9709_s18_ms_13 Q: 2

	Answer	Mark	Partial Marks
	${}_5C_3\ x^2\left(\frac{-2}{x}\right)^3$ SOI	B2,1,0	–80 www scores B3. Accept ${}_5C_2$.
	–80 Accept $\frac{-80}{x}$	B1	+80 without clear working scores SCB1
		3	

306. 9709_s18_ms_13 Q: 3

	Answer	Mark	Partial Marks
	$\left[\frac{a(1-r^n)}{1-r}\right][\div]\left[\frac{a}{1-r}\right]$	M1M1	Correct formulae **used** with/without r = 0.99 or n = 100.
		DM1	Allow numerical a (M1M1). 3rd M1 is for division $\frac{S_n}{S_\infty}$ (or ratio) SOI
	$1-0.99^{100}$ SOI OR $\frac{63(a)}{100(a)}$ SOI	A1	Could be shown multiplied by 100(%). Dep. on DM1
	63(%) Allow 63.4 or 0.63 but not 2 infringements (e.g. 0.634, 0.63%)	A1	n = 99 used scores Max M3. Condone a = 0.99 throughout $S_n = S_\infty$ (without division shown) scores 2 / 5
		5	

307. 9709_w18_ms_11 Q: 4

	Answer	Mark	Partial Marks
(i)	$S_{80} = \frac{80}{2}\left[12+79\times(-4)\right]$ or $\frac{80}{2}[6+l], l=-310$	M1A1	Correct formula (M1). Correct a, d and n (A1) .
	–12 160	A1	
		3	
(ii)	$S_\infty = \frac{6}{1-\frac{1}{3}} = 9$	M1A1	Correct formula with $\|r\|<1$ for M1
		2	

308. 9709_w18_ms_12 Q: 1

	Answer	Mark	Partial Marks
	For a correctly selected term in $\frac{1}{x^2}$: $(3x)^4$ or 3^4	**B1**	Components of coefficient added together 0/4 B1 expect 81
	$\times\left(\frac{2}{3x^2}\right)^3$ or $(2/3)^3$	**B1**	B1 expect 8/27
	$\times\ {}_7C_3$ or ${}_7C_4$	**B1**	B1 expect 35
	$\rightarrow$ **840 or** $\frac{\mathbf{840}}{\boldsymbol{x}^2}$	**B1**	All of the first three marks can be scored if the correct term is seen in an expansion **and it is selected** but then wrongly simplified.
			SC: A completely correct unsimplified term seen in an expansion but not correctly selected can be awarded B2.
		4	

309. 9709_w18_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	From the AP: $x-4=y-x$	**B1**	Or equivalent statement e.g. $y=2x-4$ or $x=\frac{y+4}{2}$.
	From the GP: $\frac{y}{x}=\frac{18}{y}$	**B1**	Or equivalent statement e.g. $y^2=18x$ or $x=\frac{y^2}{18}$.
	Simultaneous equations: $y^2-9y-36=0$ or $2x^2-17x+8=0$	**M1**	Elimination of either x or y to give a three term quadratic (= 0)
	OR		
	$4+d=x$, $4+2d=y\rightarrow\frac{4+2d}{4+d}=r$ oe	**B1**	
	$(4+d)\left(\frac{4+2d}{4+d}\right)^2=18\rightarrow 2d^2-d-28=0$	**M1**	Uses $ar^2=18$ to give a three term quadratic (= 0)
	$d=4$	**B1**	Condone inclusion of $d=\frac{-7}{2}$ oe

	Answer	Mark	Partial Marks
(i)	**OR**		
	From the GP $\frac{y}{x} = \frac{18}{y}$	B1	
	$\rightarrow x = \frac{y^2}{18} \rightarrow 4 + d = \frac{y^2}{18} \rightarrow d = \frac{y^2}{18} - 4$	B1	
	$4 + 2\left(\frac{y^2}{18} - 4\right) = y \rightarrow y^2 - 9y - 36 = 0$	M1	
	$\boldsymbol{x = 8, y = 12.}$	A1	Needs both x and y. Condone $\left(\frac{1}{2}, -3\right)$ included in final answer. Fully correct answer www 4/4.
		4	
(ii)	AP 4th term = **16**	B1	Condone inclusion of $\frac{-13}{2}$ oe
	GP 4th term = $8 \times \left(\frac{12}{8}\right)^3$	M1	A valid method using their x and y from **(i)**.
	= **27**	A1	Condone inclusion of −108
			Note: Answers from fortuitous $x = 8$, $y = 12$ in **(i)** can only score M1. Unidentified correct answer(s) with no working seen after valid $x = 8$, $y = 12$ to be credited with appropriate marks.
		3	

310. 9709_w18_ms_13 Q: 1

	Answer	Mark	Partial Marks
	7C5 $x^2(-2/x)^5$ soi	B1	Can appear in an expansion. Allow 7C2
	21×-32 soi	B1	Identified. Allow $(21x^2) \times (-32\,x^{-5})$. Implied by correct answer
	–672	B1	Allow $\frac{-672}{x^3}$. If 0/3 scored, 672 scores SCB1
		3	

311. 9709_w18_ms_13 Q: 5

	Answer	Mark	Partial Marks
	$a + (n-1)3 = 94$	B1	
	$\frac{n}{2}\left[2a + (n-1)3\right] = 1420$ OR $\frac{n}{2}\left[a + 94\right] = 1420$	B1	
	Attempt elimination of a or n	M1	
	$3n^2 - 191n + 2840\ (=0)$ OR $a^2 - 3a - 598\ (=0)$	A1	3-term quadratic (not necessarily all on the same side)
	$n = 40$ (only)	A1	
	$a = -23$ (only)	A1	Award 5/6 if a 2nd pair of solutions (71/3, 26) is given in addition or if given as the only answer.
		6	

312. 9709_m17_ms_12 Q: 2

	Answer	Mark	Partial Marks
	$5C2\left(\frac{1}{ax}\right)^3\left(2ax^2\right)^2$ soi	**B1**	Seen or implied. Can be part of an expansion.
	$10\times\frac{1}{a^3}\times4a^2=5$ soi	**M1A1**	M1 for identifying relevant term and equating to 5, all correct. Ignore extra *x*
	$a=8$ cao	**A1**	
	Total:	**4**	

313. 9709_s17_ms_11 Q: 1

	Answer	Mark	Partial Marks
	$(3-2x)^6$		
	Coeff of $x^2=3^4\times(-2)^2\times{}_6C_2=a$ Coeff of $x^3=3^3\times(-2)^3\times{}_6C_3=b$	**B3,2,1**	Mark unsimplified forms. –1 each independent error but powers must be correct. Ignore any '*x*' present.
	$\frac{a}{b}=-\frac{9}{8}$	**B1**	OE. Negative sign must appear before or in the numerator
	Total:	**4**	

314. 9709_s17_ms_11 Q: 4

	Answer	Mark	Partial Marks
(a)	$a=32,\ a+4d=22,\rightarrow d=-2.5$	**B1**	
	$a+(n-1)d=-28\rightarrow n=25$	**B1**	
	$S_{25}=\frac{25}{2}\left(64-2.5\times24\right)=50$	**M1 A1**	**M1** for correct formula with $n=24$ or $n=25$
	Total:	**4**	
(b)	$a=2000,\ r=1.025$	**B1**	$r=1+2.5\%$ ok if used correctly in S_n formula
	$S_{10}=2000(\frac{1.025^{10}-1}{1.025-1})=22400$ or a value which rounds to this	**M1 A1**	**M1** for correct formula with $n=9$ or $n=10$ and their *a* and r
			SR: correct answer only for $n=10$ **B3**, for $n=9$, **B1** (£19 900)
	Total:	**3**	

315. 9709_s17_ms_12 Q: 1

	Answer	Mark	Partial Marks
(i)	Coefficient of $x=80(x)$	**B2**	Correct value must be selected for both marks. SR +80 seen in an expansion gets **B1** or −80 gets **B1** <u>if selected.</u>
	Total:	**2**	
(ii)	Coefficient of $\frac{1}{x}=-40\left(\frac{1}{x}\right)$	**B2**	Correct value soi in (ii), if powers unsimplified only allow if selected. SR +40 soi in (ii) gets **B1**.
	Coefficient of $x=(1\times\text{their }80)+(3\times\text{their }-40)=-40(x)$	**M1 A1**	Links the appropriate 2 terms only for **M1**.
	Total:	**4**	

316. 9709_s17_ms_12 Q: 7

	Answer	Mark	Partial Marks
(a)	$(S_n =)\ \frac{n}{2}\left[32+(n-1)8\right]$ and 20000	M1	**M1** correct formula used with d from $16+d=24$
		A1	**A1** for correct expression linked to 20000.
	$\to n^2+3n-5000\ (<,=,>0)$	DM1	Simplification to a three term quadratic.
	$\to (n=69.2) \to$ 70 terms needed.	A1	Condone use of 20001 throughout. Correct answer from trial and improvement gets 4/4.
	Total:	4	

	Answer	Mark	Partial Marks
(b)	$a=6,\ \frac{a}{1-r}=18 \to r=⅔$	M1A1	Correct $S\infty$ formula used to find r.
	New progression $a=36$, $r=\frac{4}{9}$ oe	M1	Obtain new values for a and r by any valid method.
	New $S\infty = \frac{36}{1-\frac{4}{9}} \to 64.8$ or $\frac{324}{5}$ oe	A1	(Be aware that $r=-⅔$ leads to 64.8 but can only score M marks)
	Total:	4	

317. 9709_s17_ms_13 Q: 1

	Answer	Mark	Partial Marks
	$7C1\times 2^6\times a(x)$, $7C2\times 2^5\times\left[a(x)\right]^2$	B1 B1	SOI Can be part of expansion. Condone ax^2 only if followed by a^2. ALT $2^7\left[1+ax/2\right]^7 \to 7C1\left[a(x)/2\right]=7C2\left[a(x)/2\right]^2$
	$a=\frac{7\times 2^6}{21\times 2^5}=\frac{2}{3}$	B1	Ignore extra soln $a=0$. Allow $a=0.667$. Do not allow an extra x in the answer
	Total:	3	

318. 9709_s17_ms_13 Q: 2

	Answer	Mark	Partial Marks
(i)	$S=\frac{r^2-3r+2}{1-r}$	M1	
	$S=\frac{(r-1)(r-2)}{1-r}=\frac{-(1-r)(r-2)}{1-r}=2-r$ OR $\frac{(1-r)(2-r)}{1-r}=2-r$ OE	A1	**AG** Factors must be shown. Expressions requiring minus sign taken out must be shown
	Total:	2	
(ii)	Single range $1<S<3$ or (1, 3)	B2	Accept $1<2-r<3$. Correct range but with $S=2$ omitted scores SR **B1** $1\leqslant S\leqslant 3$ scores SR **B1**. $[S>1$ and $S<3]$ scores SR **B1**.
	Total:	2	

319. 9709_w17_ms_11 Q: 3

	Answer	Mark	Partial Marks
(i)	$\frac{3a}{1-r}=\frac{a}{1+2r}$	M1	Attempt to equate 2 sums to infinity. At least one correct
	$3+6r=1-r$	DM1	Elimination of 1 variable (a) at any stage and multiplication
	$r=-\frac{2}{7}$	A1	
		3	
(ii)	$½n\left[2\times15+(n-1)4\right]=½n\left[2\times420+(n-1)(-5)\right]$	M1A1	Attempt to equate 2 sum to n terms, at least one correct (**M1**). Both correct (**A1**)
	$n=91$	A1	
		3	

320. 9709_w17_ms_12 Q: 1

	Answer	Mark	Partial Marks
	EITHER: Term is ${}^9C_3\times2^6\times(-¼)^3$	(B1, B1, B1)	OE
	OR1: $\left(\frac{8x^3-1}{4x^2}\right)^9=\left(\frac{1}{4x^2}\right)^9(8x^3-1)^9\ or\ -\left(\frac{1}{4x^2}\right)^9(1-8x^3)^9$		
	Term is $-\frac{1}{4^9}\times{}^9C_3\times8^6$	(B1, B1, B1)	OE
	OR2: $(2x)^9\left(1-\frac{1}{8x^3}\right)^9$		
	Term is $2^9\times{}^9C_3\times\left(-\frac{1}{8}\right)^3$	(B1, B1, B1)	OE
	Selected term, which must be independent of x = − 84	B1	
		4	

321. 9709_w17_ms_12 Q: 3

	Answer	Mark	Partial Marks
(a)	Uses r = (1.05 or 105%)$^{9,\ 10\text{ or }11}$	B1	Used to multiply repeatedly or in any GP formula.
	New value = 10000×1.05^{10} = ($)16 300	B1	
		2	

	Answer	Mark	Partial Marks
(b)	*EITHER:* $n=1\rightarrow5$ $\quad a=5$	(B1	Uses n = 1 to find a
	$n=2\rightarrow13$	B1	Correct S_n for any other value of n (e.g. n = 2)
	$a+(a+d)=13\quad\rightarrow d=3$	M1 A1)	Correct method leading to d =
	OR: $\left(\frac{n}{2}\right)(2a+(n-1)d)=\left(\frac{n}{2}\right)(3n+7)$		$\left(\frac{n}{2}\right)$ maybe be ignored
	$\therefore dn+2a-d=3n+7\rightarrow dn=3n\ \rightarrow d=3$	(*M1A1	Method mark awarded for equating terms in n from correct S_n formula.
	$2a-(their\,3)=7,\quad a=5$	DM1 A1)	
		4	

322. 9709_w17_ms_13 Q: 1

	Answer	Mark	Partial Marks
	$\frac{1}{2}n\left[-24+(n-1)6\right] \sim 3000$ Note: ~ denotes <u>any</u> inequality or equality	M1	Use correct formula with RHS ≈ 3000 (e.g. 3010).
	$(3)\left(n^2-5n-1000\right)\ (\sim 0)$	A1	Rearrange into a 3-term quadratic.
	$n \sim 34.2\ (\&-29.2)$	A1	
	35. Allow $n \geqslant 35$	A1	
		4	

323. 9709_w17_ms_13 Q: 3

	Answer	Mark	Partial Marks
(i)	$6C3\left(\frac{2}{x}\right)^3(-3x)^3$ SOI also allowed if seen in an expansion	M1	Both x's can be missing.
	–4320 Identified as answer	A1	Cannot be earned retrospectively in **(ii)**.
		2	
(ii)	$6C2\left(\frac{2}{x}\right)^4\left[(-)3x\right]^2$ SOI clearly identified as critical term	M1	Both x's and minus sign can be missing.
	$15a\times16\times9-\mathit{their}\,4320(=0)$	A1 FT	FT on *their* 4320.
	$a=2$	A1	
		3	

324. 9709_m16_ms_12 Q: 1

	Answer	Mark	Partial Marks
(i)	$80(x^4)$, $-32(x^5)$	**B1B1** **[2]**	Fully simplified
(ii)	$(-32+80p)\left(x^5\right)=0$ $p=2/5$ or 32/80 oe	**M1** **A1**√ **[2]**	Attempt to mult. relevant terms & put $=0$

325. 9709_m16_ms_12 Q: 3

	Answer	Mark	Partial Marks
	$a+11d=17$ $\frac{31}{2}(2a+30d)=1023$ Solve simultaneous equations $d=4$, $a=-27$ 31st term = 93	**B1** **B1** **M1** **A1** **A1** **[5]**	 At least one correct

326. 9709_s16_ms_11 Q: 1

	Answer	Mark	Partial Marks
	$\left(x-\frac{3}{2x}\right)^6$ Term is ${}^6C_3 \times x^3 \times \left(\frac{-3}{2x}\right)^3$ $\rightarrow -67.5$ oe	 **B1 B1** **B1** [3]	 B1 for Bin coeff. B1 for rest.

327. 9709_s16_ms_11 Q: 9

	Answer	Mark	Partial Marks
(a)	$a = 50$, $ar^2 = 32$ $\rightarrow r = \frac{4}{5}$ (allow $-\frac{4}{5}$ for M mark) $\rightarrow S_\infty = 250$	**B1** **M1** **A1** [3]	seen or implied Finding r and use of correct S_∞ formula Only if $\|r\| < 1$
(b) (i)	$2\sin x$, $3\cos x$, $(\sin x + 2\cos x)$. $3c - 2s = (s + 2c) - 3c$ (or uses a, $a + d$, $a + 2d$) $\rightarrow 4c = 3s \rightarrow t = \frac{4}{3}$ SC uses $t = \frac{4}{3}$ to show $u_1 = \frac{8}{5}, u_2 = \frac{9}{5}, u_3 = \frac{10}{5}$, **B1** only	 **M1** **M1 A1** [3]	 Links terms up with AP, needs one expression for d. Arrives at $t = k$. ag
(ii)	$\rightarrow c = \frac{3}{5}$, $s = \frac{4}{5}$ or calculator $x = 53.1°$ $\rightarrow a = 1.6$, $d = 0.2$ $\rightarrow S_{20} = 70$	**M1** **M1** **A1** [3]	 Correct method for both a and d. (Uses S_n formula)

328. 9709_s16_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$\left(x-\frac{2}{x}\right)^6$ Term is ${}_6C_3 \times (-2)^3 = (-)160$ -160	 **B1** **B1** [2]	 ±160 seen anywhere
(ii)	$\left(2+\frac{3}{x^2}\right)\left(x-\frac{2}{x}\right)^6$ Term in $x^2 = {}_6C_2(-2)^2 x^2$ $= 60\ (x^2)$ Term independent of x: $= 2 \times (\text{their}-160) + 3 \times (\text{their } 60)$ -140	 **B1** **B1** **M1** **A1** [4]	 ±60 seen anywhere Using 2 products correctly

329. 9709_s16_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i) (a)	$a+(n-1)d = 10 + 29\times 2$ $= 68$	**M1** **A1** [2]	Use of nth term of an AP with a=±10, d=±2, n=30 or 29 Condone – 68 → 68
(b)	$\frac{1}{2}n(20 + 2(n-1)) = 2000$ or 0 $\rightarrow 2n^2 + 18n - 4000 = 0$ oe (n=) 41	**M1** **A1** **A1** [3]	Use of S_n formula for an AP with a=±10, d=±2 and equated to either 0 or 2000. Correct 3 term quadratic = 0.
(ii)	$r = 1.1$, oe Uses $S_{30} = \frac{10(1.1^{30}-1)}{1.1-1}$ (= 1645) Percentage lost = $\frac{2000-1645}{2000}\times 100$ $= 17.75$	**B1** **M1** **DM1** **A1** [4]	e.g. $\frac{11}{10}$, 110% Use of S_n formula for a GP, a=±10, n=30. Fully correct method for % left with "their 1645" allow 17.7 or 17.8.

330. 9709_s16_ms_13 Q: 1

	Answer	Mark	Partial Marks
	$5C2\left(\frac{1}{x}\right)^3\left(3x^2\right)^2$ $10(\times 1)\times 3^2$ $90\ (x)$	**B1** **B1** **B1** [3]	Can be seen in expansion Identified as leading to answer

331. 9709_s16_ms_13 Q: 4

	Answer	Mark	Partial Marks
	$r=\frac{3+2d}{3}$ or $\frac{3+12d}{3+2d}$ or $r^2=\frac{3+12d}{3}$	**B1**	1 correct equation in r and d only is sufficient
	$(3+2d)^2=3(3+12d)$ oe OR sub $2d = 3r - 3$	**M1**	Eliminate r or d using valid method
	$(4)d(d-6)=0$ OR $3r^2=18r-15 \rightarrow (r-1)(r-5)$	**DM1**	Attempt to simplify and solve quadratic
	$d=6$ $r=5$	**A1** **A1** [5]	Ignore $d = 0$ or $r = 1$ Do not allow −5 or ±5

332. 9709_w16_ms_11 Q: 2

	Answer	Mark		Partial Marks
	$8C6(2x)^6\left(\frac{1}{2x^3}\right)^2$ soi	**B1**		May be seen within a number of terms
	$28\times64\times\frac{1}{4}$ oe (powers and factorials evaluated)	**B2,1,0**		May be seen within a number of terms
	448	**B1**	[4]	Identified as answer

333. 9709_w16_ms_11 Q: 5

	Answer	Mark		Partial Marks
	$a(1+r)=50$ or $\frac{a(1-r^2)}{1-r}=50$	**B1**		
	$ar(1+r)=30$ or $\frac{a(1-r^3)}{1-r}=30+a$	**B1**		Or otherwise attempt to solve for r
	Eliminating a or r	**M1**		Any correct method
	$r=3/5$	**A1**		
	$a=125/4$ oe	**A1**		
	$S=625/8$ oe	**A1**√	[6]	Ft through on *their* r and a $(-1<r<1)$

334. 9709_w16_ms_12 Q: 4

	Answer	Mark		Partial Marks
	Term in $x = \frac{nx}{2}$	B1		Could be implied by use of a numerical n.
	$(3 - 2x)(1 + \frac{nx}{2} + \ldots) \to 7 = \frac{3n}{2} - 2$	M1		(Their 2 terms in x) = 7
	$\to n = 6$	A1		
	Term in $x^2 = \frac{n(n-1)}{2}\left(\frac{x}{2}\right)^2$	B1		May be implied by (their n) × (their n-1) ÷ 8.
	Coefficient of $x^2 = \frac{3n(n-1)}{8} - \frac{2n}{2}$	M1		Considers 2 terms in x^2.
	$= \frac{21}{4}$	A1	[6]	aef

335. 9709_w16_ms_12 Q: 8

	Answer	Mark		Partial Marks
(a) (i)	$200 + (15-1)(+/-5)$	M1		Use of nth term with $a = 200$, $n = 14$ or 15and $d = +/- 5$.
	$= 130$	A1	[2]	
(ii)	$\frac{n}{2}\left[400 + (n-1)(+/-5)\right] = (3050)$	M1		Use of S_n a=200 and $d = +/- 5$.
	$\to 5n^2 - 405n + 6100$ $(= 0)$	A1		
	$\to 20$	A1	[3]	
(b) (i)	ar^2, ar^5 $\to$ $r = ½$	M1 A1		Both terms correct.
	$\frac{63}{2} = \frac{a(1 - ½^6)}{½}$ $\to a = 16$	M1 A1	[4]	Use of $S_n = 31.5$ with a numeric r.
(ii)	Sum to infinity $= \frac{16}{½} = 32$	B1√	[1]	√ for their a and r with $\lvert r \rvert < 1$.

336. 9709_w16_ms_13 Q: 2

	Answer	Mark		Partial Marks
	$(+/-)20 \times 3^3 (x^3)$, $10a^3 (x^3)$ soi	B1B1		Each term can include x^3
	$-540 + 10a^3 = 100$ oe	M1		Must have 3 terms and include a^3 and 100
	$a = 4$	A1	[4]	

337. 9709_w16_ms_13 Q: 9

	Answer	Mark		Partial Marks
(a)	$\frac{6}{1-r}=\frac{12}{1+r}$	**M1**		
	$r=\frac{1}{3}$	**A1**		
	$S=9$	**A1**	[3]	
(b)	$\frac{13}{2}\left[2\cos\theta+12\sin^2\theta\right]=52$	**M1***		Use of correct formula for sum of AP
	$2\cos\theta+12(1-\cos^2\theta)=8\rightarrow 6\cos^2\theta-\cos\theta-2(=0)$	**DM1**		Use $s^2=1-c^2$ & simplify to 3-term quad
	$\cos\theta=2/3$ or $-1/2$ soi	**A1**		
	$\theta=0.841$, 2.09 Dep on previous A1	**A1A1**	[5]	Accept 0.268π, 2π/3. SRA1 for 48.2°, 120° Extra solutions in range –1

338. 9709_s15_ms_11 Q: 3

	Answer	Mark	Partial Marks
	$(1-x)^2(1+2x)^6$.		
(i) **(a)**	$(1-x)^6 = 1 - 6x + 15x^2$	B2,1 [2]	−1 each error
(b)	$(1+2x)^6 = 1 + 12x + 60x^2$	B2,1 [2]	−1 each error **SC** B1 only, in each part, for all 3 correct descending powers **SC** only one penalty for omission of the '1' in each expansion
(ii)	Product of **(a)** and **(b)** with >1 term → 60 – 72 + 15 = 3	M1 DM1A1 [3]	Must be 2 or more products M1 exactly 3 products. cao, condone $3x^2$

339. 9709_s15_ms_11 Q: 7

	Answer	Mark	Partial Marks
(a)	$ar^2 = \frac{1}{3}$, $ar^3 = \frac{2}{9}$		
	$\rightarrow r = \frac{2}{3}$ aef	M1	Any valid method, seen or implied. Could be answers only.
	Substituting $\rightarrow a = \frac{3}{4}$	A1	Both a and r
	$\rightarrow S_\infty = \frac{\frac{3}{4}}{\frac{1}{3}} = 2\frac{1}{4}$ aef	M1 A1 [4]	Correct formula with $\lvert r\rvert < 1$, cao
(b)	$4a = a + 4d \rightarrow 3a = 4d$	B1	May be implied in $360 = 5/2(a + 4a)$
	$360 = S_5 = \frac{5}{2}(2a + 4d)$ or $12.5a$	M1	Correct S_n formula or sum of 5 terms
	$\rightarrow a = 28.8^\circ$ aef	A1	cao, may be implied
	Largest $= a + 4d$ or $4a = 115.2^\circ$ aef	B1 [4]	(may use degrees or radians)

340. 9709_s15_ms_12 Q: 3

	Answer	Mark	Partial Marks
(i)	$(2 - x)^6$		
	Coeff of x^2 is 240	B1	co
	Coeff of x^3 is $-20 \times 8 = -160$	B2,1 [3]	B1 for +160
(ii)	$(3x + 1)(2 - x)^6$		
	Product needs exactly 2 terms	M1	3 × their 240 + their -160
	$\rightarrow 720 - 160 = 560$	A1√ [2]	√ for candidate's answers.

341. 9709_s15_ms_12 Q: 7

	Answer	Mark	Partial Marks
	$A(4, 6)$, $B(10, 2)$.		
(i)	$M = (7, 4)$	B1	co
	m of $AB = -\frac{2}{3}$	B1	co
	m of perpendicular $= \frac{3}{2}$		
	$\rightarrow y - 4 = \frac{3}{2}(x - 7)$	M1 A1 [4]	Use of $m_1m_2 = -1$ & their midpoint in the equation of a line. co
(ii)	Eqn of line parallel to AB through (3, 11)		
	$\rightarrow y - 11 = -\frac{2}{3}(x - 3)$	M1	Needs to use m of AB
	Sim eqns $\rightarrow C(9, 7)$	DM1A1 [3]	Must be using their correct lines. Co

342. 9709_s15_ms_13 Q: 3

	Answer	Mark	Partial Marks
(i)	$a^5 - 5a^4x + 10a^3x^2 - 10a^2x^3 \;+ ....$	**B2,1,0** **[2]**	Ok full expansion (ignore extra terms) Descending: Ok if full expansion but max B1 for 4 terms
(ii)	$(1-ax)(..10a^3x^2 - 10a^2x^3..) = (x^3)(-10a^4 - 10a^2)$ $-10a^4 - 10a^2 = -200$ $a^2 = 4$ ignore $a^2 = -5$ $a = \pm 2$ cao	**M1** **A1**√ **M1** **A1** **[4]**	Attempt to find coeff. of x^3 from 2 terms Ft from *their* $10a^3, -10a^2$ from part (i) Attempt soln. for a^2 from 3-term quad. in a^2 Ignore any imaginary solutions

343. 9709_s15_ms_13 Q: 9

	Answer	Mark	Partial Marks
(a)	2222/17 (=131 or 130.7) 131 × 17 (=2227) −2222 + 2227 = 5	**M1** **M1** **A1** **[3]**	Ignore signs. Allow 2239/17→131.7 or 132 Ignore signs. Use 131. 5 www gets 3/3
(b)	$r = \dfrac{2\cos\theta}{\sqrt{3}}$ soi oe $(-1<)\dfrac{2\cos\theta}{\sqrt{3}} < 1$ or $(0<)\dfrac{2\cos\theta}{\sqrt{3}} < 1$ soi $\pi/6,\ 5\pi/6$ soi (but dep. on M1) $\pi/6 < \theta < 5\pi/6$ cao	**B1** **M1**√ **A1A1** **A1** **[5]**	 Ft on *their r*. Ignore a 2nd inequality on LHS Allow 30º, 150º. Accept ⩽

344. 9709_w15_ms_11 Q: 1

	Answer	Mark	Partial Marks
	$(a+x)^5 = a^5 + {}^5C_1 a^4 x + {}^5C_2 a^3 x^2 + ...$ soi $\left(-\dfrac{2}{a} \times (their\ 5a^4) + (their\ 10a^3)\right)(x^2)$ 0	**M1** **M1** **A1** [3]	Ignore subsequent terms **AG**

345. 9709_w15_ms_11 Q: 8

	Answer	Mark	Partial Marks
(i)	$x^2 - 4x = 12$	M1	$4x - x^2 = 12$ scores M1A0
	$x = -2$ or 6	A1	
	3^{rd} term $= (-2)^2 + 12 = 16$ or $6^2 + 12 = 48$	A1A1 [4]	SC1 for 16, 48 after $x = 2, -6$
(ii)	$r^2 = \frac{x^2}{4x}\left(= \frac{x}{4}\right)$ soi	M1	
	$\frac{4x}{1 - \frac{x}{4}} = 8$	M1	Accept use of unsimplified $\frac{x^2}{4x}$ or $\frac{4x}{x^2}$ or $\frac{4}{x}$
	$x = \frac{4}{3}$ or $r = \frac{1}{3}$	A1	
	3^{rd} term $= \frac{16}{27}$ (or 0.593)	A1 [4]	
	ALT		
	$\frac{4x}{1-r} = 8 \rightarrow r = 1 - \frac{1}{2}x$ **or** $\frac{4x}{1-r} = 8 \rightarrow x = 2(1-r)$	M1	
	$x^2 = 4x\left(1 - \frac{1}{2}x\right)$ $r = \frac{2(1-r)}{4}$	M1	
	$x = \frac{4}{3}$ $r = \frac{1}{3}$	A1	

346. 9709_w15_ms_12 Q: 2

	Answer	Mark	Partial Marks
	$(x + 2k)^7$		
	Term in $x^5 = 21 \times 4k^2 = 84k^2$	B1	
	Term in $x^4 = 35 \times 8k^3 = 280k^3$	B1	
	Equate and solve $\rightarrow k = 0.3$ or $\frac{3}{10}$	M1 A1 [4]	Correct method to obtain k.

