Topical Past Paper Questions Workbook

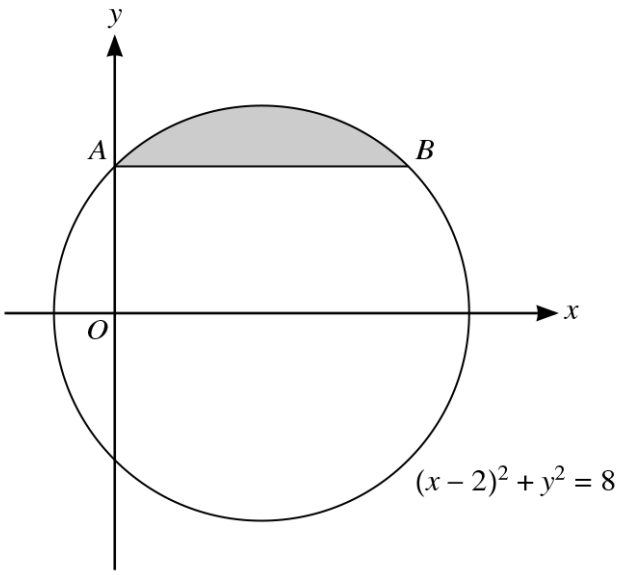
AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

Chapter 4

Circular measure

129. 9709_m22_qp_12 Q: 8

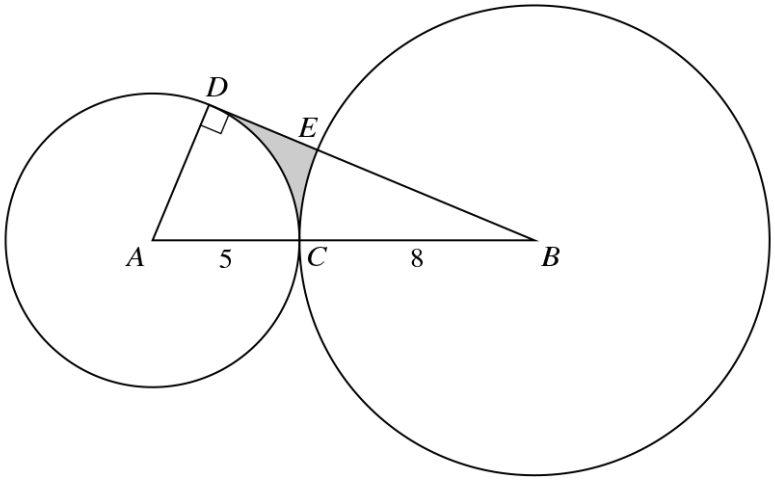


The diagram shows the circle with equation $(x-2)^2+y^2=8$. The chord AB of the circle intersects the positive y-axis at A and is parallel to the x-axis.

(a) Find, by calculation, the coordinates of A and B. [3]

(b) Find the volume of revolution when the shaded segment, bounded by the circle and the chord AB, is rotated through 360° about the x-axis. [5]

130. 9709_m22_qp_12 Q: 10



The diagram shows a circle with centre A of radius 5 cm and a circle with centre B of radius 8 cm. The circles touch at the point C so that ACB is a straight line. The tangent at the point D on the smaller circle intersects the larger circle at E and passes through B.

(a) Find the perimeter of the shaded region. [5]

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(b) Find the area of the shaded region. [3]

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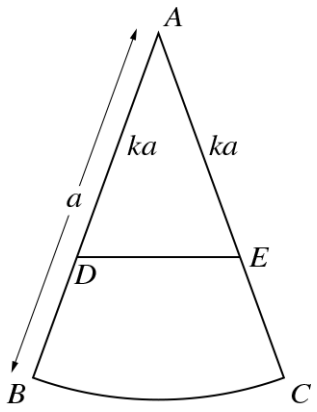
131. 9709_m21_qp_12 Q: 10