347. 9709_w15_ms_13 Q: 2

	Answer	Mark	Partial Marks
	$[7C2] \times \left[\left(\frac{x}{3}\right)^5\right] \times \left[\left(\frac{9}{x^2}\right)^2\right]$ soi	B2,1,0	Seen
	$21 \times \frac{1}{3^5}\left(x^5\right) \times 81\left(\frac{1}{x^4}\right)$ soi	B1	Identified as required term
	7	B1 [4]	Accept $7x$

348. 9709_w15_ms_13 Q: 6

	Answer	Mark	Partial Marks
(i) (a)	$1.92 + 1.84 + 1.76 + ...$ oe	**B1**	OR a=0.96, d= –.04 & ans
	$\frac{20}{2}[2\times1.92+19\times(-0.08)]$ oe	**M1**	doubled/adjusted
	23.2 cao	**A1** [3]	Corr formula used with corr d & *their* a, n $a = 1, n = 21 \rightarrow 12.6\ (25.2)$, $a = 0.96, n = 21 \rightarrow 11.76\ (23.52)$
(b)	$1.92+1.92(.96)+1.92(.96)^2+...$	**B1**	
	$\frac{1.92(1-.96^{20}}{1-.96}$	**M1**	OR a=.96, r =.96 & ans /doubled/adjusted
	26.8 cao	**A1** [3]	Corr formula used with r =.96 & *their* a, n $a = .96, n = 21 \rightarrow 13.82\ (27.63)$ $a = 1, n = 21 \rightarrow 14.39\ (28.78)$
(ii)	$\frac{1.92}{1-.96}=48$ or $\frac{0.96}{1-0.96}=24$ & then Double **AG**	**M1A1** [2]	$a = 1\rightarrow 25\ (50)$ but must be doubled for M1 $1.92\frac{(1-0.96^n)}{1-0.96}<48\rightarrow0.96^n>0$ (www) 'which is true' scores SCB1

349. 9709_m22_ms_12 Q: 11

Question	Answer	Marks	Guidance
(a)	$\frac{dy}{dx}=\{-k(3x-k)^{-2}\}\{\times3\}\{+3\}$	**B2, 1, 0**	
	$\frac{-3k}{(3x-k)^2}+3=0$ leading to $(3)(3x-k)^2=(3)k$ leading to $3x-k=[\pm]\sqrt{k}$	**M1**	Set $\frac{dy}{dx}=0$ and remove the denominator
	$x=\frac{k\pm\sqrt{k}}{3}$	**A1**	OE
		4	
(b)	$a=\frac{4\pm\sqrt{4}}{3}$ leading to $a=2$	**B1**	Substitute $x = a$ when $k = 4$. Allow $x = 2$.
	$f''(x)=f'\left[-12(3x-4)^{-2}+3\right]=72(3x-4)^{-3}$	**B1**	Allow $18k(3x-k)^{-3}$
	> 0 (or 9) when $x = 2 \rightarrow$ minimum	**B1 FT**	FT on *their* $x = 2$, providing their $x\geqslant\frac{3}{2}$ and $f''(x)$ is correct
		3	

Question	Answer	Marks	Guidance
(c)	Substitute $k=-1$ leading to $g'(x)=\frac{3}{(3x+1)^2}+3$	M1	Condone one error.
	$g'(x)>0$ or $g'(x)$ always positive, hence g is an increasing function	A1	WWW. A0 if the conclusion depends on substitution of values into $g'(x)$.
	Alternative method for question 11(c)		
	$x=\frac{k\pm\sqrt{k}}{3}$ when $k=-1$ has no solutions, so g is increasing or decreasing	M1	Allow the statement 'no turning points' for increasing or decreasing
	Show g'(x) is positive for any value of x, hence g is an increasing function	A1	Or show g(b) > g(a) for $b > a \rightarrow$ g, hence g is an increasing function
		2	

350. 9709_s21_ms_12 Q: 3

Question	Answer	Marks	Guidance
(a)	1.2679	B1	AWRT. ISW if correct answer seen. $3-\sqrt{3}$ scores B0
		1	
(b)	1.7321	B1	AWRT. ISW if correct answer seen.
		1	
(c)	Sight of 2 or 2.0000 or two in reference to the gradient	*B1	
	This is because the gradient at E is the limit of the gradients of the chords as the x-value tends to 3 or δx tends to 0.	DB1	Allow it gets nearer/approaches/tends/almost/approximately 2
		2	

351. 9709_s21_ms_13 Q: 2

Question	Answer	Marks	Guidance
	$\left[f^{-1}(x)=\right]\left((2x-1)^{1/2}\right)\times\left(\frac{1}{3}\times 2\times\frac{3}{2}\right)(-2)$	B2, 1, 0	Expect $(2x-1)^{1/2}-2$
	$(2x-1)^{1/2}-2\leqslant 0 \rightarrow 2x-1\leqslant 4$ or $2x-1<4$	M1	SOI. Rearranging and then squaring, must have power of ½ not present Allow '=0'at this stage but do not allow '≥ 0' or '>0' If '–2' missed then must see $\leqslant$ or < for the M1
	Value [of a] is 2½ or $a=2½$	A1	WWW, OE e.g. $\frac{5}{2}$, 2.5 Do not allow from '=0' unless some reference to negative gradient.
		4	

352. 9709_w21_ms_12 Q: 9

Question	Answer	Marks	Guidance
(a)	$\left[\frac{dV}{dr}=\right]\ \frac{9}{2}\left(r-\frac{1}{2}\right)^2$	B1	OE. Accept unsimplified.
	$\frac{dr}{dt}=\frac{dr}{dV}\times\frac{dV}{dt}=\frac{1.5}{their\frac{dV}{dr}}\quad\left[=\frac{1.5}{\frac{9}{2}\left(5.5-\frac{1}{2}\right)^2}=\frac{1.5}{112.5}\right]$	M1	Correct use of chain rule with 1.5, *their* differentiated expression for $\frac{dV}{dr}$ and using $r=5.5$.
	0.0133 or $\frac{3}{225}$ or $\frac{1}{75}$ [metres per second]	A1	
		3	
(b)	$\frac{dV}{dr}$ or $their\frac{dV}{dr}=\frac{1.5}{0.1}$ or 15 OR $0.1=\frac{1.5}{their\frac{dV}{dr}}\quad\left[=\frac{2\times1.5}{9\left(r-\frac{1}{2}\right)^2}\text{OE}\right]$	B1 FT	Correct statement involving $\frac{dV}{dr}$ or *their* $\frac{dV}{dr}$, 1.5 and 0.1.
	$\left[\frac{9}{2}\left(r-\frac{1}{2}\right)^2=15\Rightarrow\right]\ r=\frac{1}{2}+\sqrt{\frac{10}{3}}$	B1	OE e.g. AWRT 2.3 Can be implied by correct volume.
	[Volume =] 8.13 AWRT	B1	OE e.g. $\frac{-3+5\sqrt{30}}{3}$. CAO.
		3	

353. 9709_w21_ms_12 Q: 10

Question	Answer	Marks	Guidance
(a)	$[f'(x)=]\ 2x-\frac{k}{x^2}$	B1	
	$f'(2)=0\ \left[2\times2-\frac{k}{2^2}=0\right]\Rightarrow k=\ldots$	M1	Setting *their* 2-term f′(2) = 0, at least one term correct and attempting to solve as far as *k* = .
	$k=16$	A1	
		3	
(b)	$f''(2)=$ e.g. $2+\frac{2k}{2^3}$	M1	Evaluate a two term f″(2) with at least one term correct. Or other valid method.
	$\left[2+\frac{2k}{2^3}\right]>0\Rightarrow$ minimum or $=6\Rightarrow$ minimum	A1 FT	WWW. FT on positive *k* value.
		2	
(c)	When $x=2$, $f(x)=14$	B1	SOI
	[Range is or y or $f(x)$] $\geqslant$ *their* f(2)	B1 FT	Not $x\geqslant$ *their* f(2)
		2	

354. 9709_w21_ms_13 Q: 3

Question	Answer	Marks	Guidance
(a)	$\{5(y-3)^2\}$ $\{+5\}$	B1 B1	Accept $a = -3$, $b = 5$
		2	
(b)	$[f'(x) =] 5x^4 - 30x^2 + 50$	B1	
	$5(x^2-3)^2 + 5$ or $b^2 < 4ac$ and at least one value of f'(x) > 0	M1	
	> 0 and increasing	A1	WWW
		3	

355. 9709_m20_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$f'(x) = [-(3x+2)^{-2}] \times [3] + [2x]$	B2, 1, 0	
	< 0 hence decreasing	B1	Dependent on at least B1 for $f'(x)$ and must include < 0 or '(always) neg'
		3	

356. 9709_m20_ms_12 Q: 4

	Answer	Mark	Partial Marks
	$\frac{dy}{dx} = 2x - 2$	B1	
	$\frac{dy}{dx} = \frac{4}{6}$	B1	OE, SOI
	$their(2x-2) = their\frac{4}{6}$	M1	LHS and RHS must be *their* $\frac{dy}{dx}$ expression and value
	$x = \frac{4}{3}$ oe	A1	
		4	

357. 9709_s20_ms_11 Q: 9

(a)	$\frac{dy}{dx} = 3(3-2x)^2 \times -2 + 24 = -6(3-2x)^2 + 24$ (**B1** without ×−2. **B1** for ×−2)	B1B1
	$\frac{d^2y}{dx^2} = -12(3-2x) \times -2 = 24(3-2x)$ (**B1FT** from $\frac{dy}{dx}$ without – 2)	B1FT B1
		4
(b)	$\frac{dy}{dx} = 0$ when $6(3-2x)^2 = 24 \rightarrow 3-2x = \pm 2$	M1
	$x = ½, y = 20$ or $x = 2½, y = 52$ (**A1** for both x values or a correct pair)	A1A1
		3
(c)	If $x = ½$, $\frac{d^2y}{dx^2} = 48$ Minimum	B1FT
	If $x = 2½$, $\frac{d^2y}{dx^2} = -48$ Maximum	B1FT
		2

358. 9709_s20_ms_12 Q: 3

(a)	Volume after 30 s = 18000 $\frac{4}{3}\pi r^3 = 18000$	M1
	$r = 16.3$ cm	A1
		2
(b)	$\frac{dV}{dr} = 4\pi r^2$	B1
	$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt} = \frac{600}{4\pi r^2}$	M1
	$\frac{dr}{dt} = 0.181$ cm per second	A1
		3

359. 9709_s20_ms_12 Q: 10

(a)	$\frac{dy}{dx} = 54 - 6(2x-7)^2$	B2,1
	$\frac{d^2y}{dx^2} = -24(2x-7)$ (**FT** only for omission of '×2 ' from the bracket)	B2,1 FT
		4
(b)	$\frac{dy}{dx} = 0 \to (2x-7)^2 = 9$	M1
	$x = 5, y = 243$ or $x = 2, y = 135$	A1 A1
		3
(c)	$x = 5$ $\frac{d^2y}{dx^2} = -72 \to$ Maximum (**FT** only for omission of '×2 ' from the bracket)	B1FT
	$x = 2$ $\frac{d^2y}{dx^2} = 72 \to$ Minimum (**FT** only for omission of '×2 ' from the bracket)	B1FT
		2

360. 9709_s20_ms_13 Q: 6

(a)	$\frac{dy}{dx} = \left[\frac{1}{2}(5x-1)^{-1/2}\right] \times [5]$	B1 B1
	Use $\frac{dy}{dt} = 2 \times \left(their \frac{dy}{dx} \text{ when } x = 1 \right)$	M1
	$\frac{5}{2}$	A1
		4
(b)	$2 \times their \frac{5}{2}(5x-1)^{-1/2} = \frac{5}{8}$ oe	M1
	$(5x-1)^{1/2} = 8$	A1
	$x = 13$	A1
		3

361. 9709_w20_ms_11 Q: 3

	Answer	Mark	Partial Marks
	(Derivative =) $4\pi r^2$ ($\rightarrow$ 400π)	B1	SOI Award this mark for $\frac{\mathrm{d}r}{\mathrm{d}V}$
	$\frac{50}{\textit{their}\text{ derivative}}$	M1	Can be in terms of r
	$\frac{1}{8\pi}$ or 0.0398	A1	AWRT
		3	

362. 9709_w20_ms_11 Q: 6

	Answer	Mark	Partial Marks
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \left[\frac{1}{2}\left(25-x^2\right)^{-1/2}\right] \times [-2x]$	B1 B1	
	$\frac{-x}{\left(25-x^2\right)^{1/2}} = \frac{4}{3} \rightarrow \frac{x^2}{25-x^2} = \frac{16}{9}$	M1	Set = $\frac{4}{3}$ and square both sides
	$16\left(25-x^2\right) = 9x^2 \rightarrow 25x^2 = 400 \rightarrow x = (\pm)4$	A1	
	When $x = -4$, $y = 5$ $\rightarrow$ $(-4, 5)$	A1	
		5	

363. 9709_w20_ms_13 Q: 8

	Answer	Mark	Partial Marks
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = [2]$ $[-2(2x+1)^{-2}]$	B1 B1	
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 8(2x+1)^{-3}$	B1	
		3	
(b)	Set *their* $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and attempt solution	M1	
	$(2x+1)^2 = 1 \rightarrow 2x+1 = (\pm)\,1$ or $4x^2 + 4x = 0 \rightarrow (4)x(x+1) = 0$	M1	Solving as far as $x =$ ....
	$x = 0$	A1	WWW. Ignore other solution.
	(0, 2)	A1	One solution only. Accept $x = 0$, $y = 2$ only.
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$ from a solution $x > -\frac{1}{2}$ hence minimum	B1	Ignore other solution. Condone arithmetic slip in value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$. *Their* $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ must be of the form $k(2x+1)^{-3}$
		5	

364. 9709_m19_ms_12 Q: 4

	Answer	Mark	Partial Marks
(i)	$\mathrm{d}y/\mathrm{d}x = -2(2x-1)^{-2} + 2$	B2,1,0	Unsimplified form ok (–1 for each error in '–2', '$(2x-1)^{-2}$' and '2')
	$\mathrm{d}^2y/\mathrm{d}x^2 = 8(2x-1)^{-3}$	B1	Unsimplified form ok
		3	
(ii)	Set $\mathrm{d}y/\mathrm{d}x$ to zero and attempt to solve – at least one correct step	M1	
	$x = 0, 1$	A1	Expect $(2x-1)^2 = 1$
	When $x = 0$, $\mathrm{d}^2y/\mathrm{d}x^2 = -8$ (or < 0). Hence MAX	B1	
	When $x = 1$, $\mathrm{d}^2y/\mathrm{d}x^2 = 8$ (or > 0). Hence MIN	B1	Both final marks dependent on correct x and correct $\mathrm{d}^2y/\mathrm{d}x^2$ and no errors May use change of sign of $\mathrm{d}y/\mathrm{d}x$ but not at $x = 1/2$
		4	

365. 9709_m19_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	$\mathbf{u.v} = 8q + 2q - 2 + 6q^2 - 42$	B1	May be unsimplified
	$6q^2 + 10q - 44 = 0$ oe	M1	Simplify, set to zero and attempt to solve
	$q = 2, -11/3$	A1	Both required. Accept –3.67
		3	
(ii)	$\mathbf{u} = \begin{pmatrix} 0 \\ 2 \\ 6 \end{pmatrix}$ $\mathbf{v} = \begin{pmatrix} 8 \\ -1 \\ -7 \end{pmatrix}$ $\mathbf{u.v} = -2 - 42$	M1	Correct method for scalar product
	$\lvert\mathbf{u}\rvert \times \lvert\mathbf{v}\rvert = \sqrt{2^2 + 6^2} \times \sqrt{8^2 + 1^2 + 7^2}$	M1	Prod of mods. At least one methodically correct.
	$\cos\theta = \dfrac{-44}{\sqrt{40} \times \sqrt{114}} = \dfrac{-44}{4\sqrt{285}} = \dfrac{-4}{\sqrt{11}}$	M1	All linked correctly and inverse cos used correctly
	$\theta = 130.7°$ or 2.28(05) rads	A1	No other angles between 0° and 180°
		4	

366. 9709_s19_ms_11 Q: 7

	Answer	Mark	Partial Marks
(i)	$\overrightarrow{AM} = 1.5\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ $\overrightarrow{GM} = 6.5\mathbf{i} - 4\mathbf{j} - 5\mathbf{k}$	B3,2,1	Loses 1 mark for each error.
		3	
(ii)	$\overrightarrow{AM} . \overrightarrow{GM} = 9.75 - 16 - 25 = -31.25$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on AM and GM
	$\overrightarrow{AM} . \overrightarrow{GM} = \sqrt{(1.5^2+4^2+5^2)} \times \sqrt{(6.5^2+4^2+5^2)} \cos GMA$	M1 M1	M1 for product of 2 modulii M1 all correctly connected
	Equating → Angle $GMA = 121°$	A1	
		4	

367. 9709_s19_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	$6\times3+-2\times k+-6\times-3=0$ $(18-2k+18=0)$	M1	Use of scalar product = 0. Could be $\overrightarrow{AO}.\overrightarrow{OB}$, $\overrightarrow{AO}.\overrightarrow{BO}$ or $\overrightarrow{OA}.\overrightarrow{BO}$
	$k=18$	A1	
	Alternative method for question 8(i)		
	$76+18+k^2=18+(k+2)^2$	M1	Use of Pythagoras with appropriate lengths.
	$k=18$	A1	
		2	
(ii)	$36+4+36=9+k^2+9$	M1	Use of modulus leading to an equation and solve to $k=$ or $k^2=$
	$k=\pm\sqrt{58}$ or ±7.62	A1	Accept exact or decimal answers. Allow decimals to greater accuracy.
		2	
(iii)	$\overrightarrow{AB}=\begin{pmatrix}-3\\6\\3\end{pmatrix}\to\overrightarrow{AC}=\begin{pmatrix}-2\\4\\2\end{pmatrix}$ then $\overrightarrow{OA}+\overrightarrow{AC}$	M1	Complete method using $\overrightarrow{AC}=\pm\frac{2}{3}\overrightarrow{AB}$ And then $\overrightarrow{OA}+\text{their }\overrightarrow{AC}$
	$\overrightarrow{OC}=\begin{pmatrix}4\\2\\-4\end{pmatrix}$	A1	
	$\div\sqrt{(\text{their }4)^2+(\text{their }2)^2+(\text{their }-4)^2}$	M1	Divides by modulus of their $\overrightarrow{OC}$
	$=\frac{1}{6}\begin{pmatrix}4\\2\\-4\end{pmatrix}$ **or** $\frac{1}{6}(4i+2j-4k)$	A1	
	Alternative method for question 8(iii)		
	Let $\overrightarrow{OC}=\begin{pmatrix}p\\q\\r\end{pmatrix}\to\overrightarrow{AC}=\begin{pmatrix}p-6\\q+2\\r+6\end{pmatrix}\&\overrightarrow{CB}=\begin{pmatrix}3-p\\4-q\\-3-r\end{pmatrix}$	M1	Correct method. Equates coefficients leading to values for p, q, r
	$p-6=2(3-p)$; $q+2=2(4-q)$; $r+6=2(-3-r)$ $\to p=4$, $q=2$ & $r=-4$	A1	
	$\div\sqrt{(\text{their }4)^2+(\text{their }2)^2+(\text{their }-4)^2}$	M1	Divides by modulus of their $\overrightarrow{OC}$
	$=\frac{1}{6}\begin{pmatrix}4\\2\\-4\end{pmatrix}$ **or** $\frac{1}{6}(4i+2j-4k)$	A1	
(iii)	**Alternative method for question 8(iii)**		
	$\overrightarrow{CB}=\overrightarrow{OB}-\overrightarrow{OC}\ \therefore 2(\overrightarrow{OB}-\overrightarrow{OC})=\overrightarrow{OC}-\overrightarrow{OA}$ $\to 2\overrightarrow{OB}+\overrightarrow{OA}=3\overrightarrow{OC}\therefore\ 3\overrightarrow{OC}=\begin{pmatrix}12\\6\\-12\end{pmatrix}$	M1	Correct method. Gets to a numerical expression for k$\overrightarrow{OC}$ from $\overrightarrow{OA}$ &$\overrightarrow{OB}$.
	$\overrightarrow{OC}=\begin{pmatrix}4\\2\\-4\end{pmatrix}$	A1	
	$\div\sqrt{(\text{their }4)^2+(\text{their }2)^2+(\text{their }-4)^2}$	M1	Divides by modulus of their $\overrightarrow{OC}$
	$=\frac{1}{6}\begin{pmatrix}4\\2\\-4\end{pmatrix}$ **or** $\frac{1}{6}(4i+2j-4k)$	A1	
		4	

368. 9709_s19_ms_12 Q: 9

	Answer	Mark	Partial Marks
	For C_1: $\frac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	y – '*their 4*' = '*their* m' $(x - 3)$ or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y - 4 = 2(x - 3)$ or $y = 2x - 2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	$2x - 2 = \sqrt{4x+k}$ $(\rightarrow 4x^2 - 12x + 4 - k = 0)$	*M1	Forms an equation in one variable using tangent & C_2
	Use of $b^2 - 4ac = 0$ on a 3 term quadratic set to 0.	*DM1	Uses 'discriminant = 0'
	$144 = 16(4 - k) \rightarrow k = -5$	A1	
	$4x^2 - 12x + 4 - k = 0 \rightarrow 4x^2 - 12x + 9 = 0$	DM1	Uses k to form a 3 term quadratic in x
	$x = \frac{3}{2}\left(or \frac{1}{2}\right)$, $y = 1$(or – 1).	A1	Condone 'correct' extra solution.
	Alternative method for question 9		
	For C_1: $\frac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	y – '*their 4*' = '*their* m' $(x - 3)$ or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y - 4 = 2(x - 3)$ or $y = 2x - 2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	For C_2: $\frac{dy}{dx} = A(4x+k)^{-\frac{1}{2}}$	*M1	Finds $\frac{dy}{dx}$ for C_2 in the form $A(4x+k)^{-\frac{1}{2}}$
	At P: '*their* 2' = $A(4x+k)^{-\frac{1}{2}}$" $\rightarrow (x = \frac{1-k}{4}$ or $4x + k = 1)$	*DM1	Equating '*their* 2' to '*their* $\frac{dy}{dx}$' and simplify to form a linear equation linking $4x + k$ and a constant.
	$(2x-2)^2 = 4x + k \rightarrow (2x-2)^2 = 1 \rightarrow (4x^2 - 8x + 3 = 0)$	DM1	Using *their* $y = 2x - 2$, $y^2 = 4x + k$ and *their* $4x + k = 1$ (but not =0) to form a 3 term quadratic in x.
	$x = \frac{3}{2}\left(or \frac{1}{2}\right)$ and from $k = -5(or -1)$	A1	Needs correct values for x and k.
	from $y^2 = 4x + k$, $y = 1$(or – 1).	A1	Condone 'correct' extra solution.
	Alternative method for question 9		
	For C_1: $\frac{dy}{dx} = 2x - 4 \rightarrow m = 2$	B1	
	y – '*their 4*' = '*their* m' $(x - 3)$ or using $y = mx + c$	M1	Use of : $\frac{dy}{dx}$ and (3, their 4) to find the tangent equation.
	$y - 4 = 2(x - 3)$ or $y = 2x - 2$	A1	If using $= mx + c$, getting $c = -2$ is enough.
	For C_2: $\frac{dy}{dx} = A(4x+k)^{-\frac{1}{2}}$	*M1	Finds $\frac{dy}{dx}$ for C_2 in the form $A(4x+k)^{-\frac{1}{2}}$
	At P: '*their* 2' = $A(4x+k)^{-\frac{1}{2}}$" $\rightarrow (x = \frac{1-k}{4}$ or $4x + k = 1)$	*DM1	Equating '*their* 2' to '*their* $\frac{dy}{dx}$' and simplify to form a linear equation linking $4x + k$ and a constant.
	From $4x + k = 1$ and $y^2 = 4x + k \rightarrow y^2 = 1$	DM1	Using *their* $4x + k = 1$ (but not =0) and C_2 to form y^2 = a constant
	$y = 1$(or – 1) and $x = \frac{3}{2}\left(or \frac{1}{2}\right)$	A1	Needs correct values for y and x.
	From $4x + k = 1$, $k = -5$ (or – 1)	A1	Condone 'correct' extra solution
		8	

369. 9709_s19_ms_13 Q: 6

	Answer	Mark	Partial Marks
(i)	**MF** = –4**i** + 2**j** + 7**k**	**B1**	
		1	
(ii)	**FN** = 2**i** – **j**	**B1**	
		1	
(iii)	**MN** = –2**i** + **j** + 7**k**	**B1**	**FT** on *their* (**MF** + **FN**)
		1	
(iv)	**MF.MN** = 8 + 2 + 49 = 59	***M1**	**MF.MN** or **FM.NM** but allow if one is reversed (implied by –59)
	$\lvert\mathbf{MF}\rvert \times \lvert\mathbf{MN}\rvert = \sqrt{4^2+2^2+7^2} \times \sqrt{2^2+1^2+7^2}$	***DM1**	Product of modulus. At least one methodically correct
	$\cos FMN = \dfrac{+/-59}{\sqrt{69}\times\sqrt{54}}$	**DM1**	All linked correctly. Note $\sqrt{69}\times\sqrt{54} = 9\sqrt{46}$
	FMN = 14.9° or 0.259	**A1**	Do not allow if exactly 1 vector is reversed – even if adjusted finally
		4	

370. 9709_s19_ms_13 Q: 8

	Answer	Mark	Partial Marks
	$f'(-1)=0 \to 3-a+b=0 \quad f'(3)=0 \to 27+3a+b=0$	**M1**	Stationary points at $x = -1$ & $x = 3$ gives sim. equations in a & b
	$a = -6$	**A1**	Solve simultaneous equation
	$b = -9$	**A1**	
	Hence $f'(x)=3x^2-6x-9 \to f(x)=x^3-3x^2-9x(+c)$	**B1**	**FT** correct integration for *their a,b* (numerical a, b)
	$2=-1-3+9+c$	**M1**	Sub $x = -1$, $y = 2$ into *their* integrated f(x). c must be present
	$c = -3$	**A1**	**FT** from *their* f(x)
	$f(3)=k \to k=27-27-27-3$	**M1**	Sub $x = 3$, $y = k$ into *their* integrated f(x) (Allow c omitted)
	$k = -30$	**A1**	
		8	

371. 9709_w19_ms_11 Q: 2

	Answer	Mark	Partial Marks
	Attempt to solve $f'(x)=0$ or $f'(x)>0$ or $f'(x)\geqslant 0$	**M1**	SOI
	$(x-2)(x-4)$	**A1**	2 and 4 seen
	(Least possible value of n is) 4	**A1**	Accept $n = 4$ or $n \geqslant 4$
		3	

372. 9709_w19_ms_11 Q: 10

	Answer	Mark	Partial Marks
(i)	$\mathbf{AB} = \begin{pmatrix}2\\-3\\5\end{pmatrix} - \begin{pmatrix}-1\\3\\-4\end{pmatrix} = \begin{pmatrix}3\\-6\\9\end{pmatrix}$, $\mathbf{BC} = \begin{pmatrix}4\\-2\\5\end{pmatrix} - \begin{pmatrix}2\\-3\\5\end{pmatrix} = \begin{pmatrix}2\\1\\0\end{pmatrix}$	B1B1	Condone reversal of labels
	AB.BC = 6 – 6 → = 0 (hence perpendicular)	B1	AG
(ii)	$\mathbf{DC} = \begin{pmatrix}4\\-2\\5\end{pmatrix} - \begin{pmatrix}2\\2\\-1\end{pmatrix} = \begin{pmatrix}2\\-4\\6\end{pmatrix}$	B1	Or: $\mathbf{CD} = \begin{pmatrix}-2\\4\\-6\end{pmatrix}$
	AB = *k***DC**	M1	OE Expect $k = \frac{3}{2}$ Or: **DC.BC** = 4 – 4 = 0 hence *BC* is also perpendicular to *DC* Or: **AB.DC** = 1 or **AB.CD** = –1, angle between lines is 0 or 180
	AB is parallel to *DC*, hence *ABCD* is a trapezium	A1	
(iii)	$\lvert\mathbf{AB}\rvert = \sqrt{9+36+81} = \sqrt{126} = 11.22$ $\lvert\mathbf{DC}\rvert = \sqrt{4+16+36} = \sqrt{56} = 7.483$ $\lvert\mathbf{BC}\rvert = \sqrt{4+1+0} = \sqrt{5} = 2.236$	M1	Method for finding at least 2 magnitudes
	Area = $\frac{1}{2}\left(theirAB + theirDC\right) \times theirBC = 20.92$	M1A1	OE

373. 9709_w19_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	Use of Pythagoras → $r^2 = 15^2 - h^2$	M1	
	$V = \frac{1}{3}\pi(225 - h^2) \times h \to \frac{1}{3}\pi(225h - h^3)$	A1	AG WWW e.g. sight of $r = 15 - h$ gets A0.
		2	
(ii)	$\left(\frac{dv}{dh} = \right)\frac{\pi}{3}(225 - 3h^2)$	B1	
	Their $\frac{dv}{dh} = 0$	M1	Differentiates, sets *their* differential to 0 and attempts to solve at least as far as $h^2 \neq 0$.
	$(h =)\ \sqrt{75}$, $5\sqrt{3}$ or AWRT 8.66	A1	Ignore $-\sqrt{75}$ OE and ISW for both A marks
	$\frac{d^2h}{dh^2} = \frac{\pi}{3}(-6h)\ (\to -\text{ve})$	M1	Differentiates for a second time and considers the sign of the second differential or any other valid complete method.
	→ Maximum	A1FT	Correct conclusion from correct 2nd differential, value for *h* not required, or any other valid complete method. FT for *their* *h*, if used, as long as it is positive.
			SC Omission of π or $\frac{\pi}{3}$ throughout can score B0M1A1M1A0
		5	

374. 9709_w19_ms_12 Q: 7

	Answer	Mark	Partial Marks
(i)	$(\overrightarrow{PB}) = 5\mathbf{i} + 8\mathbf{j} - 5\mathbf{k}$	B2,1,0	B2 all correct, B1 for two correct components.
	$(\overrightarrow{PQ}) = 4\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$	B2,1,0	B2 all correct, B1 for two correct components.
			Accept column vectors. **SC** B1 for each vector if all components multiplied by –1.
		4	
(ii)	(Length of PB =) $\sqrt{(5^2 + 8^2 + 5^2)} = (\sqrt{114} \approx 10.7)$ (Length of PQ =) $\sqrt{(4^2 + 8^2 + 5^2)} = (\sqrt{105} \approx 10.2)$	M1	Evaluation of both lengths. Other valid complete comparisons can be accepted.
	P is nearer to Q.	A1	WWW
		2	
(iii)	$(\overrightarrow{PB}.\overrightarrow{PQ}) = 20 + 64 - 25$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$ on *their* $\overrightarrow{PB}$ and $\overrightarrow{PQ}$
	$(Their\sqrt{114})(their\sqrt{105})\cos BPQ = (their\ 59)$	M1	All elements present and in correct places.
	$BPQ = 57.4(°)$ or 1.00 (rad)	A1	AWRT Calculating the obtuse angle and then subtracting gets A0.
		3	

375. 9709_w19_ms_13 Q: 3

	Answer	Mark	Partial Marks
	$\frac{dy}{dx} = 3x^2 + 2x - 8$	B1	
	Set to zero (SOI) and solve	M1	
	(Min) $a = -2$, (Max) $b = 4/3$. – in terms of a and b.	A1 A1	Accept $a \geqslant -2,\ b \leqslant \frac{4}{3}$ **SC**: A1 for $a > -2,\quad b < \frac{4}{3}$ or for $-2 < x < \frac{4}{3}$
		4	

376. 9709_w19_ms_13 Q: 5

	Answer	Mark	Partial Marks
(i)	$S = 28x^2$, $V = 8x^3$	B1B1	SOI
	$7V^{\frac{2}{3}} = 7 \times 4x^2 = S$	B1	AG, WWW
		3	
(ii)	$\left(\frac{dS}{dV}\right) = \frac{14V^{-\frac{1}{3}}}{3} = \frac{14}{30}$ SOI when $V = 1000$	*M1 A1	Attempt to differentiate For M mark $\left(\frac{dS}{dV}\right)$ to be of form $kV^{-\frac{1}{3}}$
	$\left(\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS}\right)$ OE used with $\frac{dS}{dt} = 2$ **and** $\frac{1}{their\ \frac{14}{30}}$	DM1	
	$\frac{30}{7}$ or 4.29	A1	OE
	Alternative method for question 5(ii)		
	$V = \frac{S^{\frac{3}{2}}}{7\sqrt{7}} \to \left(\frac{dV}{dS}\right) = \frac{3}{2} \times S^{\frac{1}{2}} \times \frac{1}{7\sqrt{7}} = \frac{30}{14}$ SOI when $S = 700$	*M1 A1	Attempt to differentiate For M mark $\left(\frac{dV}{dS}\right)$ to be of form $kS^{\frac{1}{2}}$
	$\left(\frac{dV}{dt} = \frac{dS}{dt} \times \frac{dV}{dS}\right)$ OE used with $\frac{dS}{dt} = 2$ **and** $\frac{1}{their\ \frac{14}{30}}$	DM1	
	$\frac{30}{7}$ or 4.29	A1	OE
(ii)	**Alternative method for question 5(ii)**		
	Attempt to find either $\frac{dV}{dx}$ **or** $\left(\frac{dS}{dx}\text{ and }\frac{dV}{dS}\right)$ together with either $\frac{dx}{dt}$ **or** x	*M1	
	$\frac{dV}{dx} = 24x^2$ **or** $\left(\frac{dS}{dx} = 56x \text{ and } \frac{dV}{dS} = \frac{3x}{7}\right)$, $\frac{dx}{dt} = \frac{1}{140}$ **or** $x = 5$ (A1)	A1	
	Correct method for $\frac{dV}{dt}$	DM1	
	$\frac{30}{7}$ or 4.29	A1	OE
		4	

377. 9709_w19_ms_13 Q: 10

	Answer	Mark	Partial Marks
(i)	$\mathbf{AX}=\begin{pmatrix}6\\2\\3\end{pmatrix}$, and one of $\mathbf{AB}=\begin{pmatrix}18\\6\\9\end{pmatrix}$, $\mathbf{XB}=\begin{pmatrix}12\\4\\6\end{pmatrix}$, $\mathbf{BX}=\begin{pmatrix}-12\\-4\\-6\end{pmatrix}$	B1B1	
	State $\mathbf{AB}=3\mathbf{AX}$ (or $\mathbf{XB}=2\mathbf{AX}$ or $\mathbf{AB}=\frac{3}{2}\mathbf{XB}$ etc) hence straight line **OR** $\frac{\mathbf{AX}.\mathbf{AB}}{\lvert\mathbf{AX}\rvert\lvert\mathbf{AB}\rvert}=1\ (\to\theta=0)$ or $\frac{\mathbf{AX}.\mathbf{BX}}{\lvert\mathbf{AX}\rvert\lvert\mathbf{BX}\rvert}=-1\ (\to\theta=180)$ hence straight line	B1	WWW A conclusion (i.e. a straight line) is required.
		3	
(ii)	$\mathbf{CX}=\begin{pmatrix}-3\\6\\2\end{pmatrix}$	B1	
	$\mathbf{CX}.\mathbf{AX}=-18+12+6$	M1	
	= 0 (hence *CX* is perpendicular to *AX*)	A1	
		3	
(iii)	$\lvert\mathbf{CX}\rvert=\sqrt{3^2+6^2+2^2}$, $\lvert\mathbf{AB}\rvert=\sqrt{18^2+6^2+9^2}$ Both attempted	M1	
	Area $\Delta ABC=\frac{1}{2}\times$ *their* $21\times$ *their* $7=73\frac{1}{2}$	M1A1	Accept answers which round to 73.5
		3	

378. 9709_m18_ms_12 Q: 7

	Answer	Mark	Partial Marks
(i)	$\overrightarrow{CE}=-4\mathbf{i}-\mathbf{j}+8\mathbf{k}$	B1	
	$\lvert\overrightarrow{CE}\rvert=\sqrt{((their-4)^2+(their-1)^2+(their\,8)^2}=9$	M1A1	Could use Pythagoras' theorem on triangle *CDE*
		3	
(ii)	$\overrightarrow{CA}=3\mathbf{i}-3\mathbf{j}$ or $\overrightarrow{AC}=-3\mathbf{i}+3\mathbf{j}$	B1	
	$\overrightarrow{CE}.\overrightarrow{CA}=(-4\mathbf{i}-\mathbf{j}+8\mathbf{k}).(3\mathbf{i}-3\mathbf{j})=-12+3$ (Both vectors reversed ok)	M1	Scalar product of *their* $\overrightarrow{CE}$, $\overrightarrow{CA}$. One vector reversed ok for all **M** marks
	$\lvert\overrightarrow{CE}\rvert\times\lvert\overrightarrow{CA}\rvert=\sqrt{16+1+64}\times\sqrt{9+9}$	M1	Product of moduli of *their* $\overrightarrow{CE}$, $\overrightarrow{CA}$
	$\cos^{-1}\left(\frac{-12+3}{9\sqrt{18}}\right)=\cos^{-1}\left(\frac{-1}{\sqrt{18}}\right)$ $\left[\text{or e.g. }\cos^{-1}\left(\frac{-3}{\sqrt{162}}\right),\ \cos^{-1}\left(\frac{-9}{\sqrt{1458}}\right)\right]$ etc.	A1A1	**A1** for any correct expression, **A1** for required form Equivalent answers must be in required form $m/\sqrt{n}$ (*m*, *n* integers)
		5	

379. 9709_m18_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	$dy/dx=x-6x^{½}+8$	B2,1,0	
	Set to zero and attempt to solve a quadratic for $x^{½}$	M1	Could use a substitution for $x^{½}$ or rearrange and square correctly*
	$x^{½}=4$ or $x^{½}=2$ [$x=2$ *and* $x=4$ gets **M1 A0**]	A1	Implies **M1**. 'Correct' roots for *their* dy/dx also implies **M1**
	$x=16$ or 4	A1FT	Squares of their solutions *Then **A1**,**A1** for each answer
		5	

	Answer	Mark	Partial Marks
(ii)	$\mathrm{d}^2y/\mathrm{d}x^2 = 1 - 3x^{-1/2}$	B1FT	FT on *their* dy/dx, providing a fractional power of x is present
		1	
(iii)	(When $x = 16$) $\mathrm{d}^2y/\mathrm{d}x^2 = 1/4 > 0$ hence MIN	M1	Checking both of their values in their $\mathrm{d}^2y/\mathrm{d}x^2$
	(When $x = 4$) $\mathrm{d}^2y/\mathrm{d}x^2 = -1/2 < 0$ hence MAX	A1	All correct Alternative methods ok but must be explicit about values of x being considered
		2	

380. 9709_m18_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)(a)	$f(x) > 2$	B1	Accept $y > 2$, $(2, \infty)$, $(2, \infty]$, *range* > 2
		1	
(i)(b)	$g(x) > 6$	B1	Accept $y > 6$, $(6, \infty)$, $(6, \infty]$, *range* > 6
		1	
(i)(c)	$2 < fg(x) < 4$	B1	Accept $2 < y < 4$, $(2, 4)$, $2 < range < 4$
		1	

	Answer	Mark	Partial Marks
(ii)	The range of f is (partly) outside the domain of g	B1	
		1	
(iii)	$f'(x) = \frac{-8}{(x-2)^2}$	B1	SOI
	$y = \frac{8}{x-2} + 2 \rightarrow y - 2 = \frac{8}{x-2} \rightarrow x - 2 = \frac{8}{y-2}$	M1	Order of operations correct. Accept sign errors
	$f^{-1}(x) = \frac{8}{x-2} + 2$	A1	SOI
	$\frac{-48}{(x-2)^2} + \frac{16}{x-2} + 4 - 5 \ (<0) \quad \rightarrow \quad x^2 - 20x + 84 \quad (<0)$	M1	Formation of 3-term quadratic in $x, (x-2)$ or $1/(x-2)$
	$(x-6)(x-14)$ or 6, 14	A1	SOI
	$2 < x < 6$, $x > 14$	A1	CAO
		6	

381. 9709_s18_ms_11 Q: 2

	Answer	Mark	Partial Marks
	$y = 2x + \frac{5}{x} \rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = 2 - \frac{5}{x^2} = -3$ (may be implied) when $x = 1$.	M1 A1	Reasonable attempt at differentiation CAO (-3)
	$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t} \rightarrow -0.06$	M1 A1	Ignore notation, but needs to multiply $\frac{\mathrm{d}y}{\mathrm{d}x}$ by 0.02.
		4	

382. 9709_s18_ms_11 Q: 7

	Answer	Mark	Partial Marks
	$\overrightarrow{OA}=\begin{pmatrix}1\\-3\\2\end{pmatrix}$, $\overrightarrow{OB}=\begin{pmatrix}-1\\3\\5\end{pmatrix}$ and $\overrightarrow{OC}=\begin{pmatrix}3\\1\\-2\end{pmatrix}$		
(i)	$\overrightarrow{AC}=\begin{pmatrix}2\\4\\-4\end{pmatrix}$	B1	B1 for $\overrightarrow{AC}$.
		1	

	Answer	Mark	Partial Marks
(ii)	$\overrightarrow{OM}=\overrightarrow{OA}+\overrightarrow{AM}=\begin{pmatrix}2\\-1\\0\end{pmatrix}$ or $\frac{1}{2}[\begin{pmatrix}1\\-3\\2\end{pmatrix}+\begin{pmatrix}3\\1\\-2\end{pmatrix}]$	M1	M1 for their $\overrightarrow{OM}=\overrightarrow{OA}+\overrightarrow{AM}$ oe
	Unit vector in direction of $\overrightarrow{OM}=\frac{1}{\sqrt{5}}(\overrightarrow{OM})$	M1 A1	M1 for dividing their $\overrightarrow{OM}$ by their modulus
		3	
(iii)	$\overrightarrow{AB}=\begin{pmatrix}-2\\6\\3\end{pmatrix}$, Allow ±	B1	
	$\lvert\overrightarrow{AB}\rvert=7$, $\lvert\overrightarrow{AC}\rvert=6$ $\begin{pmatrix}-2\\6\\3\end{pmatrix}.\begin{pmatrix}2\\4\\-4\end{pmatrix}=-4+24-12=8$	M1 M1	Product of both moduli, Scalar product of ± their AB and AC
	$7\times6\cos\theta=8\rightarrow\theta=79.(0)°$	A1	1.38 radians ok
		4	

383. 9709_s18_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	$\overrightarrow{DA}=6\mathbf{i}-4\mathbf{k}$	B1	
	$\overrightarrow{CA}=6\mathbf{i}-5\mathbf{j}-4\mathbf{k}$	B1	
		2	
(ii)	Method marks awarded only for ***their*** vectors ± $\overrightarrow{CA}$ & ± $\overrightarrow{DA}$		Full marks can be obtained using $\overrightarrow{AC}$ & $\overrightarrow{AD}$
	$\overrightarrow{CA}.\overrightarrow{DA}=36+16\ (=52)$	M1	Using $x_1x_2+y_1y_2+z_1z_2$
	$\lvert\overrightarrow{DA}\rvert=\sqrt{52}$, $\lvert\overrightarrow{CA}\rvert=\sqrt{77}$	M1	Uses modulus twice
	$52=\sqrt{77}\sqrt{52}\cos C\hat{A}D$ oe	M1	All linked correctly
	$\cos C\hat{A}D=0.82178..\rightarrow C\hat{A}D=34.7°$ or 0.606^c awrt	A1	Answer must come from +ve cosine ratio
		4	

384. 9709_s18_ms_13 Q: 8

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = 3x^2 - 18x + 24$	M1A1	Attempt to differentiate. All correct for A mark
	$3x^2 - 18x + 24 = -3$	M1	Equate *their* $\frac{dy}{dx}$ to –3
	$x = 3$	A1	
	$y = 6$	A1	
	$y - 6 = -3(x - 3)$	A1FT	FT on *their A*. Expect $y = -3x + 15$
		6	
(ii)	$(3)(x-2)(x-4)$ SOI or $x = 2, 4$ Allow $(3)(x+2)(x+4)$	M1	Attempt to factorise or solve. Ignore a RHS, e.g. = 0 or > 0, etc.
	Smallest value of *k* is 4	A1	Allow $k \geqslant 4$. Allow *k* = 4. Must be in terms of *k*
		2	

385. 9709_s18_ms_13 Q: 9

	Answer	Mark	Partial Marks
(i)	**OE** = $\frac{2}{10}$(8**i** + 6**j**) = 1.6**i** + 1.2**j** **AG**	M1A1	Evidence of *OB* = 10 or other valid method (e.g. trigonometry) is required
		2	
(ii)	**OD** = 1.6i + 1.2**j** + 7k	B1	Allow reversal of one or both of **OD**, **BD.**
	BD = –8**i** – 6**j** + 1.6i + 1.2**j** + 7k OE = –6.4**i** –4.8**j** + 7**k**	M1A1	For M mark allow sign errors. Also if 2 out of 3 components correct
	Correct method for ±**OD.**±**BD** (using *their* answers)	M1	Expect 1.6 × –6.4 + 1.2 × –4.8 + 49 = 33 or $\frac{825}{25}$ 825 / 25.
	Correct method for |**OD**| or |**BD**| (using *their* answers)	M1	Expect $\sqrt{1.6^2 + 1.2^2 + 7^2}$ or $\sqrt{6.4^2 + 4.8^2 + 7^2}$ = √53 or √113
	Cos *BDO* = *their* $\frac{\mathbf{OD.BD}}{\lvert\mathbf{OD}\rvert \times \lvert\mathbf{BD}\rvert}$	DM1	Expect $\frac{33}{77.4}$. Dep. on all previous M marks and either B1 or A1
	64.8° Allow 1.13(rad)	A1	Can't score A1 if 1 vector only is reversed unless explained well
		7	

386. 9709_w18_ms_11 Q: 8

	Answer	Mark	Partial Marks
(i)	$\overline{DF} = -6\mathbf{i} + 2\mathbf{k}$	B1	
		1	
(ii)	$\overline{EF} = -6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$	B1	
	$\lvert\overline{EF}\rvert = \sqrt{(-6)^2 + (-3)^2 + 2^2}$	M1	Must use *their* $\overline{EF}$
	Unit vector = $\frac{1}{7}(-6\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$	A1	
		3	
(iii)	$\overline{DF}.\overline{EF}$ = (–6**i** + 2**k**).(–6**i** – 3**j** + 2**k**) = 36 + 4 = 40	M1	
	$\lvert\overline{DF}\rvert$ = √40, $\lvert\overline{EF}\rvert$ =7	M1	
	$\cos EFD = \frac{40}{7\sqrt{40}}$ oe	M1	
	EFD = 25.4°	A1	Special case: use of cosine rule M1(must evaluate lengths using correct method) A1 only
		4	

387. 9709_w18_ms_11 Q: 10

	Answer	Mark	Partial Marks
(i)(a)	$\frac{dy}{dx} = \left[-½(4x-3)^{-2}\right]\times[4]$	B1B1	Can gain this in part **(b)(ii)**
	When $x=1,\ m=-2$	B1FT	Ft from *their* $\frac{dy}{dx}$
	Normal is $y-½=½(x-1)$	M1	Line with gradient $-1/m$ and through A
	$y=½x$ soi	A1	Can score in part **(b)**
		5	
(i)(b)	$\frac{1}{2(4x-3)}=\frac{x}{2}\to 2x(4x-3)=2\to(2)(4x^2-3x-1)\ (=0)$	M1A1	$x/2$ seen on RHS of equation can score *previous* A1
	$x=-1/4$	A1	Ignore $x=1$ seen in addition
		3	
(ii)	*Use* of chain rule: $\frac{dy}{dt}=(their -2)\times(\pm)0.3=0.6$	M1A1	Allow +0.3 or –0.3 for M1
		2	

388. 9709_w18_ms_12 Q: 3

	Answer	Mark	Partial Marks
(i)	P is **(t, 5t)** Q is **(t, $t(9-t^2)$)** → **$4t-t^3$**	B1 B1	B1 for both y coordinates which can be implied by subsequent working. B1 for PQ allow $\lvert 4t-t^3\rvert$ or $\lvert t^3-4t\rvert$. **Note:** $4x-x^3$ from equating line and curve 0/2 even if x then replaced by t.
		[2]	

	Answer	Mark	Partial Marks
(ii)	$\frac{d(PQ)}{dt}=4-3t^2$	B1FT	B1FT for differentiation of their PQ, which MUST be a cubic expression, but can be $\frac{d}{dx}f(x)$ from **(i)** but not the equation of the curve.
	$=0\to t=+\frac{2}{\sqrt{3}}$	M1	Setting their differential of PQ to 0 and attempt to solve for t or x.
	→ **Maximum PQ** = $\frac{16}{3\sqrt{3}}$ **or** $\frac{16\sqrt{3}}{9}$	A1	Allow 3.08 awrt. If answer comes from wrong method in **(i)** award A0. Correct answer from correct expression by T&I scores 3/3.
		3	

389. 9709_w18_ms_12 Q: 7

	Answer	Mark	Partial Marks
	$\overrightarrow{PN}$ = 8**i** – 8**k**	**B1**	
	$\overrightarrow{PM}$ = 4**i** + 4**j** – 6**k**	**B2,1,0**	Loses 1 mark for each component incorrect
			SC: $\overrightarrow{PN}$ = – 8**i** + 8**k** **and** $\overrightarrow{PM}$ = – 4**i** – 4**j** +6**k** scores 2/3.
	$\overrightarrow{PN}.\overrightarrow{PM}$ = 32 + 0 + 48 = 80	**M1**	Evaluates $x_1x_2+y_1y_2+z_1z_2$ for correct vectors or one or both reversed.
	$\lvert PN\rvert\times\lvert PM\rvert = \sqrt{128}\times\sqrt{68}$ (= $16\sqrt{34}$)	**M1**	Product of their moduli – may be seen in cosine rule
	$\sqrt{128}\times\sqrt{68}\cos M\hat{P}N = 80$	**M1**	All linked correctly.
	Angle $M\hat{P}N$ = **31.0°** **awrt**	**A1**	Answer must come directly from +ve cosine ratio. Cosine rule not accepted as a complete method. Allow 0.540^c awrt. **Note:** Correct answer from incorrect vectors scores A0 (XP)
		7	

390. 9709_w18_ms_13 Q: 2

	Answer	Mark	Partial Marks
	$f'(x)=3x^2+4x-4$	**B1**	
	Factors or crit. values or sub any 2 values $(x\neq-2)$ into $f'(x)$ soi	**M1**	Expect $(x+2)(3x-2)$ or –2, ⅔ or any 2 subs (excluding $x = -2$).
	For $-2<x<\frac{2}{3}$, $f'(x)$ < 0; for $x>\frac{2}{3}$, $f'(x)$ >0 soi Allow ⩽, ⩾	**M1**	Or at least 1 specific value $(\neq-2)$ in each interval giving opp signs Or f′($\frac{2}{3}$)=0 and f″($\frac{2}{3}$)$\neq 0$ (i.e. gradient changes sign at $x = \frac{2}{3}$)
	Neither www	**A1**	Must have 'Neither'
	ALT 1 At least 3 values of f(x)	**M1**	e.g. f(0) = 7, f(1) = 6, f(2) = 15
	At least 3 correct values of f(x)	**A1**	
	At least 3 correct values of f(x) spanning x = ⅔	**A1**	
	Shows a decreasing and then increasing pattern. Neither www	**A1**	Or similar wording. Must have 'Neither'
	ALT 2 $f'(x)=3x^2+4x-4 = 3(x+\frac{2}{3})^2-\frac{16}{3}$	**B1B1**	Do not condone sign errors
	$f'(x)\geqslant-\frac{16}{3}$	**M1**	
	$f'(x)$ < 0 for some values and > 0 for other values. Neither www	**A1**	Or similar wording. Must have 'Neither'
		4	

391. 9709_w18_ms_13 Q: 6

	Answer	Mark	Partial Marks
	$(\mathbf{BO})=-8\mathbf{i}-6\mathbf{j}$	**B1**	OR (**OB**) = 8**i** + 6**j**
	$(\mathbf{BF})=-6\mathbf{j}-8\mathbf{i}+7\mathbf{k}+4\mathbf{i}+2\mathbf{j} = -4\mathbf{i}-4\mathbf{j}+7\mathbf{k}$	**B1**	OR (**FB**) = 4**i** + 4**j** – 7**k**
	$(\mathbf{BF.BO})=(-4)(-8)+(-4)(-6)$	**M1**	OR (**FB.OB**) Expect 56. Accept one reversed but award final A0
	$\lvert\mathbf{BF}\rvert\times\lvert\mathbf{BO}\rvert=\sqrt{4^2+4^2+7^2}\times\sqrt{8^2+6^2}$	**M1**	Expect 90. At least one magnitude methodically correct
	Angle $OBF = \cos^{-1}\left(\frac{their\ 56}{their\ 90}\right)=\cos^{-1}\left(\frac{56}{90}\right)$ or $\cos^{-1}\left(\frac{28}{45}\right)$	**DM1A1**	Or equivalent 'integer' fractions. All M marks dependent on use of (±)**BO** and (±)**BF**. 3rd M mark dep on both preceding M marks
		6	

392. 9709_m17_ms_12 Q: 3

	Answer	Mark	Partial Marks
(i)	$V = \frac{1}{12}h^3$ oe	B1	
	Total:	1	
(ii)	$\frac{dV}{dh} = \frac{1}{4}h^2$ or $\frac{dh}{dV} = 4(12v)^{-2/3}$	M1A1	Attempt differentiation. Allow incorrect notation for M. For A mark accept *their* letter for volume - but otherwise correct notation. Allow V'
	$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $= \frac{4}{h^2} \times 20$ soi	DM1	Use chain rule correctly with $\frac{d(V)}{dt} = 20$. Any equivalent formulation. Accept non-explicit chain rule (or nothing at all)
	$\left(\frac{dh}{dt}\right) = \frac{4}{10^2} \times 20 = 0.8$ or equivalent fraction	A1	
	Total:	4	

393. 9709_m17_ms_12 Q: 6

	Answer	Mark	Partial Marks
(i)	**BA** = **OA** – **OB** = –5**i** – **j** + 2**k**	B1	Allow vector reversed. Ignore label **BA** or **AB**
	OA.BA = –10 – 3 + 10 = –3	M1	soi by ±3
	$\lvert\mathbf{OA}\rvert \times \lvert\mathbf{BA}\rvert = \sqrt{2^2+3^2+5^2} \times \sqrt{5^2+1^2+2^2}$	M1	Prod. of mods for at least 1 correct vector or reverse.
	$\cos OAB = \frac{+/-3}{\sqrt{38} \times \sqrt{30}}$	M1	
	$OAB = 95.1°$ (or 1.66^c)	A1	
	Total:	5	
(ii)	$\Delta\, OAB = \frac{1}{2}\sqrt{38} \times \sqrt{30} \sin 95.1$. Allow $½\sqrt{38} \times \sqrt{74} \sin 39.4$	M1	Allow their moduli product from **(i)**
	= 16.8	A1	cao but NOT from sin 84.9 (1.482^c)
	Total:	2	

394. 9709_m17_ms_12 Q: 7

	Answer	Mark	Partial Marks
(i)	$f'(x) = \left[\frac{3}{2}(4x+1)^{1/2}\right] [4]$	B1B1	Expect $6(4x+1)^{1/2}$ but can be unsimplified.
	$f''(x) = 6 \times 1/2 \times (4x+1)^{-1/2} \times 4$	B1√	Expect $12(4x+1)^{-1/2}$ but can be unsimplified. Ft from *their* f ′(*x*).
	Total:	3	
(ii)	f(2), f ′(2), *k*f ″(2) = 27, 18, 4*k* OR 12	B1B1√B1√	Ft dependent on attempt at differentiation
	27/18 = 18/4*k* oe OR *k*f ″(2) = 12 $\Rightarrow k = 3$	M1A1	
	Total:	5	

395. 9709_m17_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = 2x - 2$. At $x = 2$, $m = 2$	B1B1	Numerical m
	Equation of tangent is $y - 2 = 2(x - 2)$	B1	Expect $y = 2x - 2$
	Total:	3	
(ii)	Equation of normal $y - 2 = -½(x - 2)$	M1	Through (2, 2) with gradient $= -1/m$. Expect $y = -½x + 3$
	$x^2 - 2x + 2 = -½x + 3 \rightarrow 2x^2 - 3x - 2 = 0$	M1	Equate and simplify to 3-term quadratic
	$x = -½,\ y = 3¼$	A1A1	Ignore answer of (2, 2)
	Total:	4	

	Answer	Mark	Partial Marks
(iii)	At $x = -½$, grad$= 2(-½) - 2 = -3$	B1√	Ft *their* –½.
	Equation of tangent is $y - 3¼ = -3(x + ½)$	*M1	Through *their B* with grad *their* –3 (not m_1 or m_2). Expect $y = -3x + 7/4$
	$2x - 2 = -3x + 7/4$	DM1	Equate *their* tangents or attempt to solve simultaneous equations
	$x = 3/4,\ y = -½$	A1	Both required.
	Total:	4	

396. 9709_s17_ms_11 Q: 2

	Answer	Mark	Partial Marks
	$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix}$		
(i)	Angle $AOB = 90° \rightarrow 6 + 36 - 7p = 0$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$ or Pythagoras
	$\rightarrow p = 6$	A1	
	Total:	2	

	Answer	Mark	Partial Marks
(ii)	$\overrightarrow{OC} = \frac{2}{3}\begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$	B1 FT	CAO FT on their value of p
	$\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix} 0 \\ 2 \\ 11 \end{pmatrix}$; magnitude $= \sqrt{125}$	M1 M1	Use of **c** – **b**. Allow magnitude of **b** + **c** or **b** – **c** Allow first **M1** in terms of p
	Unit vector $= \frac{1}{\sqrt{125}}\begin{pmatrix} 0 \\ 2 \\ 11 \end{pmatrix}$	A1	OE Allow ± and decimal equivalent

397. 9709_s17_ms_11 Q: 6

	Answer	Mark	Partial Marks
(i)	Volume $= \left(\frac{1}{2}\right)x^2\frac{\sqrt{3}}{2}h = 2000 \rightarrow h = \frac{8000}{\sqrt{3}x^2}$	M1	Use of (area of triangle, with attempt at ht) × h =2000, $h = \text{f}(x)$
	$A = 3xh + (2)\times\left(\frac{1}{2}\right)\times x^2 \times \frac{\sqrt{3}}{2}$	M1	Uses 3 rectangles and at least one triangle
	Sub for $h \rightarrow A = \frac{\sqrt{3}}{2}x^2 + \frac{24000}{\sqrt{3}}x^{-1}$	A1	AG
	Total:	3	
(ii)	$\frac{dA}{dx} = \frac{\sqrt{3}}{2}2x - \frac{24000}{\sqrt{3}}x^{-2}$	B1	CAO, allow decimal equivalent
	= 0 when $x^3 = 8000 \rightarrow x = 20$	M1 A1	Sets their $\frac{dA}{dx}$ to 0 and attempt to solve for x
	Total:	3	

	Answer	Mark	Partial Marks
(iii)	$\frac{d^2A}{dx^2} = \frac{\sqrt{3}}{2}2 + \frac{48000}{\sqrt{3}}x^{-3} > 0$	M1	Any valid method, ignore value of $\frac{d^2A}{dx^2}$ providing it is positive
	→ Minimum	A1 FT	FT on their x providing it is positive
	Total:	2	

398. 9709_s17_ms_12 Q: 5

	Answer	Mark	Partial Marks
(i)	Crosses x-axis at (6, 0)	B1	$x = 6$ is sufficient.
	$\frac{dy}{dx} = (0 +) -12\,(2-x)^{-2} \times (-1)$	B2,1,0	−1 for each incorrect term of the three or addition of + C.
	Tangent $y = ¾(x-6)$ or $4y = 3x - 18$	M1 A1	Must use dy/dx, x= their 6 but not x = 0 (which gives m = 3), and correct form of line equation.
			Using $y = mx + c$ gets **A1** as soon as c is evaluated.
	Total:	5	
(ii)	If x = 4, dy/dx = 3		
	$\frac{dy}{dt} = 3 \times 0.04 = 0.12$	M1 A1FT	**M1** for ("their m" from $\frac{dy}{dx}$ and x = 4) × 0.04. Be aware: use of x = 0 gives the correct answer but gets **M0**.
	Total:	2	

399. 9709_s17_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	Uses scalar product correctly: $3 \times 6 + 2 \times 6 + (-4) \times 3 = 18$	M1	Use of dot product with $\overrightarrow{OA}$ or $\overrightarrow{AO}$ & $\overrightarrow{OB}$ or $\overrightarrow{BO}$ only.
	$\lvert\overrightarrow{OA}\rvert = \sqrt{29}$, $\lvert\overrightarrow{OB}\rvert = 9$	M1	Correct method for any one of $\lvert\overrightarrow{OA}\rvert$, $\lvert\overrightarrow{AO}\rvert$, $\lvert\overrightarrow{OB}\rvert$ or $\lvert\overrightarrow{BO}\rvert$.
	$\sqrt{29} \times 9 \times \cos AOB = 18$	M1	All linked correctly.
	$\rightarrow AOB = 68.2°$ or 1.19^c	A1	Multiples of π are acceptable (e.g. $0.379\pi^c$)
	Total:	4	
(ii)	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + (3+2p)\mathbf{k}$	*M1	For use of $\overrightarrow{OB} - \overrightarrow{OA}$, allow with p = 2
	Comparing "**j**"	DM1	For comparing, $\overrightarrow{OC}$ must contain p & q. Can be implied by $\overrightarrow{AB} = 2\overrightarrow{OC}$.
	$\rightarrow p = 2\frac{1}{2}$ and $q = 4$	A1 A1	Accuracy marks only available if $\overrightarrow{AB}$ is correct.
	Total:	4	

400. 9709_s17_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = 4x^{-\frac{1}{2}} - 2$	B1	Accept unsimplified.
	$= 0$ when $\sqrt{x} = 2$		
	$x = 4$, $y = 8$	B1B1	
	Total:	3	
(ii)	$\frac{d^2y}{dx^2} = -2x^{-\frac{3}{2}}$	B1FT	FT providing –ve power of x
	$\left(\frac{d^2y}{dx^2} = -\frac{1}{4}\right) \rightarrow$ Maximum	B1	Correct $\frac{d^2y}{dx^2}$ and x=4 in (i) are required. Followed by "< 0 or negative" is sufficient" but $\frac{d^2y}{dx^2}$ must be correct if evaluated.
	Total:	2	
(iii)	*EITHER:* Recognises a quadratic in $\sqrt{x}$	(M1	Eg $\sqrt{x} = u \rightarrow 2u^2 - 8u + 6 = 0$
	1 and 3 as solutions to this equation	A1	
	$\rightarrow x = 9, x = 1.$	A1)	