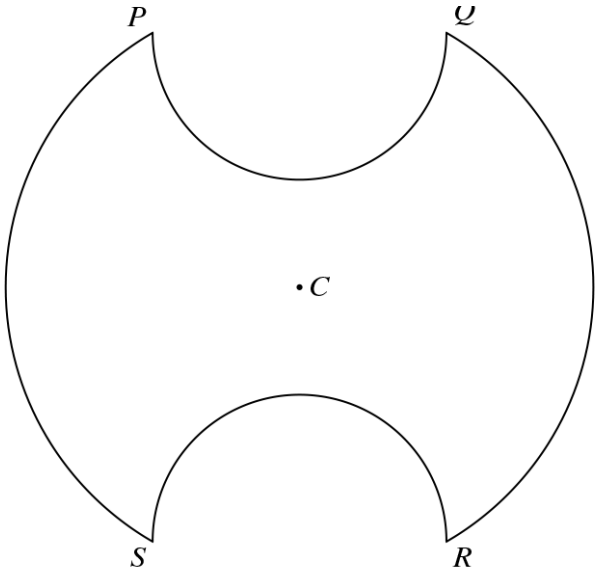
The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\dfrac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

132. 9709_s21_qp_11 Q: 8



The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

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(b) Find the exact perimeter of the plate. [3]

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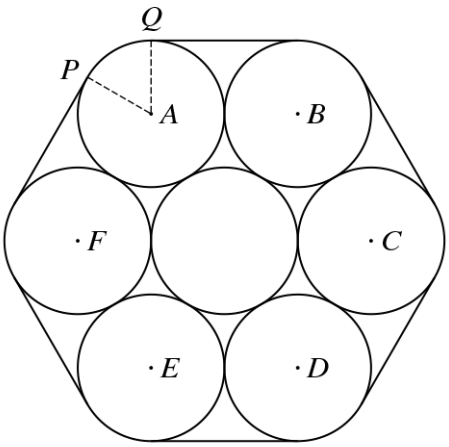
(c) Show that the area of the plate is $\left(\frac{20}{3}\pi + 8\sqrt{3}\right)$ cm^2. [5]

133. 9709_s21_qp_12 Q: 12



The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

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(b) Find the length of the rope. [4]

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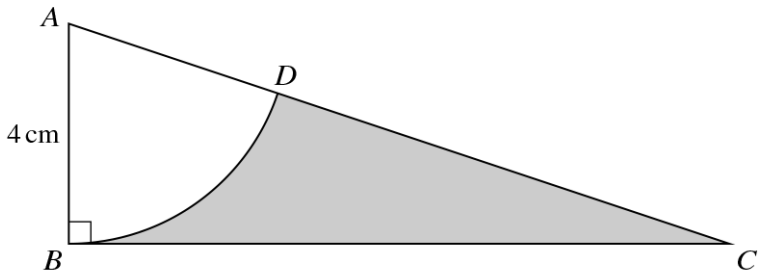
(c) Find the area of the hexagon *ABCDEF*, giving your answer in terms of $\sqrt{3}$. [2]

(d) Find the area of the complete region enclosed by the rope. [3]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

134. 9709_s21_qp_13 Q: 5



The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

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(b) Find the perimeter of the shaded region. [4]

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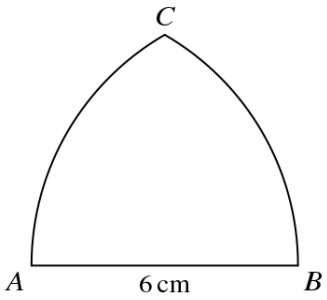
135. 9709_w21_qp_11 Q: 6