	Answer	Mark	Partial Marks
	OR: Rearranges then squares	(M1	$\sqrt{x}$ needs to be isolated before squaring both sides.
	$\rightarrow x^2 - 10x + 9 = 0$ oe	A1	
	$\rightarrow x = 9, x = 1.$	A1)	Both correct by trial and improvement gets 3/3
	Total:	3	
(iv)	$k > 8$	B1	
	Total:	1	

401. 9709_s17_ms_13 Q: 4

	Answer	Mark	Partial Marks
(i)	$\overrightarrow{OB}-\overrightarrow{OA}\ \left(=\overrightarrow{AB}\right)=\begin{pmatrix}5\\4\\-3\end{pmatrix}-\begin{pmatrix}5\\1\\3\end{pmatrix}=\begin{pmatrix}0\\3\\-6\end{pmatrix}$	B1	
	$\overrightarrow{OP}=\begin{pmatrix}5\\1\\3\end{pmatrix}+\frac{1}{3}\begin{pmatrix}0\\3\\-6\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}$	M1 A1	If $\overrightarrow{OP}$ not scored in (i) can score SR **B1** if seen correct in (ii). Other equivalent methods possible
	Total:	3	
(ii)	Distance $OP=\sqrt{5^2+2^2+1^2}\ =\ \sqrt{30}$ or 5.48	B1 FT	FT on *their* $\overrightarrow{OP}$ from (i)
	Total:	1	
(iii)	Attempt $\overrightarrow{AB}.\overrightarrow{OP}$. Can score as part of $\overrightarrow{AB}.\overrightarrow{OP}=(AB)(OP)\cos\theta$ Rare ALT: Pythagoras $\left\|\overrightarrow{OP}\right\|^2+\left\|\overrightarrow{AP}\right\|^2=5+30=\left\|\overrightarrow{OA}\right\|^2$	M1	Allow any combination of $\overrightarrow{AB}.\overrightarrow{PO}$ etc. and also if $\overrightarrow{AP}$ or $\overrightarrow{PB}$ used instead of $\overrightarrow{AB}$ giving 2–2 = 0 & 4–4 = 0 respectively. Allow notation × instead of **.** .
	(0 + 6 – 6) = 0 hence perpendicular. (Accept 90º)	A1 FT	If result not zero then 'Not perpendicular' can score **A1**FT if value is 'correct' for *their* values of $\overrightarrow{AB},\overrightarrow{OP}$ etc. from (i).
	Total:	2	

402. 9709_s17_ms_13 Q: 6

	Answer	Mark	Partial Marks
	Gradient of normal is – 1/3 → gradient of tangent is 3 SOI	B1 B1 FT	FT from *their* gradient of normal.
	$dy/dx = 2x - 5 = 3$	M1	Differentiate and set = *their* 3 (numerical).
	$x = 4$	*A1	
	Sub $x = 4$ into line → $y = 7$ & sub *their* (4, 7) into curve	DM1	OR sub $x = 4$ into curve → $y = k - 4$ and sub *their*$(4, k - 4)$ into line OR other valid methods deriving a linear equation in k (e.g. equating curve with either normal or tangent and sub $x = 4$).
	$k = 11$	A1	
	Total:	6	

403. 9709_w17_ms_11 Q: 1

	Answer	Mark	Partial Marks
	$\frac{dy}{dx}=3x^{1/2}-3-2x^{-1/2}$	B2,1,0	
	at $x = 4$, $\frac{dy}{dx}=6-3-1=2$	M1	
	Equation of tangent is $y=2(x-4)$ OE	A1FT	Equation through (4, 0) with *their* gradient
		4	

404. 9709_w17_ms_11 Q: 2

	Answer	Mark	Partial Marks
	$f'(x)=3x^2-2x-8$	M1	Attempt differentiation
	$-\frac{4}{3}$, 2 SOI	A1	
	$f'(x)>0 \Rightarrow x<-\frac{4}{3}$ SOI	M1	Accept $x > 2$ in addition. FT *their* solutions
	Largest value of a is $-\frac{4}{3}$	A1	Statement in terms of a. Accept $a\leqslant-\frac{4}{3}$ or $a<-\frac{4}{3}$. Penalise extra solutions
		4	

405. 9709_w17_ms_11 Q: 4

	Answer	Mark	Partial Marks
(i)	$V=\frac{1}{3}\pi r^2(18-r)=6\pi r^2-\frac{1}{3}\pi r^3$	B1	AG
		1	
(ii)	$\frac{dV}{dr}=12\pi r-\pi r^2=0$	M1	Differentiate and set = 0
	$\pi r(12-r)=0 \to r=12$	A1	
	$\frac{d^2V}{dr^2}=12\pi-2\pi r$	M1	
	Sub r = 12 → $12\pi-24\pi=-12\pi \to$ MAX	A1	AG
		4	
(iii)	Sub $r=12,\ h=6 \to \text{Max}\,V=288\pi$ or 905	B1	
		1	

406. 9709_w17_ms_11 Q: 8

	Answer	Mark	Partial Marks
(a)	*EITHER:* $\overrightarrow{PR}=2\overrightarrow{PQ}=2(\mathbf{q}-\mathbf{p})$	(B1	
	$\overrightarrow{OR}$=**p**+2**q**–2**p** = 2**q**–**p**	M1A1)	
	OR: $\overrightarrow{QR}=\overrightarrow{PQ}=\mathbf{q}-\mathbf{p}$	(B1	
	$\overrightarrow{OR}=\overrightarrow{OQ}+\overrightarrow{QR}$=**q**+**q**–**p**=2**q**–**p**	M1A1)	Or other valid method
		3	
(b)	$6^2+a^2+b^2=21^2$ SOI	B1	
	$18+2a+2b=0$	B1	
	$a^2+(-a-9)^2=405$	M1	Correct method for elimination of a variable. (Or same equation in b)
	$(2)(a^2+9a-162)(=0)$	A1	Or same equation in b
	$a=9$ or -18	A1	
	$b=-18$ or 9	A1	
		6	

407. 9709_w17_ms_12 Q: 7

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx}=2x-4=0$		Can use completing the square.
	$\to x=2,\ y=3$	B1 B1	
	Midpoint of AB is (3, 5)	B1 FT	**FT** on (*their* 2, *their* 3) with (4,7)
	$\to m=\frac{7}{3}$ (or 2.33)	B1	
		4	
(ii)	Simultaneous equations → $x^2-4x-mx+9(=0)$	*M1	Equates and sets to 0 must contain m
	Use of $b^2-4ac \to (m+4)^2-36$	DM1	Any use of b^2-4ac on equation set to 0 must contain m
	Solves = 0 → −10 or 2	A1	Correct end-points.
	$-10 < m < 2$	A1	Don't condone $\leqslant$ at either or both end(s). Accept $-10 < m$, $m < 2$.
		4	

408. 9709_w17_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$\overrightarrow{AB} = \begin{pmatrix}4\\3\\2\end{pmatrix}$ or $\overrightarrow{BA} = \begin{pmatrix}-4\\-3\\-2\end{pmatrix}$	M1	Use of **b** – **a** or **a** – **b**
	e.g. $\overrightarrow{AO}.\overrightarrow{AB} = -8+6+2=0 \rightarrow O\hat{A}B = 90°$ AG **OR** $\lvert\overrightarrow{OA}\rvert = 3, \lvert\overrightarrow{OB}\rvert = \sqrt{38}, \lvert\overrightarrow{AB}\rvert = \sqrt{29}$ $OA^2 + AB^2 = OB^2 \rightarrow O\hat{A}B = 90°$ AG	M1 A1	Use of dot product with either $\overrightarrow{AO}$ or $\overrightarrow{OA}$ & either $\overrightarrow{AB}$ or $\overrightarrow{BA}$. Must see 3 component products OR Correct use of Pythagoras. In both methods must state angle or $\Theta = 90°$ or similar for **A1**
		3	
(ii)	$\overrightarrow{CB} = \begin{pmatrix}6\\-6\\-3\end{pmatrix}$ or $\overrightarrow{BC} = \begin{pmatrix}-6\\6\\3\end{pmatrix}$	B1	Must correctly identify the vector.
	$\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC}$ (or $-\overrightarrow{CB}$) $= \begin{pmatrix}0\\7\\4\end{pmatrix}$	M1 A1	Correct link leading to $\overrightarrow{OC}$
		3	

	Answer	Mark	Partial Marks
(iii)	$\lvert\overrightarrow{OA}\rvert = 3, \lvert\overrightarrow{BC}\rvert = 9, \lvert\overrightarrow{AB}\rvert = \sqrt{29}$ (5.39)	B1	For any one of these
	Area = ½(3 + 9) $\sqrt{29}$ or 3 $\sqrt{29}$ + 3 $\sqrt{29}$	M1	Correct formula(e) used for trapezium or (rectangle + triangle) or two triangles using their lengths.
	= 6 $\sqrt{29}$ ($1\sqrt{1044}, 2\sqrt{261}$ *or* $3\sqrt{116}$)	A1	Exact answer in correct form.
		3	

409. 9709_w17_ms_13 Q: 4

	Answer	Mark	Partial Marks
	$f'(x) = \left[\left(\frac{3}{2}\right)(2x-1)^{1/2}\right] \times [2] - [6]$	B2, 1, 0	Deduct 1 mark for each [...] incorrect.
	$f'(x) < 0$ or $\leqslant 0$ or $= 0$ SOI	M1	
	$(2x-1)^{1/2} < 2$ or $\leqslant 2$ or $= 2$ OE	A1	Allow with k used instead of x
	Largest value of k is $\frac{5}{2}$	A1	Allow $k \leqslant \frac{5}{2}$ or $k = \frac{5}{2}$ Answer must be in terms of k (not x)
		5	

410. 9709_w17_ms_13 Q: 9

	Answer	Mark	Partial Marks
(i)	$\overrightarrow{AB} = +/- \begin{pmatrix} -18 \\ 9 \\ -18 \end{pmatrix}$, $\overrightarrow{BC} = +/- \begin{pmatrix} 12 \\ -6 \\ 12 \end{pmatrix}$,	**B1 B1**	Allow **i**, **j**, **k** form throughout.
	$\lvert\overrightarrow{AB}\rvert = 27$, $\lvert\overrightarrow{BC}\rvert = 18$	**B1 FT** **B1 FT**	FT on *their* $\overrightarrow{AB}$, *their* $\overrightarrow{OD}$.
	$\lvert\overrightarrow{CD}\rvert = \left(\frac{18}{27}\right)\times 18$ OR $\left(\frac{18}{27}\right)^2 \times 27 = 12$	**B1**	
		5	
(ii)	$\overrightarrow{CD} = (\pm)\textit{their}\,\frac{18}{27}\times \textit{their}\ \overrightarrow{BC}$ SOI	**M1**	Expect $(\pm)\begin{pmatrix} 8 \\ -4 \\ 8 \end{pmatrix}$.
	$\overrightarrow{OD} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} (\pm)\ \textit{their}\ \frac{18}{27}\begin{pmatrix} 12 \\ -6 \\ 12 \end{pmatrix} = \begin{pmatrix} 10 \\ -7 \\ 7 \end{pmatrix}, \begin{pmatrix} -6 \\ 1 \\ -9 \end{pmatrix}$	**M1 A1 A1**	Other methods possible for $\overrightarrow{OD}$, e.g. $\overrightarrow{OB} + \frac{5}{2}\overrightarrow{CD}$, $\overrightarrow{OB} + \frac{1}{2}\overrightarrow{CD}$ (One soln **M2A1**, 2nd soln **A1**) OR $\overrightarrow{OB} + \frac{5}{3}\overrightarrow{BC}$, $\overrightarrow{OB} + \frac{1}{3}\overrightarrow{BC}$ (One soln **M2A1**, 2nd soln **A1**)
		4	

411. 9709_w17_ms_13 Q: 11

	Answer	Mark	Partial Marks
(i)	Gradient of $AB = \frac{1}{2}$	**B1**	
	Equation of AB is $y = \frac{1}{2}x - \frac{1}{2}$	**B1**	
		2	
(ii)	$\frac{dy}{dx} = ½(x-1)^{-\frac{1}{2}}$	**B1**	
	$½(x-1)^{-\frac{1}{2}} = ½$. Equate *their* $\frac{dy}{dx}$ to *their* ½	***M1**	
	$x = 2,\ y = 1$	**A1**	
	$y - 1 = ½(x - 2)$ (thro' *their*(2,1) & *their* ½) → $y = ½x$	**DM1 A1**	
		5	

	Answer	Mark	Partial Marks
(iii)	*EITHER:* $\sin\theta = \frac{d}{1} \rightarrow d = \sin\theta$	(M1	Where θ is angle between AB and the x-axis
	gradient of $AB = ½ \Rightarrow \tan\theta = ½ \Rightarrow \theta = 26.5(7)°$	B1	
	$d = \sin 26.5(7)° = 0.45$ (or $\frac{1}{\sqrt{5}}$)	A1)	
	OR1: Perpendicular through O has equation $y = -2x$	(M1	
	Intersection with AB: $-2x = ½x - ½ \rightarrow \left(\frac{1}{5}, \frac{-2}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $\frac{1}{\sqrt{5}}$)	A1)	
	OR2: Perpendicular through (2, 1) has equation $y = -2x + 5$	(M1	
	Intersection with AB: $-2x + 5 = ½x - ½ \rightarrow \left(\frac{11}{5}, \frac{3}{5}\right)$	A1	
	$d = \sqrt{\left(\frac{1}{5}\right)^2 + \left(\frac{2}{5}\right)^2} = 0.45$ (or $1/\sqrt{5}$)	A1)	

	Answer	Mark	Partial Marks
(iii)	*OR3:* ΔOAC has area $\frac{1}{4}$ [where $C = (0, -\frac{1}{2})$]	(B1	
	$\frac{1}{2} \times \frac{\sqrt{5}}{2} \times d = \frac{1}{4} \rightarrow d = \frac{1}{\sqrt{5}}$	M1 A1)	
		3	

412. 9709_m16_ms_12 Q: 6

	Answer	Mark	Partial Marks
(i)	$A = 2\pi r^2 + 2\pi rh$	B1	
	$\pi r^2 h = 1000 \rightarrow h = \frac{1000}{\pi r^2}$	M1	
	Sub for h into A $\rightarrow$ $A = 2\pi r^2 + \frac{2000}{r}$ **AG**	A1 [3]	
(ii)	$\frac{dA}{dr} = 0 \Rightarrow 4\pi r - \frac{2000}{r^2} = 0$	M1A1	Attempt differentiation & set = 0
	$r = = 5.4$	DM1 A1	Reasonable attempt to solve to $r^3 =$
	$\frac{d^2 A}{dr^2} = 4\pi + \frac{4000}{r^3}$		
	> 0 hence MIN hence MOST EFFICIENT **AG**	B1 [5]	Or convincing alternative method

413. 9709_m16_ms_12 Q: 7

	Answer	Mark	Partial Marks
(i)	$CP = \frac{3}{5}CA$ soi $CP = \frac{3}{5}(4\mathbf{i} - 3\mathbf{k})$ = $2.4\mathbf{i} - 1.8\mathbf{k}$ **AG**	**M1** **A1** **[2]**	
(ii)	OP = $2.4\mathbf{i} + 1.2\mathbf{k}$ BP = $2.4\mathbf{i} - 2.4\mathbf{j} + 1.2\mathbf{k}$	**B1** **B1** **[2]**	
(iii)	$BP.CP = 5.76 - 2.16 = 3.6$ $\lvert BP\rvert\,\lvert CP\rvert = \sqrt{2.4^2+2.4^2+1.2^2}\sqrt{2.4^2+1.8^2}$ $\cos BPC = \frac{3.6}{\sqrt{12.96}\sqrt{9}}$ $\left(=\frac{1}{3}\right)$ Angle $BPC = 70.5°$ (or 1.23 rads) cao	**M1** **M1** **M1** **A1** **[4]**	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Product of moduli All linked correctly

414. 9709_s16_ms_11 Q: 5

	Answer	Mark	Partial Marks
(i)	$A = 2y \times 4x (= 8xy)$ $10y + 12x = 480$ $\rightarrow A = 384x - 9.6x^2$	**B1** **B1** **B1** [3]	answer given
(ii)	$\frac{dA}{dx} = 384 - 19.2x$ $= 0$ when $x = 20$ $\rightarrow x = 20, y = 24.$ Uses $x = -\frac{b}{2a} = \frac{-384}{-19.2} = 20$, **M1**, **A1** $y = 24$, A1 From graph: **B1** for $x = 20$, **M1**, **A1** for $y = 24$	**B1** **M1** **A1** [3]	Sets to 0 and attempt to solve oe Might see completion of square Needs both x and y Trial and improvement **B3**.

415. 9709_s16_ms_11 Q: 8

	Answer	Mark	Partial Marks
	$y = 3x - \frac{4}{x}$		
	$\frac{dy}{dx} = 3 + \frac{4}{x^2}$	B1	
	m of $AB = 4$	B1	
	Equate $\rightarrow x = \pm 2$ $\rightarrow C\ (2, 4)$ and $D\ (-2, -4)$	M1 A1	Equating + solution.
	$\rightarrow M\ (0, 0)$ or stating M is the origin	B1√	√ on their C and D
	m of $CD = 2$ Perpendicular gradient $(= -\frac{1}{2})$	M1	Use of $m_1 m_2 = -1$, must use m_{CD} (not m = 4)
	$\rightarrow y = -\frac{1}{2}x$	A1 [7]	

416. 9709_s16_ms_11 Q: 10

	Answer	Mark	Partial Marks
(i)	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$		
	$10 - 1 - 2k = 0 \rightarrow k = 4\frac{1}{2}$	M1 A1 [2]	Use of scalar product = 0.
(ii)	$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -2 \\ k+2 \end{pmatrix}$,	B1	
	$\lvert\overrightarrow{OC}\rvert = 7$ (seen or implied)	B1	
	$3^2 + (-2)^2 + (k+2)^2 = 49$ $\rightarrow k = 4$ or -8	M1 A1 [4]	Correct method. Both correct. Condone sign error in $\overrightarrow{AB}$
(iii)	$\lvert\overrightarrow{OA}\rvert = 3$		
	$\overrightarrow{OD} = 3\overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix}$ and $\overrightarrow{OE} = 2$	M1 A1	Scaling from magnitudes/unit vector – oe.
	$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 12 \\ -6 \end{pmatrix}$		
	$\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} = \begin{pmatrix} -2 \\ 9 \\ 0 \end{pmatrix}$,	M1	Correct vector subtraction.
	$\rightarrow$ Magnitude of $\sqrt{85}$.	A1 [4]	

417. 9709_s16_ms_12 Q: 3

	Answer	Mark	Partial Marks
	$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$. $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ **or** $\overrightarrow{AC} = 4\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$ $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ **OR** $\begin{pmatrix}2\\1\\4\end{pmatrix} = \begin{pmatrix}x-4\\y+4\\z-2\end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}6\\-3\\6\end{pmatrix}$ **OR** $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OB}$ $\therefore \overrightarrow{OC} = 2\overrightarrow{OB} - \overrightarrow{OA}$ $= \begin{pmatrix}8\\-8\\4\end{pmatrix} - \begin{pmatrix}2\\-5\\-2\end{pmatrix} = \begin{pmatrix}6\\-3\\6\end{pmatrix}$ Unit vector = (Their $\overrightarrow{OC}$) ÷ (Mod their $\overrightarrow{OC}$) = **(6i −3j + 6k)** ÷ 9	 **B1** **M1** **B1** **M1** **B1** **M1** **M1** **A1** [4]	 correct method for $\overrightarrow{OC}$ Divides by their mod of their $\overrightarrow{OC}$ Correct unsimplified expression

418. 9709_s16_ms_13 Q: 5

	Answer	Mark	Partial Marks
	$\frac{dy}{dx} = [8] + [-2]\,[(2x-1)^{-2}]$ $= 0 \rightarrow 4(2x-1)^2 = 1$ oe eg $16x^2 - 16x + 3 = 0$ $x = \frac{1}{4}$ and $\frac{3}{4}$ $\frac{d^2y}{dx^2} = 8(2x-1)^{-3}$ When $x = \frac{1}{4}$, $\frac{d^2y}{dx^2}(= -64)$ and/or < 0 MAX When $x = \frac{3}{4}$, $\frac{d^2y}{dx^2}(= 64)$ and/or > 0 MIN	**B2,1,0** **M1** **A1** **B1√*** **DB1** **DB1** [7]	 Set to zero, simplify and attempt to solve soi Needs both x values. Ignore y values ft to $k(2x-1)^{-3}$ where $k > 0$ Alt. methods for last 3 marks (values either side of 1/4 & 3/4) must indicate <u>which</u> x-values and cannot use x = 1/2. (M1A1A1)

419. 9709_s16_ms_13 Q: 7

	Answer	Mark	Partial Marks
	$\frac{dy}{dx} = 2x - 5x^{1/2} + 5$	B1	
	$\frac{dy}{dx} = 2$	B1	
	$2x - 5x^{1/2} + 5 = 2$	M1	Equate their dy/dx to *their* 2 or ½.
	$2x - 5x^{1/2} + 3(=0)$ or equivalent 3-term quadratic	A1	
	Attempt to solve for $x^{1/2}$ e.g. $(2x^{1/2} - 3)(x^{1/2} - 1) = 0$	DM1	Dep. on 3-term quadratic
	$x^{1/2} = 3/2$ and 1	A1	ALT
	$x = 9/4$ and 1	A1 [7]	$5x^{½} = 2x + 3 \rightarrow 25x = (2x+3)^2$ $4x^2 - 13x + 9(=0)$ $x = 9/4$ and 1

420. 9709_s16_ms_13 Q: 9

	Answer	Mark	Partial Marks
(i)	**AB** = **OB** – **OA** = $\begin{pmatrix} -1 \\ 2 \\ p+4 \end{pmatrix}$	B1	Ignore labels. Allow **BA** or **BC**
	CB = **OB** – **OC** = $\begin{pmatrix} -4 \\ 5 \\ p-2 \end{pmatrix}$	B1	
	$1 + 4 + (p+4)^2 = 16 + 25 + (p-2)^2$	M1	
	$p = 2$	A1 [4]	
(ii)	**AB.CB** = 4+10−5 = 9	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2$
	$\lvert$**AB**$\rvert = \sqrt{1+4+25} = \sqrt{30}$, $\lvert$**CB**$\rvert = \sqrt{16+25+1} = \sqrt{42}$	M1	Product of moduli
	$\cos ABC = \frac{9}{\sqrt{30}\sqrt{42}}$ or $\frac{9}{6\sqrt{35}}$	M1	Allow one of **AB**, **CB** reversed - but award **A0**
	$ABC = 75.3°$ or 1.31rads (ignore reflex angle 285°)	A1 [4]	

421. 9709_w16_ms_11 Q: 9

	Answer	Mark		Partial Marks
(i)	**XP** = −4**i** + (p – 5)**j** + 2**k** [−4**i** + (p – 5)**j** + 2**k**].(p**j** + 2**k**) = 0 $p^2 - 5p + 4 = 0$ $p = 1$ or 4	**B1** **M1** **A1** **A1**	[4]	Or **PX** Attempt scalar prod with **OP/PO** and set = 0 ($= 0$ could be implied)
(ii)	**XP** = −4**i** + 4**j** + 2**k** → |**XP**| = $\sqrt{16+16+4}$ Unit vector $= 1/6$ (−4**i** + 4**j** + 2**k**) oe	**M1** **A1**	[2]	Expect 6
(iii)	**AG** = −4**i** + 15**j** + 2**k** **XQ** = λ**AG** soi $\lambda = 2/3$ → **XQ** = $-\frac{8}{3}$**i** + 10**j** + $\frac{4}{3}$**k**	**B1** **M1** **A1**	[3]	

422. 9709_w16_ms_11 Q: 11

	Answer	Mark		Partial Marks
(i)	$\frac{dy}{dx} = -(x-1)^{-2} + 9(x-5)^{-2}$ $m_{\text{tangent}} = -\frac{1}{4} + \frac{9}{4} = 2$ Equation of normal is $y - 5 = -½(x-3)$ $x = 13$	**M1A1** **B1** **M1** **A1**	[5]	May be seen in part **(ii)** Through (3, 5) and with $m = -1/m_{tangent}$
(ii)	$(x-5)^2 = 9(x-1)^2$ $x - 5 = (\pm)3(x-1)$ or $(8)(x^2 - x - 2) = 0$ $x = -1$ or 2 $\frac{d^2y}{dx^2} = 2(x-1)^{-3} - 18(x-5)^{-3}$ When $x = -1$, $\frac{d^2y}{dx^2} = -\frac{1}{6} < 0$ MAX When $x = 2$, $\frac{d^2y}{dx^2} = \frac{8}{3} > 0$ MIN	**B1** **M1** **A1** **B1** **B1** **B1**	[6]	Set $\frac{dy}{dx} = 0$ and simplify Simplify further and attempt solution If change of sign used, x values close to the roots must be used and all must be correct

423. 9709_w16_ms_12 Q: 7

	Answer	Mark		Partial Marks
(i)	$\frac{dy}{dx} = \frac{-3}{(2x-1)^2} \times 2$	**B1** **B1**	[2]	B1for a single correct term (unsimplified) without ×2.
(ii)	e.g. Solve for $\frac{dy}{dx} = 0$ is impossible.	**B1**√	[1]	Satisfactory explanation.
(iii)	If $x = 2$, $\frac{dy}{dx} = \frac{-6}{9}$ and $y = 3$ Perpendicular has $m = \frac{9}{6}$ $\rightarrow\ y-3=\frac{3}{2}(x-2)$ Shows when x=0 then y=0 AG	**M1*** **M1*** **DM1** **A1**	[4]	Attempt at both needed. Use of $m_1m_2 = -1$ numerically. Line equation using (2, their 3) and their m.
(iv)	$\frac{dx}{dt} = -0.06$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \rightarrow -\frac{2}{3} \times -0.06 = 0.04$	**M1 A1**	[2]	

424. 9709_w16_ms_12 Q: 9

	Answer	Mark		Partial Marks
(i)	$-4-6-6=-16$ $\sqrt{x_1^2+y_1^2+z_1^2}$ or $\sqrt{x_2^2+y_2^2+z_2^2}$ $3 \times 7 \times \cos\theta = -16$ $\rightarrow\ \theta = 139.6°$ or 2.44^c or 0.776π	**M1** **M1** **M1** **A1**	[4]	Use of $x_1x_2+y_1y_2+z_1z_2$ on their $\overrightarrow{OA}$ & $\overrightarrow{OB}$ Modulus once on either their $\overrightarrow{OA}$ *or* $\overrightarrow{OB}$ All linked using their $\overrightarrow{OA}$ & $\overrightarrow{OB}$
(ii)	$\overrightarrow{AC} = \text{c} - \text{a} = \begin{pmatrix}0\\8\\6\end{pmatrix}$ Magnitude = 10 Scaling $\rightarrow \frac{15}{their 10} \times \begin{pmatrix}0\\8\\6\end{pmatrix} = \begin{pmatrix}0\\12\\9\end{pmatrix}$	**B1** **M1** **A1**	[3]	For 15 × *their* unit vector.
(iii)	$\begin{pmatrix}2+2p\\6-2p\\5-p\end{pmatrix}$ $\rightarrow -2(2+2p)+3(6-2p)+6(5-p)=0$ $\rightarrow\ p = 2\frac{3}{4}$	**B1** **M1** **A1**	[3]	Single vector soi by scalar product. Dot product of ($p\,\overrightarrow{OA} + \overrightarrow{OC}$) and $\overrightarrow{OB} = 0$.

425. 9709_w16_ms_13 Q: 4

	Answer	Mark		Partial Marks
	$f'(x) = 3x^2 - 6x - 9$ soi	B1		
	Attempt to solve $f'(x) = 0$ or $f'(x) > 0$ or $f'(x) \geqslant 0$ soi	M1		
	$(3)(x-3)(x+1)$ or 3,−1 seen or 3 only seen	A1		With or without equality/inequality signs
	Least possible value of n is 3. Accept $n = 3$. Accept $n \geqslant 3$	A1	[4]	Must be in terms of n

426. 9709_w16_ms_13 Q: 7

	Answer	Mark		Partial Marks
(i)	**AB.AC** $= 3 - 2 - 1 = 0$ hence perpendicular or 90°	B1		3 − 2 − 1 or sum of prods etc must be seen
	AB.AD $= 3 + 4 - 7 = 0$ hence perpendicular or 90°	B1		Or single statement: mutually perpendicular or 90° seen at least once .
	AC.AD $= 1 - 8 + 7 = 0$ hence perpendicular or 90° AG	B1	[3]	
(ii)	Area $ABC = (½)\sqrt{3^2 + 1^2 + 1^2} \times \sqrt{1^2 + (-2)^2 + (-1)^2}$	M1		
	$= ½\sqrt{11} \times \sqrt{6}$	A1		Expect $½\sqrt{66}$
	Vol. $= ⅓ \times$ *their* $\Delta ABC \times \sqrt{1^2 + 4^2 + (-7)^2}$	M1		
	$= \frac{1}{6}\sqrt{66} \times \sqrt{66} = 11$	A1	[4]	Not 11.0

427. 9709_s15_ms_11 Q: 2

	Answer	Mark	Partial Marks
	$y = 2x^2$, $X(-2, 0)$ and $P(p, 0)$		
(i)	$A = \frac{1}{2} \times (2 + p) \times 2p^2$ $(= 2p^2 + p^3)$	M1 A1 [2]	Attempt at base and height in terms of p and use of $\frac{bh}{2}$
(ii)	$\frac{dA}{dp} = 4p + 3p^2$	B1	cao
	$\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt} = 0.02 \times 20 = 0.4$ or $\frac{dA}{dt} = 4p\frac{dp}{dt} + 3p^2\frac{dp}{dt}$	M1 A1 [3]	any correct method, cao

428. 9709_s15_ms_11 Q: 4

	Answer	Mark	Partial Marks
	$\overrightarrow{OA}=\begin{pmatrix}3\\0\\-4\end{pmatrix}, \overrightarrow{OB}=\begin{pmatrix}6\\-3\\2\end{pmatrix}, \overrightarrow{OC}=\begin{pmatrix}k\\-2k\\2k-3\end{pmatrix}$		
(i)	$OA\cdot OB = 18-8=10$ Modulus of $OA = 5$, of $OB = 7$	M1	Use of $x_1x_2+y_1y_2+z_1z_2$
	Angle $AOB = \cos^{-1}\left(\frac{10}{35}\right)$ aef	M1	All linked with modulus
	$\rightarrow \frac{10}{35}$ or $\frac{2}{7}$	A1 [3]	cao, (if angle given, no penalty), correct angle implies correct cosine
(ii)	$\overrightarrow{AB} = \mathbf{b}-\mathbf{a} = \begin{pmatrix}3\\-3\\6\end{pmatrix}$	B1	allow for **a** – **b**
	$k^2+4k^2+(2k-3)^2 = 9+9+36$	M1	Correct use of moduli using their AB
	$\rightarrow 9k^2-12k-45(=0)$	DM1	obtains 3 term quadratic.
	$\rightarrow k=3$ or $k=-\frac{5}{3}$	A1 [4]	cao

429. 9709_s15_ms_11 Q: 9

	Answer	Mark	Partial Marks
	$y = x^3 + px^2$		
(i)	$\frac{dy}{dx} = 3x^2 + 2px$	B1	cao
	Sets to 0 → $x = 0$ or $-\frac{2p}{3}$	M1	Sets differential to 0
	→ (0, 0) or $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$	A1 A1 [4]	cao cao, first A1 for any correct turning point or any correct pair of *x* values. 2nd A1 for 2 complete TPs
(ii)	$\frac{d^2y}{dx^2} = 6x + 2p$	M1	Other methods include; clear demonstration of sign change of gradient, clear reference to the shape of the curve
	At (0, 0) → $2p$ +ve Minimum	A1	www
	At $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$ → $-2p$ –ve Maximum	A1 [3]	
(iii)	$y = x^3 + px^2 + px$ → $3x^2 + 2px + p$ (= 0)	B1	
	Uses $b^2 - 4ac$	M1	Any correct use of discriminant
	→ $4p^2 - 12p < 0$		
	→ $0 < p < 3$ aef	A1 [3]	cao (condone ⩽)

430. 9709_s15_ms_12 Q: 2

	Answer	Mark	Partial Marks
	Radius of semicircle = $\frac{1}{2}AB = r\sin\theta$	B1	aef
	Area of semicircle = $\frac{1}{2}\pi r^2\sin^2\theta = A_1$	B1√	Uses $\frac{1}{2}\pi r^2$ with $r = f(\theta)$
	Shaded area = semicircle – segment $= A_1 - \frac{1}{2}r^2 2\theta + \frac{1}{2}r^2\sin 2\theta$	B1B1 [4]	B1 (–sector), B1 for + (triangle)

431. 9709_s15_ms_12 Q: 4

	Answer	Mark	Partial Marks
	$u = 2x(y - x)$ and $x + 3y = 12$, $u = 2x\left(\frac{12 - x}{3} - x\right)$	M1 A1	Expresses *u* in terms of *x*
	$= 8x - \frac{8x^2}{3}$		
	$\frac{du}{dx} = 8 - \frac{16x}{3}$	M1	Differentiate candidate's quadratic, sets to 0 + attempt to find *x*, or other valid method
	$= 0$ when $x = 1\frac{1}{2}$	A1	
	→ ($y = 3\frac{1}{2}$) → $u = 6$	A1 [5]	Complete method that leads to *u* Co

432. 9709_s15_ms_12 Q: 9

	Answer	Mark	Partial Marks
	$\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$		
(i)	$\overrightarrow{OA}.\overrightarrow{OB} = 6 + 4 + 16 = 26$	M1	Must be numerical at some stage
	$\lvert\overrightarrow{OA}\rvert = \sqrt{36}$, $\lvert\overrightarrow{OB}\rvert = \sqrt{26}$	M1	Product of 2 moduli
	Cos $AOB = \dfrac{26}{6\sqrt{26}}$	M1	All linked correctly
	$\rightarrow$ 31.8º	A1 [4]	co
(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix}1\\-3\\0\end{pmatrix}$	B1	
	$\overrightarrow{OC} = \begin{pmatrix}2\\4\\4\end{pmatrix} + 2\overrightarrow{AB}$ or $\begin{pmatrix}3\\1\\4\end{pmatrix} + \overrightarrow{AB}$	M1	Correct link
	$\overrightarrow{OC} = \begin{pmatrix}4\\-2\\4\end{pmatrix}$		
	Unit vector ÷ modulus $\rightarrow \frac{1}{6}\begin{pmatrix}4\\-2\\4\end{pmatrix}$	M1 A1 [4]	÷ by modulus. co
(iii)	$\lvert\overrightarrow{OC}\rvert = 6$, $\lvert\overrightarrow{OA}\rvert = 6$	B1 [1]	co

433. 9709_s15_ms_13 Q: 5

	Answer	Mark	Partial Marks
(i)	$\overrightarrow{AB} = \begin{pmatrix}5\\-1\\-2\end{pmatrix} - \begin{pmatrix}3\\2\\-3\end{pmatrix} = \begin{pmatrix}2\\-3\\1\end{pmatrix}$	**B1**	Or $\overrightarrow{BA}, \overrightarrow{CB}$. Allow any combination. Ignore labels.
	$\overrightarrow{BC} = \begin{pmatrix}6\\1\\2\end{pmatrix} - \begin{pmatrix}5\\-1\\-2\end{pmatrix} = \begin{pmatrix}1\\2\\4\end{pmatrix}$	**B1**	
	$\overrightarrow{AB}.\overrightarrow{BC} = 2 - 6 + 4$ oe must be seen = 0	**M1**	Could be part of calculation for angle ABC
	hence $ABC = 90°$	**A1** **[4]**	AG Alt methods Pythag, Cosine Rule
(ii)	$\lvert\overrightarrow{AB}\rvert = \sqrt{14}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{21}$ oe	**B1**	At least one correct
	Area $= \dfrac{1}{2}\sqrt{14}\sqrt{21}$	**M1**	Reasonable attempt at vectors and their magnitudes
	8.6 oe	**A1** **[3]**	Allow $\dfrac{7\sqrt{6}}{2}$

434. 9709_s15_ms_13 Q: 8

	Answer	Mark	Partial Marks
(i)	$-(x+1)^{-2}-2(x+1)^{-3}$	**M1A1** **A1** **[3]**	M1 for recognisable attempt at differentn. Allow $\dfrac{-x^2-4x-3}{(x+1)^4}$ from Q rule. (A2,1,0)
(ii)	$f'(x)<0$ hence decreasing	**B1** **[1]**	Dep. on *their* (i) < 0 for $x >$ 1
(iii)	$\dfrac{-1}{(x+1)^2}-\dfrac{2}{(x+1)^3}=0$ **or** $\dfrac{-x^2-4x-3}{(x+1)^4}=0$ $\dfrac{-(x+1)-2}{(x+1)^3}=0 \rightarrow -x-1-2=0$ or $-x^2-4x-3=0$ $x=-3,\ y=-1/4$	**M1*** **M1** **Dep*** **A1A1** **[4]**	Set $\dfrac{dy}{dx}$ to 0 OR mult by $(x+1)^3$ or $(x+1)^5$ (i.e.×mult) × multn $\rightarrow -(x+1)^3-2(x+1)^2=0$ (−3, −1/4) www scores 4/4

435. 9709_w15_ms_11 Q: 5

	Answer	Mark	Partial Marks
(i)	$\dfrac{dy}{dx}=-\dfrac{8}{x^2}+2$ cao $\dfrac{d^2y}{dx^2}=\dfrac{16}{x^3}$ cao	**B1B1** **B1** [3]	
(ii)	$-\dfrac{8}{x^2}+2=0 \rightarrow 2x^2-8=0$ $x=\pm 2$ $y=\pm 8$ $\dfrac{d^2y}{dx^2}>0$ when $x=2$ hence MINIMUM $\dfrac{d^2y}{dx^2}<0$ when $x=-2$ hence MAXIMUM	**M1** **A1** **A1** **B1**√ **B1**√ [5]	Set = 0 and rearrange to quadratic form If A0A0 scored, SCA1 for just (2, 8) {Ft for "correct" conclusion if $\dfrac{d^2y}{dx^2}$ incorrect or any valid method inc. a good sketch}

436. 9709_w15_ms_11 Q: 10

	Answer	Mark	Partial Marks
(i)	PM = 2**i** – 10**k** + $\frac{1}{2}$(6**j** + 8**k**) oe	**M1**	Any valid method
	PM = 2**i** + 3**j** – 6**k**	**A1**	
	$\div\sqrt{4+9+36}$	**M1**	
	Unit vector = $\frac{1}{7}$(2**i** + 3**j** – 6**k**)	**A1** [4]	
(ii)	AT = 6**j** + 8**k**, PT = a**i** + 6**j** – 2**k** soi (or TA and TP)	**B1**	Allow 1 vector reversed at this stage. (**AM** or **MT** could be used for **AT**)
	$(\cos ATP) = \frac{(6\mathbf{j}+8\mathbf{k}).(a\mathbf{i}+6\mathbf{j}-2\mathbf{k})}{\sqrt{36+64}\sqrt{a^2+36+4}}$ $= \frac{36-16}{\sqrt{36+64}\sqrt{a^2+36+4}}$	**M1**	
	$\frac{20}{10\sqrt{a^2+40}}$	**A1**√	Ft from their **AT** and **PT**
	$\frac{2}{\sqrt{a^2+40}} = \frac{2}{7}$ oe and attempt to solve	**M1**	
	$a = 3$	**A1** [5]	Withheld if only 1 vector reversed
	ALT Alt (Cosine Rule) Vectors (AT, PT etc.)	**B1**	
	$\cos ATP = \frac{a^2+36+4+36+64-(100+a^2)}{2\sqrt{(a^2+40)}\sqrt{100}}$	**M1A1**	
	then as above		

437. 9709_w15_ms_12 Q: 3

	Answer	Mark	Partial Marks
(i)	$\tan 60 = \frac{x}{h} \rightarrow x = h\tan 60$	**B1**	Any correct unsimplified length
	$A = h \times x$	**M1**	Correct method for area
	$V = 40\sqrt{(3h^2)}$	**A1** [3]	ag
(ii)	$\frac{\mathrm{d}V}{\mathrm{d}h} = 80\sqrt{(3h)}$	**B1**	B1
	If $h = 5$, $\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{1}{2\sqrt{(3)}}$ or 0.289	**M1A1** [3]	M1 (must be ÷, not ×).

438. 9709_w15_ms_12 Q: 7

	Answer	Mark	Partial Marks
	$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix}$, $\overrightarrow{OC} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}$.		
(i)	$\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}$ $\overrightarrow{AC} \begin{pmatrix} 3 \\ p-2 \\ q+3 \end{pmatrix}$ $\overrightarrow{BC} \begin{pmatrix} 1 \\ p-5 \\ q+2 \end{pmatrix}$	**B1B1**	Any 2 of 3 relevant vectors
	$\rightarrow$ $p = 6½$ and $q = -1½$	**B1 B1** [4]	
(ii)	$6 + 3p - 6 + q + 3 = 0$ $\rightarrow$ $q = -3p - 3$	**M1** **A1** [2]	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$
(iii)	$AB^2 = 4 + 9 + 1$ $AC^2 = 9 + 1 + (q+3)^2$ $\rightarrow$ $(q+3)^2 = 4$ $\rightarrow q = -1$ or -5	**M1** **A1 A1** [3]	For attempt at either

439. 9709_w15_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$f''(x) = \frac{12}{x^3}$		
	$f'(x) = -\frac{6}{x^2}$ $(+c)$	**B1**	Correct integration
	$= 0$ when $x = 2$ $\rightarrow$ $c = \frac{3}{2}$	**M1 A1**	Uses $x = 2$, $f'(x = 0)$
	$f(x) = \frac{6}{x} + \frac{3x}{2}$ $(+A)$	**B1√B1√**	For each integral
	$= 10$ when $x = 2$ $\rightarrow$ $A = 4$	**A1** [6]	
(ii)	$-\frac{6}{x^2} + \frac{3}{2} = 0$ $\rightarrow$ $x = \pm 2$ Other point is $(-2, -2)$	**M1** **A1** [2]	Sets their 2 term $f'(x)$ to 0.
(iii)	At $x = 2$, $f''(x) = 1.5$ Min At $x = -2$, $f''(x) = -1.5$ Max	**B1** **B1** [2]	

440. 9709_w15_ms_13 Q: 5

	Answer	Mark	Partial Marks
(i)	$-2p^2+16p-24+2p^2-6p+2$ Set scalar product = 0 and attempt solution $p=2.2$	**M1** **DM1** **A1** [3]	Good attempt at scalar product
(ii)	$4-2p=2(p-6)$ *or* $p=2(2p-6)$	**M1**	
	$p=4\rightarrow\overrightarrow{OA}=\begin{pmatrix}-2\\2\\1\end{pmatrix}$ $\overrightarrow{OB}=\begin{pmatrix}-4\\4\\2\end{pmatrix}$	**A1**	At least one of **OA** and **OB** correct
	$\left\lvert\overrightarrow{OA}\right\rvert=\sqrt{(-2)^2+2^2+1^2}=3$	**M1A1** [4]	For M1 accept a numerical p
	ALT 1 Compare AB with $OA\rightarrow 10-3p=p-6$ or $6-p=2p-6$. Similarly cf AB with OB	**M1**	
	ALT 2 $(OA.OB)/(\lvert OA\rvert\times\lvert OB\rvert)=1$ or $-1\rightarrow$ $10p-22=\sqrt{5p^2-36p}+$ $73\sqrt{5p^2-16p+20}$ $\rightarrow 125p^4-260p^3+941p^2-1448p+976=0\rightarrow p=4$. Similarly with $OA.AB$ or $OB.AB$.	**M1**	
	ALT 3 OA & OB have equal unit vectors. (Similarly with OA & AB or OB & AB.) Hence $\frac{1}{\sqrt{5p^2-36p+73}}\begin{pmatrix}p-6\\2p-6\\1\end{pmatrix}$ $=\frac{1}{\sqrt{5p^2-16p+20}}\begin{pmatrix}4-2p\\p\\2\end{pmatrix}$ $\rightarrow\frac{1}{\sqrt{5p^2-36p+73}}=\frac{2}{\sqrt{5p^2-16p+20}}$ $\rightarrow 15p^2-128p+272=0$ $\rightarrow(p-4)(15p-68)=0$ $\rightarrow p=4(or\,68/15)$	**M1**	

441. 9709_m22_ms_12 Q: 1

Question	Answer	Marks	Guidance
	$\left[f(x)=\right]\frac{2x^{\frac{2}{3}}}{\frac{2}{3}}-\frac{x^{\frac{4}{3}}}{\frac{4}{3}}\ [+c]$	B1 B1	$\frac{2}{3}$ and $\frac{4}{3}$ may be seen as sums of 1 and a fraction.
	$5 = 12 - 12 + c$	M1	Substituting (8,5) into an integral.
	$\left[f(x)=\right]3x^{\frac{2}{3}}-\frac{3}{4}x^{\frac{4}{3}}+5$	A1	Fractions in the denominators scores A0.
		4	

442. 9709_m21_ms_12 Q: 6

Question	Answer	Marks	Guidance
(a)	At $x = 1$, $\frac{dy}{dx}=6$	B1	
	$\frac{dx}{dt}=\left(\frac{dx}{dy}\times\frac{dy}{dt}\right)=\frac{1}{6}\times3=\frac{1}{2}$	M1 A1	Chain rule used correctly. Allow alternative and minimal notation.
		3	

Question	Answer	Marks	Guidance
(b)	$[y=]\left(\frac{6(3x-2)^{-2}}{-2}\right)\div(3)\ [+c]$	B1 B1	
	$-3=-1+c$	M1	Substitute $x = 1,\ y = -3$. c must be present.
	$y=-(3x-2)^{-2}-2$	A1	OE. Allow f(x)=
		4	

443. 9709_m21_ms_12 Q: 11

Question	Answer	Marks	Guidance
(a)	$9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)=0$ leading to $9x^{-\frac{3}{2}}(x-4)=0$	M1	OE. Set y to zero and attempt to solve.
	$x = 4$ **only**	A1	From use of a correct method.
		2	
(b)	$\frac{dy}{dx}=9\left(-\frac{1}{2}x^{-\frac{3}{2}}+6x^{-\frac{5}{2}}\right)$	B2, 1, 0	B2; all 3 terms correct: 9, $-\frac{1}{2}x^{-\frac{3}{2}}$ and $6x^{-\frac{5}{2}}$ B1; 2 of the 3 terms correct
	At $x = 4$ gradient = $9\left(-\frac{1}{16}+\frac{6}{32}\right)=\frac{9}{8}$	M1	Using *their* $x = 4$ in *their* differentiated expression and attempt to find equation of the tangent.
	Equation is $y=\frac{9}{8}(x-4)$	A1	or $y=\frac{9x}{8}-\frac{9}{2}$ OE
		4	
(c)	$9x^{-\frac{5}{2}}\left(-\frac{1}{2}x+6\right)=0$	M1	Set *their* $\frac{dy}{dx}$ to zero and an attempt to solve.
	$x=12$	A1	Condone $(\pm)12$ from use of a correct method.
		2	

Question	Answer	Marks	Guidance
(d)	$\int 9\left(x^{-\frac{1}{2}}-4x^{-\frac{3}{2}}\right)\mathrm{d}x = 9\left(\frac{x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}\right)$	B2, 1, 0	B2; all 3 terms correct: 9, $\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$, $\frac{-4x^{-\frac{1}{2}}}{-\frac{1}{2}}$ B1; 2 of the 3 terms correct
	$9\left[\left(6+\frac{8}{3}\right)-(4+4)\right]$	M1	Apply limits *their* 4 → 9 to an integrated expression with no consideration of other areas.
	6	A1	Use of π scores A0
		4	

444. 9709_s21_ms_11 Q: 1

Question	Answer	Marks	Guidance
	$[y=]-\frac{1}{x^3}+8x^4$ $[+c]$	B1 B1	OE. Accept unsimplified.
	$4=-8+\frac{1}{2}+c$	M1	Substituting $\left(\frac{1}{2},4\right)$ into an integrated expression
	$y=-\frac{1}{x^3}+8x^4+\frac{23}{2}$	A1	OE. Accept $-x^{-3}$; must be 8; $y=$ must be seen in working.
		4	

445. 9709_s21_ms_11 Q: 11

Question	Answer	Marks	Guidance
(a)	$\frac{\mathrm{d}y}{\mathrm{d}x}=3(3x+4)^{-0.5}-1$	B1 B1	B1 All correct with 1 error, B2 if all correct
	Gradient of tangent = $-\frac{1}{4}$ and Gradient of normal = 4	*M1	Substituting $x=4$ into a differentiated expression and using $m_1 m_2=-1$
	Equation of line is $(y-4)=4(x-4)$ **or** evaluate c	DM1	With (4, 4) and *their* gradient of normal
	So $y=4x-12$	A1	
		5	
(b)	$3(3x+4)^{-0.5}-1=0$	M1	Setting *their* $\frac{\mathrm{d}y}{\mathrm{d}x}=0$
	Solving as far as $x=$	M1	Where $\frac{\mathrm{d}y}{\mathrm{d}x}$ contains $a(bx+c)^{-0.5}$ a, b, c any values
	$x=\frac{5}{3}$, $y=2\left(3\times\frac{5}{3}+4\right)^{0.5}-\frac{5}{3}=\frac{13}{3}$	A1	
		3	
(c)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{9}{2}(3x+4)^{-1.5}$	M1	Differentiating *their* $\frac{\mathrm{d}y}{\mathrm{d}x}$ OR checking $\frac{\mathrm{d}y}{\mathrm{d}x}$ to find +ve and -ve either side of their $x=\frac{5}{3}$
	At $x=\frac{5}{3}$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ is negative so the point is a maximum	A1	
		2	

Question	Answer	Marks	Guidance
(d)	Area $= \left[\int 2(3x+4)^{0.5} - x\,dx = \right] \frac{4}{9}(3x+4)^{1.5} - \frac{1}{2}x^2$	**B1 B1**	B1 for each correct term (unsimplified)
	$\left(\frac{4}{9}(16)^{1.5} - \frac{1}{2}(4)^2\right) - \frac{4}{9}(4)^{1.5} = \frac{256}{9} - 8 - \frac{32}{9}$	**M1**	Substituting limits 0 and 4 into an expression obtained by integrating y
	$16\frac{8}{9}$	**A1**	Or $\frac{152}{9}$
		4	

446. 9709_s21_ms_12 Q: 9

Question	Answer	Marks	Guidance
	Curve intersects $y = 1$ at (3, 1)	B1	**Throughout Question 9: 1 < *their* 3 < 5** Sight of $x = 3$
	Volume $= [\pi]\int(x-2)[dx]$	M1	M1 for showing the intention to integrate $(x-2)$. Condone missing π or using 2π.
	$[\pi]\left[\frac{1}{2}x^2 - 2x\right]$ or $[\pi]\left[\frac{1}{2}(x-2)^2\right]$	A1	Correct integral. Condone missing π or using 2π.
	$= [\pi]\left[\left(\frac{5^2}{2} - 2\times5\right) - \left(\frac{their\,3^2}{2} - 2\times their\,3\right)\right]$ $= [\pi]\left[\frac{5}{2} + \frac{3}{2}\right]$ as a minimum requirement for *their* values	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
	Volume of cylinder $= \pi\times1^2\times(5 - their\,3)[= 2\pi]$	B1 FT	Or by integrating 1 to obtain x (condone y if 5 and *their* 3 used).
	[Volume of solid $= 4\pi - 2\pi =$]2π or 6.28	A1	AWRT

Question	Answer	Marks	Guidance
	Alternative method for Question 9		
	Curve intersects $y = 1$ at (3, 1)	B1	Sight of $x = 3$
	Volume of solid $= \pi\int(x-2)-1[dx]$	M1 B1	M1 for showing the intention to integrate $(x-2)$ B1 for correct integration of –1. Condone missing π or 2π for M1 but not for B1.
	$[\pi]\left[\frac{1}{2}x^2 - 3x\right]$ or $[\pi]\left[\frac{1}{2}(x-3)^2\right]$	A1	Correct integral, allow as two integrals. Condone missing π or using 2π.
	$= [\pi]\left[\left(\frac{5^2}{2} - 3\times5\right) - \left(\frac{their\,3^2}{2} - 3\times their\,3\right)\right]$	M1	Correct use of '*their* 3' and 5 in an integrated expression. Condone missing π or using 2π. Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
	[Volume of solid $= 4\pi - 2\pi =$]2π or 6.28	A1	AWRT
		6	

447. 9709_s21_ms_12 Q: 11

Question	Answer	Marks	Guidance
(a)	At stationary point $\frac{dy}{dx}=0$ so $6(3\times2-5)^3-k\times2^2=0$	M1	Setting given $\frac{dy}{dx}=0$ and substituting $x=2$ into it.
	$[k=]\frac{3}{2}$	A1	OE
		2	
(b)	$[y=]\frac{6}{4\times3}(3x-5)^4-\frac{1}{3}kx^3\ [+c]$.	*M1 A1FT	Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2}(3x-5)^4-\frac{1}{2}x^3$. FT *their* non zero k.
	$-\frac{7}{2}=\frac{1}{2}(3\times2-5)^4-\frac{1}{3}\times\frac{3}{2}\times2^3+c$ [leading to $-3.5+c=-3.5$]	DM1	Using (2,-3.5) in an integrated expression. + c needed. Substitution needs to be seen, simply stating c = 0 is DM0.
	$y=\frac{1}{2}(3x-5)^4-\frac{1}{2}x^3$	A1	y = or f(x)= must be seen somewhere in solution.

Question	Answer	Marks	Guidance
(b)	**Alternative method for Question 11(b)**		
	$[y=]\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x(+c)$ or $-270x^3-k\frac{x^3}{3}$	*M1 A1 FT	From $\frac{dy}{dx}=162x^3-810x^2-kx^2-1350x-750$. FT *their* k
	$-\frac{7}{2}=\frac{81}{2}\times2^4-\frac{541}{2}\times2^3+675\times2^2-750\times2+c$	DM1	Using (2, -3.5) in an integrated expression. + c needed
	$y=\frac{81}{2}x^4-\frac{541}{2}x^3+675x^2-750x+\frac{625}{2}$	A1	y = or f(x)= must be seen somewhere in solution.
		4	
(c)	$[3\times]\left[18(3x-5)^2\right][-2kx]$	B2,1,0 FT	FT *their k*. Square brackets indicate each required component. B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors.
	Alternative method for Question 11(c)		
	$486x^2-1623x+1350$ or $-1620x-2kx$	B2,1,0 FT	FT *their k*. B2 for fully correct, B1 for one error, B0 for 2 or more errors.
		2	
(d)	[At x = 2] $\left[\frac{d^2y}{dx^2}=\right]54(3\times2-5)^2-4k$ or 48	M1	OE. Substituting x = 2 into *their* second differential or other valid method.
	[> 0] Minimum	A1	WWW
		2	

448. 9709_s21_ms_13 Q: 1

Question	Answer	Marks	Guidance
	$[f(x)=]\ 2x^3+\frac{8}{x}\ [+c]$	**B1**	Allow any correct form
	$7=16+4+c$	**M1**	Substitute f(2) = 7 into an integral. c must be present. Expect $c=-13$
	$f(x)=\ 2x^3+\frac{8}{x}-13$	**A1**	Allow $y=$, $f(x)$ or y can appear earlier in answer
		3	

449. 9709_s21_ms_13 Q: 11

Question	Answer	Marks	Guidance
(a)	$\frac{dy}{dx}=\frac{1}{2}x^{-1/2}-\frac{1}{2}k^2x^{-3/2}$	B1 B1	Allow any correct unsimplified form
	$\frac{1}{2}x^{-1/2}-\frac{1}{2}k^2x^{-3/2}=0$ leading to $\frac{1}{2}x^{-1/2}=\frac{1}{2}k^2x^{-3/2}$	M1	OE. Set to zero and one correct algebraic step towards the solutions. $\frac{dy}{dx}$ must only have 2 terms.
	$(k^2, 2k)$	A1	
		4	
(b)	When $x = 4k^2$, $\frac{dy}{dx}=\left[\frac{1}{4k}-\frac{1}{16k}=\right]\frac{3}{16k}$	B1	OE
	$y=\left[2k+k^2\times\frac{1}{2k}\right]=\frac{5k}{2}$	B1	OE. Accept $2k+\frac{k}{2}$
	Equation of tangent is $y-\frac{5k}{2}=\frac{3}{16k}(x-4k^2)$ or $y=mx+c\to\frac{5k}{2}=\frac{3}{16k}(4k^2)+c$	M1	Use of line equation with *their* gradient and $(4k^2, \textit{their } y)$,
	When $x=0$, $y=\left[\frac{5k}{2}-\frac{3k}{4}=\right]\frac{7k}{4}$ or from $y=mx+c$, $c=\frac{7k}{4}$	A1	OE
		4	

Question	Answer	Marks	Guidance
(c)	$\int\left(x^{\frac{1}{2}}+k^2x^{-\frac{1}{2}}\right)dx=\frac{2x^{\frac{3}{2}}}{3}+2k^2x^{\frac{1}{2}}$	B1	Any unsimplified form
	$\left(\frac{16k^3}{3}+4k^3\right)-\left(\frac{9k^3}{4}+3k^3\right)$	M1	Apply limits $\frac{9}{4}k^2\to 4k^2$ to an integration of y. M0 if volume attempted.
	$\frac{49k^3}{12}$	A1	OE. Accept $4.08k^3$
		3	

450. 9709_w21_ms_11 Q: 9

Question	Answer	Marks	Guidance
(a)	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}$ $[+c]$	B2, 1, 0	Allow terms on different lines; allow unsimplified.
	$-\frac{1}{3}=\frac{2}{3}-7+4+c$ leading to $c=[2]$	M1	Substitute $f(1)=-\frac{1}{3}$ into an integrated expression and evaluate c.
	$f(x)=\frac{2}{3}x^3-7x+4x^{-1}+2$	A1	OE.
		4	

Question	Answer	Marks	Guidance
(b)	$2x^4 - 7x^2 - 4\ [=0]$	M1	Forms 3-term quadratic in x^2 with all terms on one side. Accept use of substitution e.g. $2y^2 - 7y - 4[=0]$.
	$(2x^2+1)(x^2-4)\ [=0]$	M1	Attempt factors or use formula or complete the square. Allow ± sign errors. Factors must expand to give *their* coefficient of x^2 or e.g. y. Must be quartic equation. Accept use of substitution e.g. $(2y+1)(y-4)$.
	$x = [\pm]2$	A1	If M0 for solving quadratic, **SC B1** can be awarded for $[\pm]2$.
	$\left[\frac{2}{3}(2)^3 - 7(2) + \frac{4}{2} + 2 \quad \text{leading to}\right]\left(2, -\frac{14}{3}\right)$ $\left[\frac{2}{3}(-2)^3 - 7(-2) + \frac{4}{-2} + 2 \quad \text{leading to}\right]\left(-2, \frac{26}{3}\right)$	B1 B1	B1 B1 for correct coordinates clearly paired; B1 for each correct point; B1 B0 if additional point.
		5	
(c)	$f''(x) = 4x + 8x^{-3}$	B1	OE
		1	

Question	Answer	Marks	Guidance
(d)	$f''(2) = 9 > 0$ MINIMUM at $x = \textit{their}\ 2$	B1 FT	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = 2$.
	$f''(-2) = -9 < 0$ MAXIMUM at $x = \textit{their} -2$	B1 FT	FT on *their* $x = [\pm]2$ provided $f''(x)$ is correct. Must have correct value of $f''(x)$ if $x = -2$. **Special case:** If values not shown and B0B0 scored, **SC B1** for $f''(2) > 0$ MIN and $f''(-2) < 0$ MAX
	Alternative method for question 9(d)		
	Evaluate $f'(x)$ for x-values either side of 2 and –2	M1	FT on *their* $x = [\pm]2$
	MINIMUM at $x = \textit{their}\ 2$, MAXIMUM at $x = \textit{their}\ 2$	A1 FT	FT on *their* $x = [\pm]2$. Must have correct values of $f'(x)$ if shown. **Special case:** If values not shown and M0A0 scored **SC B1** $f'(2)$ −/0/+ MIN and $f'(-2)$ +/0/− MAX
	Alternative method for question 9(d)		
	Justify maximum and minimum using correct sketch graph	B1 B1	Need correct coordinates in **(b)** for this method.
		2	

451. 9709_w21_ms_11 Q: 10

Question	Answer	Marks	Guidance
(a)	$\left\{\frac{(3x-2)^{-\frac{1}{2}}}{-1/2}\right\} \div \{3\}$	B2, 1, 0	Attempt to integrate
	$-\frac{2}{3}[0-1]$	M1	M1 for applying limits $1 \to \infty$ to an integrated expression (either correct power or dividing by their power).
	$\frac{2}{3}$	A1	
		4	
(b)	$[\pi]\int y^2 dx = [\pi]\int (3x-2)^{-3} dx = [\pi]\frac{(3x-2)^{-2}}{-2\times 3}$	*M1 A1	M1 for attempt to integrate y^2 (power increases); allow 1 error. A1 for correct result in any form.
	$[\pi]\left[-\frac{1}{6}\right]\left[\frac{1}{16}-1\right]$	DM1	Apply limits 1 and 2 to an integrated expression and subtract correctly; allow 1 error.
	$\frac{5\pi}{32}$	A1	OE
		4	

Question	Answer	Marks	Guidance
(c)	$\frac{dy}{dx} = -\frac{3}{2}\times 3(3x-2)^{-\frac{5}{2}}$	M1	M1 for attempt to differentiate (power decreases); allow 1 error.
	At $x = 1$, $\frac{dy}{dx} = -\frac{9}{2}$	*M1	Substitute $x = 1$ into *their* differentiated expression; allow 1 error.
	[Equation of normal is] $y-1=\frac{2}{9}(x-1)$ OR evaluates c	DM1	Forms equation of line or evaluates c using (1, 1) and gradient $\frac{-1}{their\frac{dy}{dx}}$.
	At A, $y=\frac{7}{9}$	A1	OE e.g. AWRT 0.778; must clearly identify y-intercept
		4	

452. 9709_w21_ms_12 Q: 4

Question	Answer	Marks	Guidance
	$y = -\frac{\frac{8}{3}}{(3x+2)}[+c]$	*B1	For $(3x+2)^{-1}$
		DB1	For $-\frac{8}{3}$
	$5\frac{2}{3} = -\frac{\frac{8}{3}}{(3\times 2+2)}+c$	M1	Substituting $\left(2,\ 5\frac{2}{3}\right)$ into *their* integrated expression – defined by power = -1, or dividing by their power. + c needed
	$y = -\frac{8}{3(3x+2)}+6$	A1	OE e.g. $y=-\frac{8}{3}(3x+2)^{-1}+6$
		4	

453. 9709_w21_ms_12 Q: 11

Question	Answer	Marks	Guidance
(a)	$\frac{dy}{dx} = \frac{1}{2} + \frac{1}{3(x-2)^{\frac{4}{3}}}$	**B1**	OE. Allow unsimplified.
	Attempt at evaluating *their* $\frac{dy}{dx}$ at $x = 3$ $\left[\frac{1}{2} + \frac{1}{3(3-2)^{\frac{4}{3}}} = \frac{5}{6}\right]$	***M1**	Substituting $x = 3$ into *their* differentiated expression – defined by one of 3 original terms with correct power of x.
	Gradient of normal = $\frac{-1}{their \frac{dy}{dx}}$ $\left[= -\frac{6}{5}\right]$	***DM1**	Negative reciprocal of *their* evaluated $\frac{dy}{dx}$.
	Equation of normal $y - \frac{6}{5} = (their \text{ normal gradient})(x-3)$ $\left[y = -\frac{6}{5}x + 4.8 \Rightarrow 5y = -6x + 24\right]$	**DM1**	Using *their* normal gradient and *A* in the equation of a straight line. Dependent on *M1 and *DM1.
	[When $y = 0$,] $x = 4$	**A1**	or (4, 0)
		5	

Question	Answer	Marks	Guidance
(b)	Area under curve = $\int \left(\frac{1}{2}x + \frac{7}{10} - \frac{1}{(x-2)^{\frac{1}{3}}}\right)$[d$x$]	**M1**	For intention to integrate the curve (no need for limits). Condone inclusion of π for this mark.
	$\frac{1}{4}x^2 + \frac{7}{10}x - \frac{3(x-2)^{\frac{2}{3}}}{2}$	**A1**	For correct integral. Allow unsimplified. Condone inclusion of π for this mark.
	$\left(\frac{9}{4} + 2.1 - \frac{3}{2}\right) - \left(\frac{6.25}{4} + 1.75 - \frac{3 \times 0.5^{\frac{2}{3}}}{2}\right)$	**M1**	Clear substitution of 3 and 2.5 into *their* integrated expression (with at least one correct term) and subtracting.
	0.48[24]	**A1**	If M1A1M0 scored then SC B1 can be awarded for correct answer.
	[Area of triangle =] 0.6	**B1**	OE
	[Total area =] 1.08	**A1**	Dependent on the first M1 and WWW.
		6	

454. 9709_w21_ms_13 Q: 8

Question	Answer	Marks	Guidance
(a)	$\int\left(\frac{5}{2}-x^{\frac{1}{2}}-x^{-\frac{1}{2}}\right)\mathrm{d}x$	M1	OR as 2 separate integrals $\int\left(\frac{5}{2}-x^{1/2}\right)\mathrm{d}x-\int\left(x^{-1/2}\right)\mathrm{d}x$
	$\left\{\frac{5}{2}x-\frac{2}{3}x^{\frac{3}{2}}\right\}\{-\}\left\{2x^{\frac{1}{2}}\right\}$	A1 A1 A1	If two separate integrals with no subtraction **SC B1** for each correct integral.
	$\left(10-\frac{16}{3}-4\right)-\left(\frac{5}{8}-\frac{1}{12}-1\right)$	DM1	Substitute limits $\frac{1}{4}\rightarrow 4$ at least once, must be seen.
	$\frac{9}{8}$ or 1.125	A1	WWW. Cannot be awarded if π appears in any integral.
		6	
(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]-\frac{1}{2}x^{-\frac{3}{2}}$	B1	
	When $x = 1$, $m = -\frac{1}{2}$	M1	Substitute $x = 1$ into a differential.
	[Equation of normal is] $y-1=2(x-1)$	M1	Through (1, 1) with gradient $-\frac{1}{m}$ or $\frac{1-p}{1}=2$
	[When $x = 0$,] $p=-1$	A1	WWW
		4	

455. 9709_w21_ms_13 Q: 10

Question	Answer	Marks	Guidance
(a)	$\mathrm{f}''(x)=-\left(\frac{1}{2}x+k\right)^{-3}$	B1	
	$\mathrm{f}''(2)>0\Rightarrow-(1+k)^{-3}>0$	M1	Allow for solving *their* $\mathrm{f}''(2)>0$
	$k<-1$	A1	WWW
		3	
(b)	$\left[\mathrm{f}(x)=\int\left(\left(\frac{1}{2}x-3\right)^{-2}-(-2)^{-2}\right)\mathrm{d}x=\right]\left\{\frac{\left(\frac{1}{2}x-3\right)^{-1}}{-1\times\frac{1}{2}}\right\}\left\{-\frac{x}{4}\right\}$	B1 B1	Allow $-2\left(\frac{1}{2}x+k\right)^{-1}$ OE for 1st B1 and $-(1+k)^{-2}x$ OE for 2nd B1
	$3\frac{1}{2}=1-\frac{1}{2}+c$	M1	Substitute $x = 2$, $y = 3\frac{1}{2}$ into *their* integral with c present.
	$\mathrm{f}(x)=\frac{-2}{\left(\frac{1}{2}x-3\right)}-\frac{x}{4}+3$	A1	OE
		4	
(c)	$\left(\frac{1}{2}x-3\right)^{-2}-(-2)^{-2}=0$	M1	Substitute $k=-3$ and set to zero.
	leading to $\left(\frac{1}{2}x-3\right)^{2}=4$ $\left[\frac{1}{2}x-3=(\pm)2\right]$ leading to $x=10$	A1	
	$(10, -\frac{1}{2})$	A1	Or when $x = 10$, $y = -1 - 2\frac{1}{2} + 3 = -\frac{1}{2}$
	$\mathrm{f}''(10)\left[=-(5-3)^{-3}\rightarrow\right]<0\rightarrow$ MAXIMUM	A1	WWW
		4	

456. 9709_m20_ms_12 Q: 3

	Answer	Mark	Partial Marks
	$(\pi)\int(y-1)\mathrm{d}y$	*M1	SOI Attempt to integrate x^2 or $(y-1)$
	$(\pi)\left[\frac{y^2}{2}-y\right]$	A1	
	$(\pi)\left[\left(\frac{25}{2}-5\right)-\left(\frac{1}{2}-1\right)\right]$	DM1	Apply limits $1 \to 5$ to an integrated expression
	8π or AWRT 25.1	A1	
		4	

457. 9709_m20_ms_12 Q: 10

	Answer	Mark	Partial Marks
(a)	$2(a+3)^{\frac{1}{2}}-a=0$	M1	SOI. Set $\frac{\mathrm{d}y}{\mathrm{d}x}=0$ when $x=a$. Can be implied by an answer in terms of a
	$4(a+3)=a^2 \to a^2-4a-12=0$	M1	Take a to RHS and square. Form 3-term quadratic
	$(a-6)(a+2) \to a=6$	A1	Must show factors, or formula or completing square. Ignore $a=-2$ **SC** If a is never used maximum of M1A1 for $x=6$,with visible solution
		3	
(b)	$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=(x+3)^{-\frac{1}{2}}-1$	B1	
	Sub *their a* $\to \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{1}{3}-1=-\frac{2}{3}$ $(or<0) \to$ MAX	M1A1	A mark only if completely correct If the second differential is not $-\frac{2}{3}$ correct conclusion must be drawn to award the M1
		3	
(c)	$(y=)\frac{2(x+3)^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{2}x^2 \quad (+c)$	B1B1	
	Sub $x=$ *their a* and $y=14$ $\to 14=\frac{4}{3}(9)^{\frac{3}{2}}-18+c$	M1	Substitute into an integrated expression. c must be present. Expect $c=-4$
	$y=\frac{4}{3}(x+3)^{\frac{3}{2}}-\frac{1}{2}x^2-4$	A1	Allow $f(x)=\ldots$.
		4	

458. 9709_s20_ms_11 Q: 11

(a)	Simultaneous equations $\frac{8}{x+2} = 4 - ½x$	M1
	$x = 0$ or $x = 6 \rightarrow A\ (0, 4)$ and $B\ (6, 1)$	B1A1
	At C $\frac{-8}{(x+2)^2} = -\frac{1}{2} \rightarrow C\ (2, 2)$ (**B1** for the differentiation. **M1** for equating and solving)	B1 M1A1
		6
(b)	Volume under line = $\pi\int\left(-\frac{1}{2}x+4\right)^2 dx = \pi\left[\frac{x^3}{12} - 2x^2 + 16x\right] = (42\pi)$ (**M1** for volume formula. **A2,1** for integration)	M1 A2,1
	Volume under curve = $\pi\int\left(\frac{8}{x+2}\right)^2 dx = \pi\left[\frac{-64}{x+2}\right] = (24\pi)$	A1
	Subtracts and uses 0 to 6 $\rightarrow 18\pi$	M1A1
		6

459. 9709_s20_ms_12 Q: 8

(a)	Volume = $\pi\int x^2 dy = \pi\int\frac{36}{y^2}dy$	*M1
	$= \pi\left[\frac{-36}{y}\right]$	A1
	Uses limits 2 to 6 correctly $\rightarrow (12\pi)$	DM1
	Vol of cylinder = $\pi.1^2.4$ or $\int 1^2.dy$ = $[y]$ from 2 to 6	M1
	Vol = $12\pi - 4\pi = 8\pi$	A1
		5
(b)	$\frac{dy}{dx} = \frac{-6}{x^2}$	B1
	$\frac{-6}{x^2} = -2 \rightarrow x = \sqrt{3}$	M1
	$y = \frac{6}{\sqrt{3}} = 2\sqrt{3}$ Lies on $y = 2x$	A1
		3

460. 9709_s20_ms_13 Q: 2

	$(y) = \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{3x^{\frac{1}{2}}}{\frac{1}{2}}\ (+c)$	B1 B1
	$7 = 16 - 12 + c$ (**M1** for subsituting $x = 4$, $y = 7$ into *their* integrated expansion)	M1
	$y = 2x^{\frac{3}{2}} - 6x^{\frac{1}{2}} + 3$	A1
		4

461. 9709_s20_ms_13 Q: 11

(a)	$\frac{dy}{dx} = 3x^2 - 4bx + b^2$	B1
	$3x^2 - 4bx + b^2 = 0 \rightarrow (3x-b)(x-b)\ (=0)$	M1
	$x = \frac{b}{3}$ or b	A1
	$a = \frac{b}{3} \rightarrow b = 3a$ **AG**	A1
	Alternative method for question 11(a)	
	$\frac{dy}{dx} = 3x^2 - 4bx + b^2$	B1
	Sub $b = 3a$ & obtain $\frac{dy}{dx} = 0$ when $x = a$ and when $x = 3a$	M1
	$\frac{d^2y}{dx^2} = 6x - 12a$	A1
	< 0 Max at $x = a$ and > 0 Min at $x = 3a$. Hence $b = 3a$ **AG**	A1
		4
(b)	Area under curve = $\int (x^3 - 6ax^2 + 9a^2x)dx$	M1
	$\frac{x^4}{4} - 2ax^3 + \frac{9a^2x^2}{2}$	B2,1,0
	$\frac{a^4}{4} - 2a^4 + \frac{9a^4}{2}\ \left(= \frac{11a^4}{4}\right)$ (**M1** for applying limits $0 \rightarrow a$)	M1
	When $x = a$, $y = a^3 - 6a^3 + 9a^3 = 4a^3$	B1
	Area under line = $\frac{1}{2}a \times \textit{their}\ 4a^3$	M1
	Shaded area = $\frac{11a^4}{4} - 2a^4 = \frac{3}{4}a^4$	A1
		7

462. 9709_w20_ms_11 Q: 2

	Answer	Mark	Partial Marks
	$(y =) \left[-(x-3)^{-1}\right]\left[+\frac{1}{2}x^2\right]\ (+c)$	B1 B1	
	$7 = 1 + 2 + c$	M1	Substitute $x = 2$, $y = 7$ into an integrated expansion (c present). Expect $c = 4$
	$y = -(x-3)^{-1} + \frac{1}{2}x^2 + 4$	A1	OE
		4	

463. 9709_w20_ms_11 Q: 12

	Answer	Mark	Partial Marks
(a)	$4x^{\frac{1}{2}} - 2x = 3 - x \rightarrow x - 4x^{\frac{1}{2}} + 3(=0)$	*M1	3-term quadratic. Can be expressed as e.g. $u^2 - 4u + 3$ (=0)
	$\left(x^{\frac{1}{2}} - 1\right)\left(x^{\frac{1}{2}} - 3\right)(=0)$ or $(u-1)(u-3)(=0)$	DM1	Or quadratic formula or completing square
	$x^{\frac{1}{2}} = 1, 3$	A1	SOI
	$x = 1, 9$	A1	
	Alternative method for question 12(a)		
	$\left(4x^{\frac{1}{2}}\right)^2 = (3+x)^2$	*M1	Isolate $x^{\frac{1}{2}}$
	$16x = 9 + 6x + x^2 \rightarrow x^2 - 10x + 9\ (=0)$	A1	3-term quadratic
	$(x-1)(x-9)\ (=0)$	DM1	Or formula or completing square on a quadratic obtained by a correct method
	$x = 1, 9$	A1	
		4	
(b)	$\frac{dy}{dx} = 2x^{1/2} - 2$	*B1	
	$\frac{dy}{dx}$ or $2x^{1/2} - 2 = 0$ when $x = 1$ hence B is a stationary point	DB1	
		2	
(c)	Area of correct triangle = $\frac{1}{2}$ (9 – 3) × 6	M1	or $\int_3^9 (3-x)(dx) = \left[3x - \frac{1}{2}x^2\right] \rightarrow -18$
	$\int(4x^{\frac{1}{2}} - 2x)(dx) = \left[\frac{4x^{\frac{3}{2}}}{\frac{3}{2}} - x^2\right]$	B1 B1	
	$(72-81) - \left(\frac{64}{3} - 16\right)$	M1	Apply limits 4 → *their* 9 to an integrated expression
	$-14\frac{1}{3}$	A1	OE
	Shaded region = $18 - 14\frac{1}{3} = 3\frac{2}{3}$	A1	OE
		6	

464. 9709_w20_ms_12 Q: 7

	Answer	Mark	Partial Marks
(a)	$f'(4)\left(=\frac{5}{2}\right)$	*M1	Substituting 4 into $f'(x)$
	$\left(\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\right)\to\left(\frac{dy}{dt}\right)=\frac{5}{2}\times 0.12$	DM1	Multiplies *their* $f'(4)$ by 0.12
	$\left(\frac{dy}{dt}=\right)0.3$	A1	OE
		3	
(b)	$\frac{6x^{\frac{1}{2}}}{\frac{1}{2}}-\frac{4x^{-\frac{1}{2}}}{-\frac{1}{2}}(+c)$	B1 B1	B1 for each unsimplified integral.
	Uses (4, 7) leading to $c = (-21)$	M1	Uses (4, 7) to find a c value
	y or $f(x)=12x^{\frac{1}{2}}+8x^{-\frac{1}{2}}-21$ or $12\sqrt{x}+\frac{8}{\sqrt{x}}-21$	A1	Need to see y or f(x) = somewhere in *their* solution and 12 and 8
		4	

465. 9709_w20_ms_12 Q: 10

	Answer	Mark	Partial Marks
(a)	$\left(\frac{dy}{dx}\right)=[8]\times[(3-2x)^{-3}]+[-1]$ $\left(=\frac{8}{(3-2x)^3}-1\right)$	B2, 1, 0	B2 for all three elements correct, B1 for two elements correct, B0 for only one or no elements correct.
	$\frac{d^2y}{dx^2}=-3\times8\times(3-2x)^{-4}\times(-2)$ $\left(=\frac{48}{(3-2x)^4}\right)$	B1 FT	FT providing *their* bracket is to a negative power
	$\int y\,dx=[(3-2x)^{-1}]\ [2\div(-1\times-2)]\ [-\tfrac{1}{2}x^2]\ (+c)$ $\left(=\frac{1}{3-2x}-\frac{1}{2}x^2+c\right)$	B1 B1 B1	Simplification not needed, B1 for each correct element
		6	
(b)	$\frac{dy}{dx}=0\to(3-2x)^3=8\to3-2x=k\to x=$	M1	Setting *their* 2-term differential to 0 and attempts to solve as far as x =
	$\frac{1}{2}$	A1	
	Alternative method for question 10(b)		
	$y=0\to\frac{2}{(3-2x)^2}-x=0\to(x-2)(2x-1)^2=0\to x=$	M1	Setting y to 0 and attempts to solve a cubic as far as x = (3 factors needed)
	$\frac{1}{2}$	A1	
		2	
(c)	Area under curve = *their* $\left[\frac{1}{3-2\times\left(\frac{1}{2}\right)}-\frac{\left(\frac{1}{2}\right)^2}{2}\right]-\left[\frac{1}{3-2\times0}-0\right]$	M1	Using *their* integral, *their* positive x limit from **part (b)** and 0 correctly.
	$\frac{1}{24}$	A1	
		2	

466. 9709_w20_ms_13 Q: 2

	Answer	Mark	Partial Marks
(a)	$\dfrac{-2}{x+2}$	B1	Integrate f(x). Accept $-2(x+2)^{-1}$. Can be unsimplified.
	$0-\left(-\dfrac{2}{3}\right) = \dfrac{2}{3}$	M1 A1	Apply limit(s) to an integrated expansion. CAO for A1
		3	
(b)	$-1=-2+c$	M1	Substitute $x=-1, y=-1$ into *their* integrated expression (c present)
	$y=\dfrac{-2}{x+2}+1$	A1	Accept $y=-2(x+2)^{-1}+1$. –2 must be resolved.
		2	

467. 9709_w20_ms_13 Q: 10

	Answer	Mark	Partial Marks
(a)	$\dfrac{dy}{dx}=\left[\dfrac{x^{-1/2}}{2k}\right]-\left[\dfrac{x^{-3/2}}{2}\right]+([0])$	B2, 1, 0	$([0])$ implies that more than 2 terms counts as an error
	Sub $\dfrac{dy}{dx}=3$ when $x=\dfrac{1}{4}$ Expect $3=\dfrac{1}{k}-4$	M1	
	$k=\dfrac{1}{7}$ (or 0.143)	A1	
		4	
(b)	$\int\dfrac{1}{k}x^{1/2}+x^{-1/2}+\dfrac{1}{k^2} = \left[\dfrac{2x^{3/2}}{3k}\right]+\left[2x^{1/2}\right]+\left[\dfrac{x}{k^2}\right]$	B2, 1, 0	OE
	$\left(\dfrac{2k^2}{3}+2k+1\right)-\left(\dfrac{k^2}{12}+k+\dfrac{1}{4}\right)$	M1	Apply limits $\dfrac{k^2}{4}\to k^2$ to an integrated expression. Expect $\dfrac{7}{12}k^2+k+\dfrac{3}{4}$
	$\dfrac{7}{12}k^2+k+\dfrac{3}{4}=\dfrac{13}{12}$	M1	Equate to $\dfrac{13}{12}$ and simplify to quadratic. OE, expect $7k^2+12k-4\ (=0)$
	$k=\dfrac{2}{7}$ only (or 0.286)	A1	Dependent on $(7k-2)(k+2)\ (=0)$ or formula or completing square.
		5	

468. 9709_m19_ms_12 Q: 2

	Answer	Mark	Partial Marks
	$y=\frac{1}{3}kx^3-x^2\ (+c)$	M1A1	Attempt integration for M mark
	Sub (0, 2)	DM1	Dep on c present. Expect $c=2$
	Sub (3, –1) → $-1=9k-9+$ *their* c	DM1	
	$k=2/3$	A1	
		5	

469. 9709_m19_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$V=(\pi)\int\left(x^3+x^2\right)(\mathrm{d}x)$	M1	Attempt $\int y^2\mathrm{d}x$
	$(\pi)\left[\frac{x^4}{4}+\frac{x^3}{3}\right]_0^3$	A1	
	$(\pi)\left[\frac{81}{4}+9\quad(-0)\right]$	DM1	May be implied by a correct answer
	$\frac{117\pi}{4}$ oe	A1	Accept 91.9 If additional areas rotated about x-axis, maximum of M1A0DM1A0
		4	
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2}\left(x^3+x^2\right)^{-1/2}\times\left(3x^2+2x\right)$	B2,1,0	Omission of $3x^2+2x$ is one error
	(At $x=3$,) $y=6$	B1	
	At $x=3$, $m=\frac{1}{2}\times\frac{1}{6}\times 33=\frac{11}{4}$ soi	DB1ft	Ft on *their* dy / dx providing differentiation attempted
	Equation of normal is $y-6=-\frac{4}{11}(x-3)$	DM1	Equation through (3, *their* 6) and with gradient −1/*their* m
	When $x=0$, $y=7\frac{1}{11}$ oe	A1	
		6	

470. 9709_s19_ms_11 Q: 10

	Answer	Mark	Partial Marks
(i)	integrating → $\frac{\mathrm{d}y}{\mathrm{d}x}=x^2-5x\ (+c)$	B1	
	= 0 when $x=3$	M1	Uses the point to find c after $\int = 0$.
	$c=6$	A1	
	integrating again → $y=\frac{x^3}{3}-\frac{5x^2}{2}+6x\quad(+d)$	B1	**FT** Integration again FT if a numerical constant term is present.
	use of (3, 6)	M1	Uses the point to find d after $\int = 0$.
	$d=1\frac{1}{2}$	A1	
		6	
(ii)	$\frac{\mathrm{d}y}{\mathrm{d}x}=x^2-5x+6=0\rightarrow x=2$	B1	
		1	
(iii)	$x=3$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=1$ and/or +ve Minimum. $x=2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-1$ and/or −ve Maximum	B1	www
	May use shape of '$+x^3$' curve or change in sign of $\frac{dy}{dx}$	B1	www SC: $x=3$, minimum, $x=2$, maximum, B1
		2	

471. 9709_s19_ms_11 Q: 11

	Answer	Mark	Partial Marks
(i)	$3 \times -½ \times (1+4x)^{-\frac{3}{2}}$	B1	
	$\frac{dy}{dx} = 3 \times -½ \times (1+4x)^{-\frac{3}{2}} \times 4$	B1	Must have '× 4'
	If $x = 2$, $m = -\frac{2}{9}$, Perpendicular gradient = $\frac{9}{2}$	M1	Use of $m_1.m_2 = -1$
	Equation of normal is $y-1=\frac{9}{2}(x-2)$	M1	Correct use of line eqn (could use y=0 here)
	Put $y = 0$ or on the line before → $\frac{16}{9}$	A1	AG
		5	
(ii)	Area under the curve = $\int_0^2 \frac{3}{\sqrt{1+4x}}\,dx = \frac{3\sqrt{1+4x}}{\frac{1}{2}} \div 4$	B1 B1	Correct without '÷4'. For 2nd B1, ÷4'.
	Use of limits 0 to 2 → 4½ − 1½	M1	Use of correct limits in an integral.
	3	A1	
	Area of the triangle = ½ × 1 × $\frac{2}{9} = \frac{1}{9}$ or attempt to find $\int_{16/9}^{2}\left(\frac{9}{2}x-8\right)dx$	M1	Any correct method.
	Shaded area = $3 - \frac{1}{9} = 2\frac{8}{9}$	A1	
		6	

472. 9709_s19_ms_12 Q: 3

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = 7 \times -0.05$	M1	Multiply numerical gradient at $x = 2$ by ±0.05.
	−0.35 (units/s) **or** Decreasing at a rate of (+) 0.35	A1	Ignore notation and omission of units
		2	
(ii)	$(y) = \frac{x^4}{4} + \frac{4}{x}$ $(+c)$ oe	B1	Accept unsimplified
	Uses (2, 9) in an integral to find c.	M1	The power of at least one term increase by 1.
	$c = 3$ **or** $(y =)\frac{x^4}{4} + \frac{4}{x} + 3$ oe	A1	A0 if candidate continues to a final equation that is a straight line.
		3	

473. 9709_s19_ms_12 Q: 11

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(4x+1)^{-\frac{1}{2}}\right]$ [× 4] $\left[-\frac{9}{2}(4x+1)^{-\frac{3}{2}}\right]$ [× 4]	B1B1B1	B1 B1 for each, without × 4. B1 for ×4 twice.
	$\left(\frac{2}{\sqrt{4x+1}} - \frac{18}{\left(\sqrt{4x+1}\right)^3} \text{ or } \frac{8x-16}{(4x+1)^{\frac{3}{2}}}\right)$		SC If no other marks awarded award B1 for both powers of (4*x* +1) correct.
	$\int y dx = \left[\frac{(4x+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$ [÷ 4] + $\left[\frac{9(4x+1)^{\frac{1}{2}}}{\frac{1}{2}}\right]$ [÷ 4] (+C)	B1B1B1	B1 B1 for each, without ÷ 4. B1 for ÷4 twice. + C not required.
	$\left(\frac{\left(\sqrt{4x+1}\right)^3}{6} + \frac{9}{2}\left(\sqrt{4x+1}\right)(+C)\right)$		SC If no other marks awarded , B1 for both powers of (4*x* +1) correct.
		6	
(ii)	$\frac{dy}{dx} = 0 \rightarrow \frac{2}{\sqrt{4x+1}} - \frac{18}{(4x+1)^{\frac{3}{2}}} = 0$	M1	Sets their $\frac{dy}{dx}$ to 0 (and attempts to solve
	$4x + 1 = 9$ or $(4x + 1)^2 = 81$	A1	Must be from correct differential.
	$x = 2$, $y = 6$ **or** M is (2, 6) only.	A1	Both values required. Must be from correct differential.
		3	
(iii)	Realises area is $\int y\, dx$ **and** attempt to use their 2 and sight of 0.	*M1	Needs to use their integral and to see '*their 2*' substituted.
	Uses limits 0 to 2 correctly → [4.5 + 13.5] – [$\frac{1}{6}$ + 4.5] (= 13⅓)	DM1	Uses both 0 and '*their 2*' and subtracts. Condone wrong way round.
	(Area =) 1⅓ **or** 1.33	A1	Must be from a correct differential and integral.
		3	13⅓ or 1⅓ with little or no working scores M1DM0A0.

474. 9709_s19_ms_13 Q: 10

	Answer	Mark	Partial Marks
(i)	$\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]$	B1	oe
	$\frac{dy}{dx}=\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]\times 3$	B1	Must have '×3'
	At $x=4$, $\frac{dy}{dx}=\frac{3}{8}$ soi	B1	
	Line through (4, *their*4) with gradient *their* $\frac{3}{8}$	M1	If $y \neq 4$ is used then clear evidence of substitution of $x=4$ is needed
	Equation of tangent is $y-4=\frac{3}{8}(x-4)$ or $y=\frac{3}{8}x+\frac{5}{2}$	A1	oe
		5	
(ii)	Area under line $=\frac{1}{2}\left(4+\frac{5}{2}\right)\times 4=13$	B1	OR $\int_0^4 \frac{3}{8}x+\frac{5}{2}=\left[\frac{3}{16}x^2+\frac{5}{2}x\right]=[3+10]=13$
	Area under curve: $\int(3x+4)^{\frac{1}{2}}=\left[\frac{(3x+4)^{3/2}}{3/2}\right][\div 3]$	B1B1	Allow if seen as part of the difference of 2 integrals First B1 for integral without $[\div 3]$ Second B1 must have $[\div 3]$
	$\frac{128}{9}-\frac{16}{9}=\frac{112}{9}=12\frac{4}{9}$	M1	Apply limits 0 → 4 to an integrated expression
	Area = $13-12\frac{4}{9}=\frac{5}{9}$ (or 0.556)	A1	
	Alternative method for question 10(ii)		
	Area for line = 1/2 × 4 × 3/2 = 3	B1	OR $\int_{5/2}^{4}\frac{1}{3}(8y-20)=\frac{1}{3}\left[4y^2-20\right]=\frac{1}{3}[-16+25]=3$
	Area for curve = $\int \frac{1}{3}(y^2-4)=\left[\frac{y^3}{9}\right]-\left[\frac{4y}{3}\right]$	B1B1	
	$\left(\frac{64}{9}-\frac{16}{3}\right)-\left(\frac{8}{9}-\frac{8}{3}\right)=\frac{32}{9}$	M1	Apply limits 2 → 4 to an integrated expression for curve
	Area = $\frac{32}{9}-3=\frac{5}{9}$ (or 0.556)	A1	
		5	
(iii)	$\frac{dy}{dx}=\frac{1}{2}$	B1	
	$\frac{3}{2}(3x+4)^{-\frac{1}{2}}=\frac{1}{2}$	M1	Allow M1 for $\frac{3}{2}(3x+4)^{-\frac{1}{2}}=2$.
	$(3x+4)^{\frac{1}{2}}=3 \rightarrow 3x+4=9 \rightarrow x=\frac{5}{3}$ oe	A1	
		3	

475. 9709_w19_ms_11 Q: 9

	Answer	Mark	Partial Marks
(i)	$y = [(5x-1)^{1/2} \div \frac{3}{2} \div 5]\ [-2x]$	B1 B1	
	$3 = \frac{27}{(3/2)\times 5} - 4 + c$	M1	Substitute $x = 2$, $y = 3$
	$c = 7 - \frac{18}{5} = \frac{17}{5} \rightarrow \left(y = \frac{2(5x-1)^{\frac{3}{2}}}{15} - 2x + \frac{17}{5} \right)$	A1	
(ii)	$\mathrm{d}^2y/\mathrm{d}x^2 = \left[\frac{1}{2}(5x-1)^{-1/2}\right]\ [\times 5]$	B1 B1	
(iii)	$(5x-1)^{1/2} - 2 = 0 \rightarrow 5x - 1 = 4$ $x = 1$	M1A1	Set $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ and attempt solution (M1)
	$y = \frac{16}{25} - 2 + \frac{17}{5} = \frac{37}{15}$	A1	Or 2.47 or $\left(1, \frac{37}{15}\right)$
	$\frac{\mathrm{d}^2y}{\mathrm{d}x^x} = \frac{5}{2} \times \frac{1}{2} = \frac{5}{4}$ (> 0) hence minimum	A1	OE

476. 9709_w19_ms_11 Q: 11

	Answer	Mark	Partial Marks
(i)	$(y =)\ (x+2)^2 - 1$	B1 DB1	2nd B1 dependent on 2 in bracket
	$x + 2 = (\pm)(y+1)^{1/2}$	M1	
	$x = -2 + (y+1)^{1/2}$	A1	
(ii)	$x^2 = 4 + (y+1) - / + 4(y+1)^{\frac{1}{2}}$	*M1A1	SOI. Attempt to find x^2. The last term can be – or + at this stage
	$(\pi)\int x^2\,(\mathrm{d}y) = (\pi)\left[5y + \frac{y^2}{2} - \frac{4(y+1)^{\frac{3}{2}}}{\frac{3}{2}}\right]$	A2,1,0	
	$(\pi)\left[15 + \frac{9}{2} - \frac{64}{3} - \left(-5 + \frac{1}{2}\right)\right]$	DM1	Apply y limits
	$\frac{8\pi}{3}$ or 8.38	A1	

477. 9709_w19_ms_12 Q: 3

	Answer	Mark	Partial Marks
	$(y =)\ \frac{kx^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} \left(= \frac{k\sqrt{x}}{\frac{1}{2}} \right) (+c)$	B1	OE
	Substitutes both points into an integrated expression with a '$+c$' and solve as far as a value for one variable.	M1	Expect to see $-1 = 2k + c$ and $4 = 4k + c$
	$k = 2\frac{1}{2}$ **and** $c = -6$	A1	WWW
	$y = 5\sqrt{x} - 6$	A1	OE From correct values of both k & c and correct integral.
		4	

478. 9709_w19_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = [0] + \left[(2x+1)^{-3}\right] \times [+16]$	B2,1,0	OE. Full marks for 3 correct components. Withhold one mark for each error or omission.
	$\int y dx = [x] + \left[(2x+1)^{-1}\right] \times [+2]$ (+c)	B2,1,0	OE. Full marks for 3 correct components. Withhold one mark for each error or omission.
		4	
(ii)	At A, $x = ½$.	B1	Ignore extra answer $x = -1.5$
	$\frac{dy}{dx} = 2 \rightarrow$ Gradient of normal $(=-½)$	*M1	With *their* positive value of x at A and *their* $\frac{dy}{dx}$, uses $m_1 m_2 = -1$
	Equation of normal: $y - 0 = -½(x - ½)$ or $y - 0 = -½\ (0 - ½)$ or $0 = -½ \times ½ + c$	DM1	Use of *their* x at A and *their* normal gradient.
	B (0, ¼)	A1	
		4	
(iii)	$\int_0^{\frac{1}{2}} 1 - \frac{4}{(2x+1)^2}(dx)$	*M1	$\int y\,dx$ SOI with 0 and *their* positive x coordinate of A.
	[½ + 1] – [0 + 2] = (–½)	DM1	Substitutes both 0 and *their* ½ into *their* $\int y dx$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)
	Alternative method for question 10(iii)		
	$\int_{-3}^{0} \frac{1}{(1-y)^{\frac{1}{2}}} - \frac{1}{2}(dy)$	*M1	$\int x\,dy$ SOI. Where x is of the form $k\left(1-y)^{-\frac{1}{2}} + c\right)$ with 0 and *their* negative y intercept of curve.
	$[-2] - \left[-4 + \frac{3}{2}\right] = (½)$	DM1	Substitutes both 0 and *their* –3 into *their* $\int x dy$ and subtracts.
	Area of triangle above x-axis = ½ × ½ × ¼ $\left(=\frac{1}{16}\right)$	B1	
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)
	Alternative method for question 10(iii)		
	$\int_0^{\frac{1}{2}} -\frac{1}{2}x + \frac{1}{4} - y\,dx$	*M1	$\int$(*their* normal curve) with 0 and *their* positive x coordinate of A.
	Curve [½ + 1] – [0 + 2] = (–½)	DM1	Substitutes both 0 and *their* ½ into *their* $\int y dx$ and subtracts.
	$\int_0^{\frac{1}{2}} -\frac{1}{2}x + \frac{1}{4}dx = \frac{-x^2}{4} + \frac{x}{4} = \left[\frac{-1}{16} + \frac{1}{8}\right] - [0]\ \left(=\frac{1}{16}\right)$	B1	Substitutes both 0 and ½ into the correct integral and subtracts.
	Total area of shaded region = $\frac{9}{16}$	A1	OE (including AWRT 0.563)
		4	

479. 9709_w19_ms_13 Q: 8

	Answer	Mark	Partial Marks
(i)	$(2x-1)^{\frac{1}{2}} < 2$ or $3(2x-1)^{\frac{1}{2}} < 6$	M1	SOI
	$2x-1<4$	A1	SOI
	$\frac{1}{2} < x < \frac{5}{2}$	A1 A1	Allow 2 separate statements
		4	
(ii)	f(x) = $[3(2x-1)^{3/2} \div \left(\frac{3}{2}\right) \div (2)]$ $[-6x]$ (+c)	B1 B1	
	Subsitute $x = 1$, $y = -3$ into an integrated expression.	M1	Dependent on c being present ($c = 2$)
	f(x) = $(2x-1)^{\frac{3}{2}} - 6x + 2$	A1	
		4	

480. 9709_w19_ms_13 Q: 11

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = -2(x-1)^{-3}$	B1	
	When $x = 2$, $m = -2 \rightarrow$ gradient of normal = $-\frac{1}{m}$	M1	m must come from differentiation
	Equation of normal is $y-3 = ½(x-2) \rightarrow y = ½x + 2$	A1	AG Through (2, 3) with gradient $-\frac{1}{m}$. Simplify to AG
		3	
(ii)	$(\pi)\int y_1^2\,(dx), (\pi)\int y_2^2\,(dx)$	*M1	Attempt to integrate y^2 for at least one of the functions
	$(\pi)\int\left(\frac{1}{2}x+2\right)^2$ or $\left(\frac{1}{4}x^2+2x+4\right)$ $(\pi)\int\left((x-1)^{-4}+4(x-1)^{-2}+4\right)$	A1A1	A1 for $\left(\frac{1}{2}x+2\right)^2$ depends on an attempt to integrate this form later
	$(\pi)\left[\frac{2}{3}\left(\frac{1}{2}x+2\right)^3 \text{ or } \frac{1}{12}x^3+x^2+4x\right]$ $(\pi)\left[\frac{(x-1)^{-3}}{-3}+\frac{4(x-1)^{-1}}{-1}+4x\right]$	A1A1	Must have at least 2 terms correct for each integral
	$(\pi)\left\{18-\frac{125}{12} or \frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)\right\}$ $\left\{\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)\right\}$	DM1	Apply limits to at least 1 integrated expansion
	Attempt to add 2 volume integrals (or 1 volume integral + frustum) $\pi\left\{7\frac{7}{12}+6\frac{7}{24}\right\}$	DM1	
	$13\frac{7}{8}\pi$ or $\frac{111}{8}\pi$ or 13.9π or 43.6	A1	$\frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)$ $\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)$
		8	

481. 9709_m18_ms_12 Q: 1

	Answer	Mark	Partial Marks
	$(y) = \frac{x^{½}}{½} - 3x$ $(+c)$	B1B1	
	Sub (4, –6) $-6 = 4 - 12 + c \rightarrow c = 2$	M1A1	Expect $(y) = 2x^{½} - 3x + 2$
		4	

482. 9709_m18_ms_12 Q: 11

	Answer	Mark	Partial Marks
(i)	$dy/dx = [-2] - [3(1-2x)^2] \times [-2]$ $(= 4 - 24x + 24x^2)$	B2,1,0	Award for the accuracy within each set of square brackets
	At $x = ½$ $dy/dx = -2$	B1	
	Gradient of line $y = 1 - 2x$ is –2 (hence *AB* is a tangent) AG	B1	
		4	

	Answer	Mark	Partial Marks
(ii)	Shaded region = $\int_0^{½}(1-2x) - \int_0^{½}[1-2x-(1-2x)^3]$ oe	M1	Note: If area triangle OAB – area under the curve is used the first part of the integral for the area under the curve must be evaluated
	$= \int_0^{½}(1-2x)^3\, dx$ AG	A1	
		2	
(iii)	Area = $\left[\frac{(1-2x)^4}{4}\right][\div -2]$	*B1B1	
	0 – (–1/8) = 1/8	DB1	OR $\int 1 - 6x + 12x^2 - 8x^3 = x - 3x^2 + 4x^3 - 2x^4$ **(B2,1,0)** Applying limits 0 → ½
		3	

483. 9709_s18_ms_11 Q: 3

	Answer	Mark	Partial Marks
	$\frac{dy}{dx} = \frac{12}{(2x+1)^2}$ $\rightarrow y = \frac{-12}{2x+1}$ $\div 2$ $(+c)$	B1 B1	Correct without " ÷ 2". For " ÷ 2". Ignore "*c*".
	Uses (1, 1) $\rightarrow c = 3$ $(\rightarrow y = \frac{-6}{2x+1} + 3)$	M1 A1	Finding "*c*" following integration. CAO
	Sets *y* to 0 and attempts to solve for $x \rightarrow$ $x = \frac{1}{2}$ $\rightarrow ((\frac{1}{2}, 0))$	DM1 A1	Sets *y* to 0. $x = \frac{1}{2}$ is sufficient for A1.
		6	

484. 9709_s18_ms_11 Q: 10

	Answer	Mark	Partial Marks
	$y = x^3 - 2x^2 + 5x$		
(i)	$\frac{dy}{dx} = 3x^2 - 4x + 5$	B1	CAO
	Using $b^2 - 4ac \rightarrow 16 - 60 \rightarrow$ negative $\rightarrow$ some explanation or completed square and explanation	M1 A1	Uses discriminant on equation (set to 0). CAO
		3	
(ii)	$m = 3x^2 - 4x + 5$ $\frac{dm}{dx} = 6x - 4$ $(= 0)$ (must identify as $\frac{dm}{dx}$)	B1FT	FT providing differentiation is equivalent
	$\rightarrow x = \frac{2}{3}$, $m = \frac{11}{3}$ or $\frac{dy}{dx} = \frac{11}{3}$ Alt1: $m = 3\left(x - \frac{2}{3}\right)^2 + \frac{11}{3}$, $m = \frac{11}{3}$ Alt2: $3x^2 - 4x + 5 - m = 0$, $b^2 - 4ac = 0$, $m = \frac{11}{3}$	M1 A1	Sets to 0 and solves. A1 for correct m. Alt1: B1 for completing square, M1A1 for ans Alt2: B1 for coefficients, M1A1 for ans
	$\frac{d^2m}{dx^2} = 6$ +ve $\rightarrow$ Minimum value or refer to sketch of curve or check values of m either side of $x = \frac{2}{3}$,	M1 A1	M1 correct method. A1 (no errors anywhere)
		5	

	Answer	Mark	Partial Marks
(iii)	Integrate $\rightarrow \frac{x^4}{4} - \frac{2x^3}{3} + \frac{5x^2}{2}$	B2,1	Loses a mark for each incorrect term
	Uses limits 0 to 6 $\rightarrow$ 270 (may not see use of lower limit)	M1 A1	Use of limits on an integral. CAO Answer only 0/4
		4	

485. 9709_s18_ms_12 Q: 9

	Answer	Mark	Partial Marks
(i)	$y = \frac{2}{3}(4x+1)^{\frac{3}{2}} \div 4$ $(+ C)$ $\left(= \frac{(4x+1)^{\frac{3}{2}}}{6}\right)$	B1 B1	B1 without ÷ 4. B1 for ÷ 4 oe. Unsimplified OK
	Uses $x = 2$, $y = 5$	M1	Uses (2, 5) in an integral (indicated by an increase in power by 1).
	$\rightarrow c = ½$ oe isw	A1	No isw if candidate now goes on to produce a straight line equation
		4	
(ii)	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$		
	$\frac{dx}{dt} = 0.06 \div 3$	M1	Ignore notation. Must be 0.06÷3 for M1.
	= 0.02 oe	A1	Correct answer with no working scores 2/2
		2	
(iii)	$\frac{d^2y}{dx^2} = ½\left(4x+1\right)^{-½} \times 4$	B1	
	$\frac{d^2y}{dx^2} \times \frac{dy}{dx} = \frac{2}{\sqrt{4x+1}} \times \sqrt{4x+1}$ $(= 2)$	B1FT	Must either show the algebraic product and state that it results in a constant or evaluate it as '= 2'. Must not evaluate at $x = 2$. ft to apply only if $\frac{d^2y}{dx^2}$ is of the form $k\left(4x+1\right)^{-½}$
		2	

486. 9709_s18_ms_12 Q: 11

	Answer	Mark	Partial Marks
(i)	$y = \frac{x}{2} + \frac{6}{x} = 4 \rightarrow x = 2$ or 6	B1 B1	Inspection or guesswork OK
	$\frac{dy}{dx} = \frac{1}{2} - \frac{6}{x^2}$	B1	Unsimplified OK
	When $x = 2$, $m = -1 \rightarrow x + y = 6$ When $x = 6$, $m = \frac{1}{3} \rightarrow y = \frac{1}{3}x + 2$	*M1	Correct method for either tangent
	Attempt to solve simultaneous equations	DM1	Could solve BOTH equations separately with $y = x$ and get $x = 3$ both times.
	(3,3)	A1	Statement about $y = x$ not required.
		6	

	Answer	Mark	Partial Marks
(ii)	$V = (\pi) \int \left(\frac{x^2}{4} + 6 + \frac{36}{x^2}\right) (dx)$	*M1	Integrate using $\pi \int y^2 dx$ (doesn't need π or dx). Allow incorrect squaring. Not awarded for $\pi \int \left\{4 - \left(\frac{x}{2} + \frac{6}{x}\right)\right\}^2 dx$. Integration indicated by increase in any power by 1.
	Integration $\rightarrow \frac{x^3}{12} + 6x - \frac{36}{x}$	A2,1	3 things wanted —1 each error, allow + C. (Doesn't need π)
	Using limits '*their 2*' to '*their 6*' ($53\frac{1}{3}\pi$, $\frac{160}{3}\pi$, 168 awrt)	DM1	Evidence of their values 6 and 2 from **(i)** substituted into their integrand and then subtracted. $48 - \left(-\frac{16}{3}\right)$ is enough.
	Vol for line: integration or cylinder ($\rightarrow 64\pi$)	M1	Use of $\pi r^2 h$ or integration of 4^2 (could be from $\left\{4 - \left(\frac{x}{2} + \frac{6}{x}\right)\right\}^2$)
	Subtracts $\rightarrow 10\frac{2}{3}\pi$ oe $\left(\text{e.g.} \frac{32}{3}\pi, 33.5\text{awrt}\right)$	A1	

	Answer	Mark	Partial Marks
(ii)	**OR**		
	$V = (\pi) \int 4^2 - \left(\frac{x}{2} + \frac{6}{x}\right)^2 (dx)$	M1 *M1	Integrate using $\pi \int y^2 dx$ (doesn't need π or dx) Integration indicated by increase in any power by 1.
	$= (\pi) \int 16 - \left(\frac{x^2}{4} + 6 + \frac{36}{x^2}\right) (dx)$		
	$= (\pi) \left[16x - \left(\frac{x^3}{12} + 6x - \frac{36}{x}\right)\right] (dx)$	A2,1	Or $\left[10x - \frac{x^3}{12} + \frac{36}{x}\right]$
	$= (\pi)$ (48 - 37⅓)	DM1	Evidence of their values 6 and 2 from (i) substituted
	$= 10\frac{2}{3}\pi$ oe $\left(\text{eg} \frac{32}{3}\pi, 33.5\text{awrt}\right)$	A1	
		6	

487. 9709_s18_ms_13 Q: 4

	Answer	Mark	Partial Marks
	$f(x)=\left[\frac{(3x-1)^{\frac{2}{3}}}{\frac{2}{3}}\right][\div 3]\ (+c)$	B1B1	
	$1=\frac{8^{\frac{2}{3}}}{2}+c$	M1	Sub $y=1, x=3$ Dep. on attempt to integrate and c present
	$c=-1 \rightarrow y=\frac{1}{2}(3x-1)^{\frac{2}{3}}-1$ SOI	A1	
	When $x=0$, $y=\frac{1}{2}(-1)^{\frac{2}{3}}-1 \quad =-\frac{1}{2}$	DM1A1	Dep. on previous M1
		6	

488. 9709_s18_ms_13 Q: 11

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx}=2(x+1)-(x+1)^{-2}$	B1	
	Set = 0 and obtain $2(x+1)^3=1$ convincingly www AG	B1	
	$\frac{d^2y}{dx^2}=2+2(x+1)^{-3}$ www	B1	
	Sub, e.g., $(x+1)^{-3}=2$ OE or $x=\left(\frac{1}{2}\right)^{\frac{1}{3}}-1$	M1	Requires exact method – otherwise scores M0
	$\frac{d^2y}{dx^2}=6$ CAO www	A1	and exact answer – otherwise scores A0
		5	

	Answer	Mark	Partial Marks
(ii)	$y^2=(x+1)^4+(x+1)^{-2}+2(x+1)$ SOI	B1	OR $y^2=(x^4+4x^3+6x^2+4x+1)+(2x+2)+(x+1)^{-2}$
	$(\pi)\int y^2dx=(\pi)\left[\frac{(x+1)^5}{5}\right]+\left[\frac{(x+1)^{-1}}{-1}\right]+\left[\frac{2(x+1)^2}{2}\right]$ OR $(\pi)\left[\frac{x^5}{5}+x^4+2x^3+2x^2+x\right]+\left[x^2+2x\right]+\left[-\frac{1}{x+1}\right]$	B1B1B1	Attempt to integrate y^2. Last term might appear as (x^2+2x)
	$(\pi)\left[\frac{32}{5}-\frac{1}{2}+4-\left(\frac{1}{5}-1+1\right)\right]$	M1	Substitute limits $0 \rightarrow 1$ into an attempted integration of y^2. Do not condone omission of value when $x=0$
	9.7π or 30.5	A1	Note: omission of $2(x+1)$ in first line $\rightarrow 6.7\pi$ scores 3/6. Ignore initially an extra volume, e.g. $(\pi)\int(4\frac{1}{2})^2$. Only take into account for the final answer
		6	

489. 9709_w18_ms_11 Q: 6

	Answer	Mark	Partial Marks
(i)	$0 = 9a + 3a^2$	M1	Sub $\frac{dy}{dx} = 0$ and $x = 3$
	$a = -3$ only	A1	
		2	
(ii)	$\frac{dy}{dx} = -3x^2 + 9x \rightarrow y = -x^3 + \frac{9x^2}{2}$ $(+c)$	M1A1FT	Attempt integration. $\frac{1}{3}ax^3 + \frac{1}{2}a^2x^2$ scores M1. Ft on *their a*.
	$9\frac{1}{2} = -27 + 40\frac{1}{2} + c$	DM1	Sub $x = 3, y = 9\frac{1}{2}$. Dependent on c present
	$c = -4$	A1	Expect $y = -x^3 + \frac{9x^2}{2} - 4$
		4	
(iii)	$\frac{d^2y}{dx^2} = -6x + 9$	M1	$2ax + a^2$ scores M1
	At $x = 3$, $\frac{d^2y}{dx^2} = -9 < 0$ MAX www	A1	Requires at least one of -9 or < 0. Other methods possible.
		2	

490. 9709_w18_ms_11 Q: 7

	Answer	Mark	Partial Marks
(i)	$2 = k(8 - 28 + 24) \rightarrow k = 1/2$	B1	
		1	
(ii)	When $x = 5$, $y = [\frac{1}{2}](125 - 175 + 60) = 5$	M1	Or solve $[\frac{1}{2}](x^3 - 7x^2 + 12x) = x \Rightarrow x = 5$ $[x = 0, 2]$
	Which lies on $y = x$, oe	A1	
		2	
(iii)	$\int [\frac{1}{2}(x^3 - 7x^2 + 12x) - x]dx$.	M1	Expect $\int \frac{1}{2}x^3 - \frac{7}{2}x^2 + 5x$
	$\frac{1}{8}x^4 - \frac{7}{6}x^3 + \frac{5}{2}x^2$	B2,1,0FT	Ft on their k
	$2 - 28/3 + 10$	DM1	Apply limits $0 \rightarrow 2$
	$8/3$	A1	
	OR $\frac{1}{8}x^4 - \frac{7}{6}x^3 + 3x^2$	B2,1,0FT	Integrate to find area under curve, Ft on their k
	$2 - 28/3 + 12$	M1	Apply limits $0 \rightarrow 2$. Dep on integration attempted
	Area $\Delta = \frac{1}{2} \times 2 \times 2$ or $\int_0^2 x dx = \left[\frac{1}{2}x^2\right] = 2$	M1	
	$8/3$	A1	
		5	

491. 9709_w18_ms_12 Q: 2

	Answer	Mark	Partial Marks
	Integrate → $\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$ (+C)	**B1 B1**	B1 for each term correct – allow unsimplified. C not required.
	$\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}} + 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^4 \to \frac{40}{3} - \frac{14}{3}$	**M1**	Evidence of 4 and 1 used correctly in their integrand ie at least one power increased by 1.
	$= \frac{26}{3}$ **oe**	**A1**	Allow 8.67 awrt. No integrand implies use of integration function on calculator 0/4. Beware a correct answer from wrong working.
		4	

492. 9709_w18_ms_12 Q: 11

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = \left[\frac{3}{2} \times (4x+1)^{-\frac{1}{2}}\right] [\times 4] [-2] \left(\frac{6}{\sqrt{4x+1}} - 2\right)$	**B2,1,0**	Looking for 3 components
	$\int y\,dx = \left[3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}\right] [\div 4] [-\frac{2x^2}{2}]$ (+ C) $\left(= \frac{(4x+1)^{\frac{3}{2}}}{2} - x^2\right)$	**B1 B1 B1**	B1 for $3(4x+1)^{\frac{3}{2}} \div \frac{3}{2}$ B1 for '÷4'. B1 for '$-\frac{2x^2}{2}$'. Ignore omission of + C. If included isw any attempt at evaluating.
		5	
(ii)	At M, $\frac{dy}{dx} = 0 \to \frac{6}{\sqrt{4x+1}} = 2$	**M1**	Sets their 2 term $\frac{dy}{dx}$ to 0 and attempts to solve (as far as x = k)
	$\boldsymbol{x = 2, y = 5}$	**A1 A1**	
		3	

	Answer	Mark	Partial Marks
(iii)	Area under the curve $=\left[\frac{1}{2}(4x+1)^{\frac{3}{2}}-x^2\right]_0^2$	M1	Uses their integral and their '2' and 0 correctly
	(13.5 – 4) – 0.5 or 9.5 – 0.5 = 9	A1	No working implies use of integration function on calculator M0A0.
	Area under the chord = trapezium = ½ × 2 × (3 + 5) = 8 Or $\left[\frac{x^2}{2}+3x\right]_0^2=8$	M1	Either using the area of a trapezium with their 2, 3 and 5 or $\int(their\, x+3)dx$ using their '2' and 0 correctly.
	(Shaded area = 9 – 8) = **1**	A1	Dependent on both method marks,
	OR Area between the chord and the curve is:		
	$\int_0^2 3\sqrt{4x+1}-2x-(x+3)dx$ $=\int_0^2 3\sqrt{4x+1}-3x-3dx$	M1	Subtracts their line from given curve and uses their '2' and 0 correctly.
	$=3\left[\frac{1}{6}(4x+1)^{\frac{3}{2}}-\frac{x^2}{2}-x\right]_0^2$	A1	All integration correct and limits 2 and 0.
	$=3\left\{\left(\frac{27}{6}-2-2\right)-\left(\frac{1}{6}\right)\right\}$	M1	Evidence of substituting their '2' and 0 into their integral.
	$=3\left\{\frac{1}{2}-\frac{1}{6}\right\}=3\left\{\frac{1}{3}\right\}=1$	A1	No working implies use of a calculator M0A0.
		[4]	

493. 9709_w18_ms_13 Q: 8

	Answer	Mark	Partial Marks
(i)	$y=⅓\,ax^3+½bx^2-4x\ \ (+c)$	B1	
	$11=0+0+0+c$	M1	Sub $x = 0$, $y = 11$ into an integrated expression. c must be present
	$y=⅓ax^3+½bx^2-4x+11$	A1	
		3	
(ii)	$4a+2b-4=0$	M1	Sub $x=2$, $dy/dx=0$
	$⅓(8a)+2b-8+11=3$	M1	Sub $x=2$, $y=3$ into an integrated expression. Allow if 11 missing
	Solve simultaneous equations	DM1	Dep. on both M marks
	$a=3$, $b=-4$	A1A1	Allow if no working seen for simultaneous equations
		5	

494. 9709_w18_ms_13 Q: 10

	Answer	Mark	Partial Marks
(i)	$V=4(\pi)\int(3x-1)^{-2/3}\,dx\ \ =4(\pi)\left[\frac{(3x-1)^{1/3}}{1/3}\right][\div 3]$	M1A1A1	Recognisable integration of y^2 (M1) Independent A1, A1 for [ ] [ ]
	$4(\pi)[2-1]$	DM1	Expect $4(\pi)(3x-1)^{⅓}$
	4π or 12.6	A1	Apply limits ⅔ → 3. Some working must be shown.
		5	

	Answer	Mark	Partial Marks
(ii)	$dy/dx = (-2/3)(3x-1)^{-4/3} \times 3$	B1	Expect $-2(3x-1)^{-4/3}$
	When $x = 2/3,\ y = 2$ soi $dy/dx = -2$	B1B1	2nd B1 dep. on correct expression for dy/dx
	Equation of normal is $y - 2 = \frac{1}{2}(x - \frac{2}{3})$	M1	Line through (⅔, *their* 2) and with grad $-1/m$. Dep on m from diffn
	$y = \frac{1}{2}x + \frac{5}{3}$	A1	
		5	

495. 9709_m17_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$2x - 2/x^3 = 0$	M1	Set = 0.
	$x^4 = 1 \Rightarrow x = 1$ at A cao	A1	Allow 'spotted' $x = 1$
	Total:	2	
(ii)	$f(x) = x^2 + 1/x^2 (+c)$ cao	B1	
	$\frac{189}{16} = 16 + 1/16 + c$	M1	Sub (4, $\frac{189}{16}$). c must be present. Dep. on integration
	$c = -17/4$	A1	
	Total:	3	

	Answer	Mark	Partial Marks
(iii)	$x^2 + 1/x^2 - 17/4 = 0 \Rightarrow 4x^4 - 17x^2 + 4\ (= 0)$	M1	Multiply by $4x^2$ (or similar) to transform into 3-term quartic.
	$(4x^2 - 1)(x^2 - 4)\ (= 0)$	M1	Treat as quadratic in x^2 and attempt solution or factorisation.
	$x = \frac{1}{2},\ 2$	A1A1	Not necessary to distinguish. Ignore negative values. No working scores 0/4
	Total:	4	
(iv)	$\int(x^2 + x^{-2} - 17/4)dx = \frac{x^3}{3} - \frac{1}{x} - \frac{17x}{4}$	B2,1,0√	Mark final integral
	$(8/3 - 1/2 - 17/2) - (1/24 - 2 - 17/8)$	M1	Apply *their* limits from **(iii)** (Seen). Dep. on integration of at least 1 term of y
	Area = $9/4$	A1	Mark final answer. $\int y^2$ scores 0/4
	Total:	4	

496. 9709_s17_ms_11 Q: 7

	Answer	Mark	Partial Marks
	$\frac{dy}{dx} = 7 - x^2 - 6x$		
(i)	$y = 7x - \frac{x^3}{3} - \frac{6x^2}{2}$ $(+c)$	B1	CAO
	Uses (3, − 10) → $c = 5$	M1 A1	Uses the given point to find c
	Total:	3	
(ii)	$7 - x^2 - 6x = 16 - (x+3)^2$	B1 B1	**B1** $a = 16$, **B1** $b = 3$.
	Total:	2	
(iii)	$16 - (x+3)^2 > 0 \rightarrow (x+3)^2 < 16$, and solve	M1	or factors $(x + 7)(x - 1)$
	End-points $x = 1$ or -7	A1	
	$\rightarrow -7 < x < 1$	A1	needs <, not ⩽. (SR $x < 1$ only, or $x > -7$ only **B1** i.e. 1/3)
	Total:	3	

497. 9709_s17_ms_11 Q: 10

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = \frac{-4}{(5-3x)^2} \times (-3)$	B1 B1	**B1** without ×(−3) **B1** For ×(−3)
	Gradient of tangent = 3, Gradient of normal – ⅓	*M1	Use of $m_1 m_2 = -1$ after calculus
	→ eqn: $y - 2 = -\frac{1}{3}(x-1)$	DM1	Correct form of equation, with (1, their y), not (1,0)
	$\rightarrow y = -\frac{1}{3}x + \frac{7}{3}$	A1	This mark needs to have come from $y = 2$, y must be subject
	Total:	5	
(ii)	$\text{Vol} = \pi \int_0^1 \frac{16}{(5-3x)^2} dx$	M1	Use of $V = \pi \int y^2 dx$ with an attempt at integration
	$\pi \left[\frac{-16}{(5-3x)} \div -3 \right]$	A1 A1	**A1** without(÷ −3), **A1** for (÷ −3)
	$= (\pi \left(\frac{16}{6} - \frac{16}{15} \right)) = \frac{8\pi}{5}$ (if limits switched must show – to +)	M1 A1	Use of both correct limits **M1**
	Total:	5	

498. 9709_s17_ms_12 Q: 6

	Answer	Mark	Partial Marks
	$\text{Vol} = \pi \int (5-x)^2 dx - \pi \int \frac{16}{x^2} dx$	M1*	Use of volume formula at least once, condone omission of π and limits and dx.
		DM1	Subtracting volumes somewhere must be after squaring.
	$\int (5-x)^2 dx = \frac{(5-x)^3}{3} \div -1$	B1 B1	**B1** Without ÷ (−1). **B1** for ÷(−1)
	(or $25x - 10x^2/2 + \frac{1}{3}x^3$)	(B2,1,0)	−1 for each incorrect term
	$\int \frac{16}{x^2} dx = -\frac{16}{x}$	B1	
	Use of limits 1 and 4 in an integrated expression and subtracted.	DM1	Must have used "y^2" at least once. Need to see values substituted.
	→ 9π or 28.3	A1	
	Total:	7	

499. 9709_s17_ms_13 Q: 10

	Answer	Mark	Partial Marks
(a)(i)	Attempt to integrate $V = (\pi)\int(y+1)\mathrm{d}y$	M1	Use of h in integral e.g. $\int(h+1) = ½h^2 + h$ is **M0**. Use of $\int y^2\mathrm{d}x$ is **M0**
	$= (\pi)\left[\frac{y^2}{2} + y\right]$	A1	
	$= \pi\left[\frac{h^2}{2} + h\right]$	A1	**AG**. Must be from clear use of limits 0→ h somewhere.
	Total:	3	
(ii)	$\int(y+1)^{1/2}\mathrm{d}y$ **ALT** $6 - \int(x^2 - 1)\mathrm{d}x$	M1	Correct variable and attempt to integrate
	$⅔(y+1)^{3/2}$ oe **ALT** $6 - (⅓x^3 - x)$ CAO	*A1	Result of integration must be shown
	$⅔[8-1]$ **ALT** $6 - [\left(\frac{8}{3} - 1\right) - \left(\frac{1}{3} - 1\right)]$	DM1	Calculation seen with limits 0→3 for y. For ALT, limits are 1→2 and rectangle.
	14/3 **ALT** 6 – 4/3 = 14/3	A1	16/3 from $⅔ \times 8$ gets **DM1A0** provided work is correct up to applying limits.
	Total:	4	

	Answer	Mark	Partial Marks
(b)	Clear attempt to differentiate wrt h	M1	Expect $\frac{\mathrm{d}V}{\mathrm{d}h} = \pi(h+1)$. Allow h + 1. Allow h.
	Derivative = 4π SOI	*A1	
	$\frac{2}{\textit{their}\text{ derivative}}$. Can be in terms of h	DM1	
	$\frac{2}{4\pi}$ or $\frac{1}{2\pi}$ or 0.159	A1	
	Total:	4	

500. 9709_s17_ms_13 Q: 11

	Answer	Mark	Partial Marks
(i)	$\mathrm{f}'(x) = [(4x+1)^{1/2} \div ½]\,[\div 4]\,(+c)$	B1 B1	Expect $½(4x+1)^{1/2}\ (+c)$
	$\mathrm{f}'(2) = 0 \Rightarrow \frac{3}{2} + c = 0 \Rightarrow c = -\frac{3}{2}$ (Sufficient)	B1 FT	Expect $½(4x+1)^{1/2} - \frac{3}{2}$. FT on *their* $\mathrm{f}'(x) = k(4x+1)^{1/2} + c$. (i.e. $c = -3k$)
	Total:	3	
(ii)	$\mathrm{f}''(0) = 1$ SOI	B1	
	$\mathrm{f}'(0) = 1/2 - 1½ = -1$ SOI	B1 FT	Substitute x = 0 into *their* f ′(x) but must not involve c otherwise **B0B0**
	f(0) = – 3	B1 FT	FT for 3 terms in AP. FT for 3rd **B1** dep on 1st **B1**. Award marks for the AP method only.
	Total:	3	
(iii)	$\mathrm{f}(x) = \left[½(4x+1)^{3/2} \div 3/2 \div 4\right] - \left[1½x\right]\,(+k)$	B1 FT B1 FT	Expect $(1/12)(4x+1)^{3/2} - 1½x\ (+k)$. FT from *their* f ′($x$) but c numerical.
	$-3 = 1/12 - 0 + k \Rightarrow k = -37/12$ CAO	M1A1	Sub $x = 0, y = \textit{their}\ \mathrm{f}(0)$ into *their* f(x). Dep on cx & k present (c numerical)
	Minimum value = f(2) = $\frac{27}{12} - 3 - \frac{37}{12} = -\frac{23}{6}$ or – 3.83	A1	
	Total:	5	

501. 9709_w17_ms_11 Q: 10

	Answer	Mark	Partial Marks
(i)	Area $= \int ½\left(x^4 - 1\right)dx = ½\left[\frac{x^5}{5} - x\right]$	*B1	
	$½\left[\frac{1}{5} - 1\right] - 0 \quad = \quad (-)\frac{2}{5}$	DM1A1	Apply limits 0→1
		3	
(ii)	Vol $= \pi \int y^2 dx \quad = \quad ¼(\pi)\int\left(x^8 - 2x^4 + 1\right)dx$	M1	(If middle term missed out can only gain the M marks)
	$¼(\pi)\left[\frac{x^9}{9} - \frac{2x^5}{5} + x\right]$	*A1	
	$¼(\pi)[\left(\frac{1}{9} - \frac{2}{5} + 1\right] - 0$	DM1	
	$\frac{8\pi}{45}$ or 0.559	A1	
		4	

	Answer	Mark	Partial Marks
(iii)	Vol $= \pi \int x^2 dy \quad = \quad (\pi)\int(2y+1)^{1/2}dy$	M1	Condone use of x if integral is correct
	$(\pi)\left[\frac{(2y+1)^{3/2}}{3/2}\right][\div 2]$	*A1A1	Expect $(\pi)\left[\frac{(2y+1)^{3/2}}{3}\right]$
	$(\pi)\left[\frac{1}{3} - 0\right]$	DM1	
	$\frac{\pi}{3}$ or 1.05	A1	Apply $-\frac{1}{2} \to 0$
		5	

502. 9709_w17_ms_12 Q: 8

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = 0$	M1	Sets $\frac{dy}{dx}$ to 0 and attempts to solve leading to two values for x.
	$x = 1, x = 4$	A1	Both values needed
		2	

	Answer	Mark	Partial Marks
(ii)	$\frac{d^2y}{dx^2} = -2x + 5$	B1	
	Using both of their x values in their $\frac{d^2y}{dx^2}$	M1	Evidence of any valid method for both points.
	$x = 1 \to (3) \to$ Minimum, $x = 4 \to (-3) \to$ Maximum	A1	
		3	
(iii)	$y = -\frac{x^3}{3} + \frac{5x^2}{2} - 4x \quad (+c)$	B2, 1, 0	$+c$ not needed. –1 each error or omission.
	Uses x =6, y = 2 in an integrand to find c → c = 8	M1 A1	Statement of the final equation not required.
		4	

503. 9709_w17_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = \frac{1}{2} \times (5x-1)^{-\frac{1}{2}} \times 5 \quad (= \frac{5}{6})$	B1 B1	**B1** Without × 5 **B1** × 5 of an attempt at differentiation
	m of normal = $-\frac{6}{5}$	M1	Uses $m_1m_2 = -1$ with their numeric value from their dy/dx
	Equation of normal $y - 3 = -\frac{6}{5}(x-2)$ OE or $5y + 6x = 27$ or $y = \frac{-6}{5}x + \frac{27}{5}$	A1	Unsimplified. Can use $y = mx + c$ to get $c = 5.4$ ISW

	Answer	Mark	Partial Marks
(ii)	*EITHER:* For the curve $(\int)\sqrt{5x-1}dx = \frac{(5x-1)^{\frac{3}{2}}}{\frac{3}{2}} \div 5$	(B1 B1	Correct expression without ÷5 For dividing an attempt at integration of y by 5
	Limits from $\frac{1}{5}$ to 2 used → 3.6 or $\frac{18}{5}$ OE	M1 A1	Using $\frac{1}{5}$ and 2 to evaluate an integrand (may be $\int y^2$)
	Normal crosses x-axis when $y = 0$, → x= (4½)	M1	Uses their equation of normal, NOT tangent
	Area of triangle = 3.75 or $\frac{15}{4}$ OE	A1	This can be obtained by integration
	Total area=3.6 + 3.75 = 7.35, $\frac{147}{20}$ OE	A1)	
	OR: For the curve: $(\int)\frac{1}{5}(y^2+1)dy = \frac{1}{5}\left(\frac{y^3}{3} + y\right)$	(B2, 1, 0	−1 each error or omission.
	Limits from 0 to 3 used → 2.4 or $\frac{12}{5}$ OE	M1 A1	Using 0 and 3 to evaluate an integrand
	Uses their equation of normal, NOT tangent.	M1	Either to find side length for trapezium or attempt at integrating between 0 and 3
	Area of trapezium = $\frac{1}{2}(2 + 4½) \times 3 = \frac{39}{4}$ *or* $9\frac{3}{4}$	A1	This can be obtained by integration
	Shaded area = $\frac{39}{4} - \frac{12}{5} = 7.35, \frac{147}{20}$ OE	A1)	

504. 9709_w17_ms_13 Q: 8

	Answer	Mark	Partial Marks
(i)	*EITHER:* $4 - 3\sqrt{x} = 3 - 2x \rightarrow 2x - 3\sqrt{x} + 1$ (=0) or e.g. $2k^2 - 3k + 1$ (=0)	(M1	Form 3-term quad & attempt to solve for $\sqrt{x}$.
	$\sqrt{x} = ½, 1$	A1	Or $k = ½$ or 1 (where $k = \sqrt{x}$).
	$x = ¼, 1$	A1)	
	OR1: $(3\sqrt{x})^2 = (1+2x)^2$	(M1	
	$4x^2 - 5x + 1$ (=0)	A1	
	$x = ¼, 1$	A1)	
	OR2: $\frac{3-y}{2} = \left(\frac{4-y}{3}\right)^2 \quad \left(\rightarrow 2y^2 - 7y + 5(=0)\right)$	(M1	Eliminate x
	$y = \frac{5}{2}, 1$	A1	
	$x = ¼, 1$	A1)	
		3	

	Answer	Mark	Partial Marks
(ii)	*EITHER:* Area under line = $\int(3-2x)\,dx = 3x - x^2$	(B1	
	$= \left[(3-1) - \left(\frac{3}{4} - \frac{1}{16}\right)\right]$	M1	Apply *their* limits (e.g. ¼ → 1) after integn.
	Area under curve $= \int\left(4 - 3x^{1/2}\right)dx = 4x - 2x^{3/2}$	B1	
	$\left[(4-2) - (1 - ¼)\right]$	M1	Apply *their* limits (e.g. ¼ → 1) after integration.
	Required area = $\frac{21}{16} - \frac{5}{4} = \frac{1}{16}$ (or 0.0625)	A1)	
	OR: $+/- \int(3-2x) - \left(4 - 3x^{\frac{1}{2}}\right) = +/- \int(-1 - 2x + 3x^{\frac{1}{2}})$	(*M1	Subtract functions and then attempt integration
	$+/-\left[-x - x^2 + \frac{3x^{3/2}}{3/2}\right]$	A2, 1, 0 FT	FT on *their* subtraction. Deduct 1 mark for each term incorrect
	$+/-\left[-1 - 1 + 2 - \left(-\frac{1}{4} + \frac{1}{16} + \frac{1}{8}\right)\right] = \frac{1}{16}$ (or 0.0625)	DM1 A1)	Apply *their* limits ¼ → 1
		5	

505. 9709_w17_ms_13 Q: 10

	Answer	Mark	Partial Marks
(i)	$ax^2+bx=0 \to x(ax+b)=0 \to x=\frac{-b}{a}$	B1	
	Find $f''(x)$ and attempt sub *their* $\frac{-b}{a}$ into *their* $f''(x)$	M1	
	When $x=\frac{-b}{a}$, $f''(x)=2a\left(\frac{-b}{a}\right)+b=-b$ MAX	A1	
		3	
(ii)	Sub $f'(-2)=0$	M1	
	Sub $f'(1)=9$	M1	
	$a=3 \quad b=6$	*A1	Solve simultaneously to give both results.
	$f'(x)=3x^2+6x \to f(x)=x^3+3x^2\ (+c)$	*M1	Sub *their a, b* into $f'(x)$ and integrate 'correctly'. Allow $\frac{ax^3}{3}+\frac{bx^2}{2}(+c)$
	$-3=-8+12+c$	DM1	Sub $x=-2,\ y=-3$. Dependent on *c* present. Dependent also on *a*, *b* substituted.
	$f(x)=x^3+3x^2-7$	A1	
		6	

506. 9709_m16_ms_12 Q: 2

	Answer	Mark	Partial Marks
	$y=\frac{3x^3}{3}-\frac{2x^{-2}}{-2}\quad(+c)$	B1B1	
	$3=-1+1+c$	M1	Sub $x=-1, y=3$. c must be present
	$y=x^3+x^{-2}+3$	A1 [4]	Accept $c=3$ www

507. 9709_m16_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$x=1/3$	**B1** **[1]**	
(ii)	$\frac{dy}{dx}=\left[\frac{2}{16}(3x-1)\right][3]$	**B1B1**	
	When $x=3$ $\frac{dy}{dx}=3$ soi	**M1**	
	Equation of QR is $y-4=3(x-3)$	**M1**	
	When $y=0$ $x=5/3$	**A1** **[5]**	
(iii)	Area under curve $=\left[\frac{1}{16\times3}(3x-1)^3\right]\left[\times\frac{1}{3}\right]$	**B1B1**	
	$\frac{1}{16\times9}\left[8^3-0\right]=\frac{32}{9}$	**M1A1**	Apply limits: *their* $\frac{1}{3}$ and 3
	Area of $\Delta=8/3$	**B1**	
	Shaded area $=\frac{32}{9}-\frac{8}{3}=\frac{8}{9}$ (or 0.889)	**A1** **[6]**	

508. 9709_s16_ms_11 Q: 3

	Answer	Mark	Partial Marks
	$x=\frac{12}{y^2}-2.$ Vol = (π) × ∫ x^2 dy $\rightarrow\left[\frac{-144}{3y^3}+4y+\frac{48}{y}\right]$	**M1** **3 × A1**	Ignore omission of π at this stage Attempt at integration Un-simplified
	Limits 1 to 2 used $\rightarrow 22\pi$	**A1** [5]	only from correct integration

509. 9709_s16_ms_11 Q: 4

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx}=2-8(3x+4)^{-½}$ $(x=0,\rightarrow\ \frac{dy}{dx}=-2\ )$ $\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}\ \rightarrow -0.6$	**M1A1** [2]	Ignore notation. Must be $\frac{dy}{dx}$ × 0.3
(ii)	$y=\{2x\}\left\{-\frac{8\sqrt{3x+4}}{\frac{1}{2}}\div 3\right\}\ (+c)$	**B1 B1**	No need for $+c$.
	$x=0, y=\frac{4}{3}\ \rightarrow c=12.$	**M1 A1** [4]	Uses x, y values after ∫ with c

510. 9709_s16_ms_12 Q: 2

	Answer	Mark	Partial Marks
	$f'(x)=\frac{8}{(5-2x)^2}$		
	$f(x)=\frac{8(5-2x)^{-1}}{-1}\ \div -2\ (+c)$	**B1** **B1**	Correct without (÷ by −2) An attempt at integration (÷ by−2)
	Uses $x=2$, $y=7$,	**M1**	Substitution of correct values into an integral to find c
	$c=3$	**A1** [4]	

511. 9709_s16_ms_12 Q: 10

	Answer	Mark	Partial Marks
	$y=\frac{8}{x}+2x.$		
(i)	$\frac{dy}{dx}=-8x^{-2}+2$	B1	unsimplified ok
	$\frac{d^2y}{dx^2}=16x^{-3}$	B1	unsimplified ok
	$\int y^2 dx=-64x^{-1}$ oe+ $32x$ oe + $\frac{4x^3}{3}$ **oe** $(+c)$	3 × B1 [5]	B1 for each term – unsimplified ok
(ii)	sets $\frac{dy}{dx}$ to 0 $\rightarrow x=\pm 2$	M1	Sets to 0 and attempts to solve
	$\rightarrow M(2, 8)$	A1	Any pair of correct values A1
	Other turning point is $(-2, -8)$	A1	Second pair of values A1
	If $x=-2$, $\frac{d^2y}{dx^2}<0$	M1	Using their $\frac{d^2y}{dx^2}$ if kx^{-3} and $x<0$
	∴Maximum	A1 [5]	
(iii)	Vol = π × [part (i)] from 1 to 2	M1	Evidence of using limits 1&2 in their integral of y^2 (ignore π)
	$\frac{220\pi}{3}, 73.3\pi, 230$	A1 [2]	

512. 9709_s16_ms_13 Q: 2

	Answer	Mark	Partial Marks
	$(\pi)\int(x^3+1)\,dx$	M1	Attempt to resolve y^2 and attempt to integrate
	$(\pi)\left[\frac{x^4}{4}+x\right]$	A1	
	6π or 18.8	DM1A1 [4]	Applying limits 0 and 2. (Limits reversed: Allow M mark and allow A mark if final answer is 6π)

513. 9709_s16_ms_13 Q: 3

	Answer	Mark	Partial Marks
(i)	$6+k=2 \rightarrow k=-4$	B1 [1]	
(ii)	$(y)=\frac{6x^3}{3}-\frac{4}{-2}x^{-2}$ $(+c)$	B1B1√	ft on *their k*. Accept $+\frac{k}{-2}x^{-2}$
	$9=2+2+c$ *c* must be present	M1	Sub (1,9) with numerical *k*. Dep on attempt $\int$
	$(y)=2x^3+2x^{-2}+5$	A1 [4]	Equation needs to be seen Sub (2, 3) $\rightarrow c=-13\frac{1}{2}$ scores M1A0

514. 9709_w16_ms_11 Q: 7

	Answer	Mark		Partial Marks
(i)	$A = (½, 0)$	**B1**	[1]	Accept $x = 0$ *at* $y = 0$
(ii)	$\int (1-2x)^{\frac{1}{2}} \, dx \;=\; \left[\dfrac{(1-2x)^{3/2}}{3/2}\right]\left[\div(-2)\right]$ $\int (2x-1)^2 \, dx \;=\; \left[\dfrac{(2x-1)^3}{3}\right][\div 2]$ $[0-(-1/3)]-[0-(-1/6)]$ 1/6	**B1B1** **B1B1** **M1** **A1**	[6]	May be seen in a single expression May use $\int_a^1 x \, dy$, may expand $(2x-1)^2$ Correct use of *their* limits

515. 9709_w16_ms_11 Q: 10

	Answer	Mark		Partial Marks
(i)	$3z - \dfrac{2}{z} = -1 \;\Rightarrow\; 3z^2 + z - 2 = 0$ $x^{1/2}(\text{or } z) = 2/3 \text{ or } -1$ $x = 4/9$ only	**M1** **A1** **A1**	[3]	Express as 3-term quad. Accept $x^{1/2}$ for z (OR $3x - 1 = -\sqrt{x},\ 9x^2 - 13x + 4 = 0$ **M1, A1,A1** $x = 4/9$)
(ii)	$f(x) = \dfrac{3x^{3/2}}{3/2} - \dfrac{2x^{1/2}}{1/2} \quad (+c)$ Sub $x = 4, y = 10 \quad 10 = 16 - 8 + c \;\Rightarrow\; c = 2$ When $x = \dfrac{4}{9},\ y = 2\left(\dfrac{4}{9}\right)^{3/2} - 4\left(\dfrac{4}{9}\right)^{1/2} + 2$ $-2/27$	**B1B1** **M1A1** **M1** **A1**	[6]	 c must be present Substituting x value from part **(i)**

516. 9709_w16_ms_12 Q: 1

	Answer	Mark		Partial Marks
	$(y) = 8(4x+1)^{\frac{1}{2}} \;\div ½ \div 4\ (+c)$ Uses $x = 2$ and $y = 5$ $c = -7$	**B1** **B1** **M1** **A1**	[4]	Correct integrand (unsimplified) without ÷4 ÷4. Ignore c. Substitution of correct values into an integrand to find c. $y = 4\sqrt{4x+1} - 7$

517. 9709_w16_ms_13 Q: 10

	Answer	Mark		Partial Marks
(i)	at $x=a^2$, $\frac{dy}{dx}=\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}\left(=\frac{3}{a^2}\text{ or }3a^{-2}\right)$	**B1**		$\frac{2}{a^2}+\frac{1}{a^2}$ or $2a^{-2}+a^{-2}$ seen anywhere in **(i)**
	$y-3=\frac{3}{a^2}\left(x-a^2\right)$ or $y=\frac{3}{a^2}x+c\to 3=\frac{3}{a^2}a^2+c$	**M1**		Through $(a^2,3)$ & with *their* grad as f(a)
	$y=\frac{3}{a^2}x$ or $3a^{-2}x$ cao	**A1**	[3]	
(ii)	$(y)=\frac{2}{a}\frac{x^{½}}{½}+\frac{ax^{-½}}{-½}$ $(+c)$	**B1B1**		
	sub $x=a^2$, $y=3$ into $\int dy/dx$	**M1**		*c* must be present. Expect $3=4-2+c$
	$c=1$ $\left(y=\frac{4x^{½}}{a}-2ax^{-½}+1\right)$	**A1**	[4]	
(iii)	sub $x=16$, $y=8\to 8=\frac{4}{a}\times 4-2a\times\frac{1}{4}+1$	***M1**		Sub into *their y*
	$a^2+14a-32(=0)$	**A1**		
	$a=2$	**A1**		Allow −16 in addition
	$A=(4, 3)$, $B=(16, 8)$ $AB^2=12^2+5^2\to AB=13$	**DM1A1**	[5]	

518. 9709_w16_ms_13 Q: 11

	Answer	Mark		Partial Marks
(i)	Attempt diffn. and equate to 0 $\frac{dy}{dx}=-k(kx-3)^{-2}+k=0$	***M1**		Must contain $(kx-3)^{-2}$ + other term(s)
	$(kx-3)^2=1$ or $k^3x^2-6k^2x+8k(=0)$	**DM1**		Simplify to a quadratic
	$x=\frac{2}{k}$ or $\frac{4}{k}$	***A1*A1**		Legitimately obtained
	$\frac{d^2y}{dx^2}=2k^2(kx-3)^{-3}$	**B1**√		Ft must contain $Ak^2(kx-3)^{-3}$ where A>0
	When $x=\frac{2}{k}$, $\frac{d^2y}{dx^2}=\left(-2k^2\right)<0$ MAX All previous	**DB1**		Convincing alt. methods (values either side) must show which values used & cannot use $x=3/k$
	When $x=\frac{4}{k}$, $\frac{d^2y}{dx^2}=\left(2k^2\right)>0$ MIN working correct	**DB1**	[7]	
(ii)	$V=(\pi)\int\left[(x-3)^{-1}+(x-3)\right]^2dx$	***M1**		Attempt to expand y^2 and then integrate
	$=(\pi)\int[(x-3)^{-2}+(x-3)^2+2]dx$	**A1**		
	$=(\pi)\left[-(x-3)^{-1}+\frac{(x-3)^3}{3}(+2x)\right]$ Condone missing $2x$	**A1**		Or $\left[-(x-3)^{-1}+\frac{x^3}{3}-3x^2+9x+2x\right]$
	$=(\pi)\left[1-\frac{1}{3}+4-\left(\frac{1}{3}-9+0\right)\right]$	**DM1**		Apply limits 0→2
	$=40\pi/3$ oe or 41.9	**A1**	[5]	2 missing → $28\pi/3$ scores M1A0A1M1A0

519. 9709_s15_ms_11 Q: 10

	Answer	Mark	Partial Marks
(i)	$y = \dfrac{8}{\sqrt{3x+4}}$		
	$\dfrac{dy}{dx} = \dfrac{-4}{(3x+4)^{\frac{3}{2}}} \times 3$ aef	B1 B1	Without the "×3" For "×3" even if 1st B mark lost.
	$\rightarrow m_{(x=0)} = -\dfrac{3}{2}$ Perpendicular $m_{(x=0)} = \dfrac{2}{3}$	M1	Use of $m_1 m_2 = -1$ after attempting to find $\dfrac{dy}{dx}_{(x=0)}$
	Eqn of normal $y - 4 = \dfrac{2}{3}(x - 0)$	M1	Unsimplified line equation
	Meets $x = 4$ at $B\left(4, \dfrac{20}{3}\right)$	A1 [5]	cao
(ii)	$\displaystyle\int \dfrac{8}{\sqrt{(3x+4)}}\,dx = \dfrac{8\sqrt{(3x+4))}}{\frac{1}{2}} \div 3$	B1 B1	Without "÷3". For "÷3"
	Limits from 0 to 4 $\rightarrow$ Area $P = \dfrac{32}{3}$	M1 A1	Correct use of correct limits. cao
	Area Q = Trapezium – P Area of Trapezium = $\dfrac{1}{2}\left(4 + \dfrac{20}{3}\right) \times 4 = \dfrac{64}{3}$	M1	Correct method for area of trapezium
	$\rightarrow$ Areas of P and Q are both $\dfrac{32}{3}$	A1 [6]	All correct.

520. 9709_s15_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$y = \frac{4}{2x-1}$.	B1	Correct without the ÷2
	$\int \frac{16}{(2x-1)^2} dx = \frac{-16}{2x-1} \div 2$	B1	For the ÷2 even if first B1 is lost
	Vol = $\pi \left[\frac{-8}{2x-1} \right]$ with limits 1 and 2	M1	Use of limits in a changed expression.
	$\rightarrow \frac{16\pi}{3}$	A1 [4]	co
(ii)	$m = \frac{1}{2} m$ of tangent $= -2$	M1	Use of $m_1m_2 = -1$
	$\frac{dy}{dx} = \frac{-4}{(2x-1)^2} \times 2$	B1	Correct without the ×2
		B1	For the ×2 even if first B1 is lost
	Equating their $\frac{dy}{dx}$ to -2	DM1	
	$\rightarrow x = \frac{3}{2}$ or $-\frac{1}{2}$	A1	co
	($y = 2$ or -2)		
	$\rightarrow c = \frac{5}{2}$ or $-\frac{7}{2}$	A1 [6]	co

521. 9709_s15_ms_13 Q: 2

	Answer	Mark	Partial Marks
	$\left[\frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}} \right]$ [÷2] (+c)	**B1B1**	
	$7 = 9 + c$	**M1**	Attempt subst $x = 4$, $y = 7$. c must be there. Dep. on attempt at integration.
	$y = \frac{(2x+1)^{\frac{3}{2}}}{3} - 2$ or unsimplified	**A1** **[4]**	$c = -2$ sufficient

522. 9709_s15_ms_13 Q: 10

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = 6 - 6x$ At $x = 2$, gradient $= -6$ soi $y - 9 = -6(x - 2)$ oe Expect $y = -6x + 21$ When $y = 0$, $x = 3\frac{1}{2}$ cao	**B1** **B1**√ **M1** **A1** **[4]**	Line through (2, 9) and with gradient *their* −6
(ii)	Area under curve: $\int 9 + 6x - 3x^2 dx = 9x + 3x^2 - x^3$ $(27 + 27 - 27) - (18 + 12 - 8)$ Area under tangent: $\frac{1}{2} \times \frac{3}{2} \times 9 \; (= \frac{27}{4})$ Area required $\frac{27}{4} - 5 = \frac{7}{4}$	**B2,1,0** **M1** **B1**√ **A1** **[5]**	Allow unsimplified terms Apply limits 2,3. Expect 5 OR $\int_2^{7/2} (-6x + 21)\,dx \; (\to \frac{27}{4})$. Ft on *their* $-6x + 21$ and/or *their* 7/2.

523. 9709_w15_ms_11 Q: 2

	Answer	Mark	Partial Marks
	$f(x) = x^3 - 7x \; (+c)$ $5 = 27 - 21 + c$ $c = -1 \to f(x) = x^3 - 7x - 1$	**B1** **M1** **A1** [3]	Sub $x = 3$, $y = 5$. Dep. on c present

524. 9709_w15_ms_11 Q: 11

	Answer	Mark	Partial Marks
(i)	$\frac{dy}{dx} = \left[\frac{1}{2}(1+4x)^{-1/2}\right] \times [4]$	**B1B1**	
	At $x = 6, \frac{dy}{dx} = \frac{2}{5}$	**B1**	
	Gradient of normal at $P = -\frac{1}{2}$	**B1**√	**OR** eqn of norm $y - 5 = their - \frac{5}{2}(x-6)$
	Gradient of $PQ = -\frac{5}{2}$ hence PQ is a normal, or $m_1 m_2 = -1$	**B1** [5]	When $y = 0, x = 8$ hence result
(ii)	Vol for curve $= (\pi)\int(1+4x)$ and attempt to integrate y^2	**M1**	
	$= (\pi)\left[x + 2x^2\right]$ ignore '+ c'	**A1**	
	$= (\pi)[6 + 72 - 0]$	**DM1**	Apply limits $0 \to 6$ (allow reversed if corrected later)
	$= 78(\pi)$	**A1**	
	Vol for line $= \frac{1}{3} \times (\pi) \times 5^2 \times 2$	**M1**	**OR** $(\pi)\left[\frac{\left(-\frac{5}{2}x + 20\right)^3}{3 \times -\frac{5}{2}}\right]_6^8$
	$= \frac{50}{3}(\pi)$	**A1**	
	Total Vol $= 78\pi + 50\pi/3 = 94\frac{2}{3}\pi$ (or $284\pi/3$)	**A1** [7]	

525. 9709_w15_ms_12 Q: 10

	Answer	Mark	Partial Marks
(i)	$y = \sqrt{(9 - 2x^2)}$ $P\,(2, 1)$		
	$\frac{dy}{dx} = \frac{1}{2\sqrt{(9-2x^2)}} \times -4x$	**B1** **B1**	Without " $\times -4x$" Allow even if B0 above.
	At P, $x = 2$, $m = -4$ Normal grad = ¼	**M1**	For $m_1 m_2 = -1$ calculus needed
	Eqn AP $y - 1 = \frac{1}{4}(x - 2)$	**M1**	Normal, not tangent
	$\rightarrow A\,(-2, 0)$ or $B\,(0, \frac{1}{2})$	**A1**	
	Midpoint AP also $(0, \frac{1}{2})$	**A1** [6]	Full justification.
(ii)	$\int x^2 dy = \int\left(\frac{9}{2} - \frac{y^2}{2}\right)dy$	**M1**	Attempt to integrate x^2
	$= \frac{9y}{2} - \frac{y^3}{6}$	**A1**	Correct integration
	Upper limit = 3	**B1**	Evaluates upper limit
	Uses limits 1 to 3	**DM1**	Uses both limits correctly
	$\rightarrow$ volume $= 4\frac{2}{3}\pi$	**A1** [5]	

526. 9709_w15_ms_13 Q: 9

	Answer	Mark	Partial Marks
(i)	At $x=4,\ \frac{dy}{dx}=2$	**B1**	
	$\frac{dy}{dt}=\frac{dy}{dx}\times\frac{dx}{dt}=2\times3=6$	**M1A1** [3]	Use of Chain rule
(ii)	$(y)=x+4x^{\frac{1}{2}}(+c)$	**B1**	
	Sub $x=4,\ y=6\rightarrow6=4+(4\times4^{\frac{1}{2}})+c$	**M1**	Must include *c*
	$c=-6\rightarrow(y=x+4x^{\frac{1}{2}}-6$	**A1** [3]	
(iii)	Eqn of tangent is $y-6=2(x-4)$ or $(6-0)/(4-x)=2$	**M1** **A1**	Correct eqn thru (4, 6) & with $m=$ *their* 2
	$B=(1,0)$ (Allow $x=1$) Gradient of normal $=-1/2$ $C=(16,0)$ (Allow $x=16$)	**M1** **A1** **A1**	[Expect eqn of normal: $y=-½x+8$]
	Area of triangle $=\frac{1}{2}\times15\times6=45$	[5]	Or $AB=\sqrt{45}$, $AC=\sqrt{180}\rightarrow$ Area = 45.0

527. 9709_w15_ms_13 Q: 10

	Answer	Mark	Partial Marks
(i)	$f'(x)=2-2(x+1)^{-3}$	**B1**	
	$f''(x)=6(x+1)^{-4}$	**B1**	
	$f'0=0$ hence stationary at $x=0$	**B1**	**AG**
	$f''0=6>0$ hence minimum	**B1** [4]	www. Dependent on correct f ″(*x*) except $-6(x+1)^{-4}\rightarrow<0$ MAX scores SC1
(ii)	$AB^2=(3/2)^2+(3/4)^2$	**M1**	
	$AB=1.68$ or $\sqrt{45/4}$ oe	**A1** [2]	
(iii)	Area under curve = $\int f(x)=x^2-(x+1)^{-1}$	**B1**	Ignore +*c* even if evaluated Do not penalise reversed limits
	$=\left(1-\frac{1}{2}\right)-\left(\frac{1}{4}-2\right)=9/4$ (Apply limits $-½\rightarrow1$)	**M1A1**	Allow reversed subtn if final ans positive
	Area trap. $=\frac{1}{2}(3+\frac{9}{4})\times\frac{3}{2}$	**M1**	
	$=63/16$ or 3.94	**A1**	
	Shaded area $63/16-9/4+27/16$ or 1.69	**A1** [6]	
	ALT eqn *AB* is $y=-½x+11/4$	**B1**	
	Area = $\int-½x+11/4-\int2x+(x+1)^{-2}$	**M1**	Attempt integration of at least one
	$=\left[-\frac{1}{4}x^2+\frac{11}{4}x\right]-\left[x^2-(x+1)^{-1}\right]$	**A1A1**	Ignore +*c* even if evaluated Dep. on integration having taken place
	Apply limits $-½\rightarrow1$ to both integrals	**M1**	Allow reversed subtn if final ans positive
	27/16 or 1.69	**A1**	