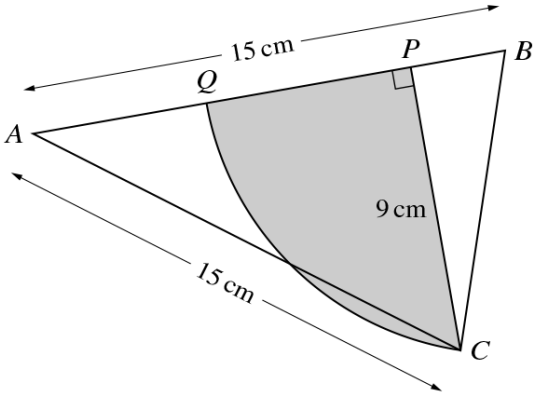


The diagram shows a metal plate ABC in which the sides are the straight line AB and the arcs AC and BC. The line AB has length 6 cm. The arc AC is part of a circle with centre B and radius 6 cm, and the arc BC is part of a circle with centre A and radius 6 cm.

(a) Find the perimeter of the plate, giving your answer in terms of π. [3]

(b) Find the area of the plate, giving your answer in terms of π and $\sqrt{3}$. [4]

136. 9709_w21_qp_12 Q: 7

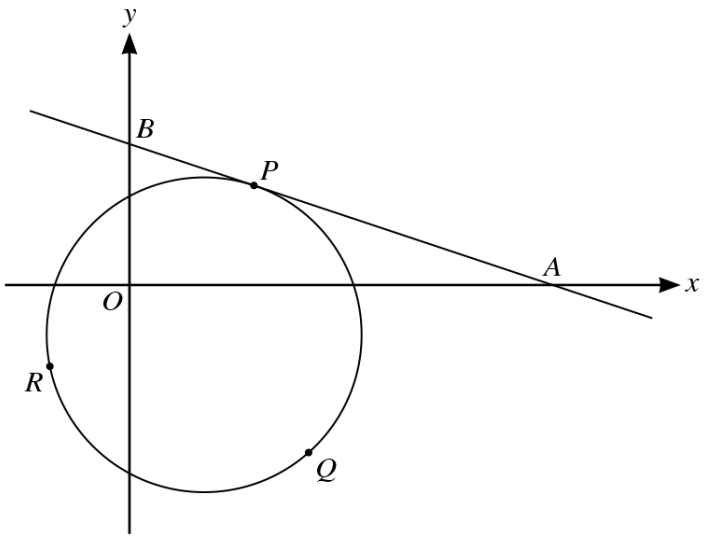


In the diagram the lengths of AB and AC are both 15 cm. The point P is the foot of the perpendicular from C to AB. The length $CP = 9$ cm. An arc of a circle with centre B passes through C and meets AB at Q.

(a) Show that angle $ABC = 1.25$ radians, correct to 3 significant figures. [2]

(b) Calculate the area of the shaded region which is bounded by the arc CQ and the lines CP and PQ. [4]

137. 9709_w21_qp_12 Q: 12



The diagram shows the circle with equation $x^2 + y^2 - 6x + 4y - 27 = 0$ and the tangent to the circle at the point $P(5, 4)$.

(a) The tangent to the circle at P meets the x-axis at A and the y-axis at B.

Find the area of triangle OAB, where O is the origin. [5]

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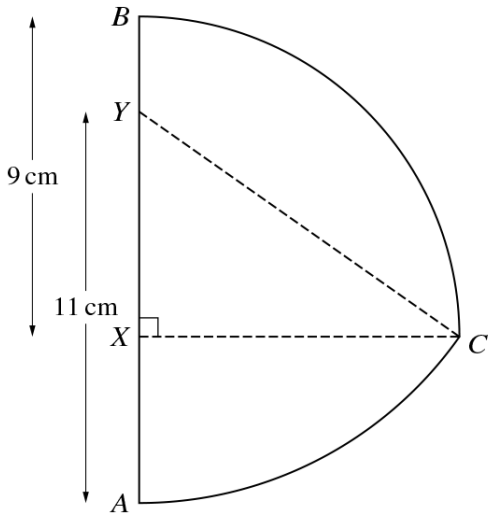
(b) Points Q and R also lie on the circle, such that PQR is an equilateral triangle.

Find the exact area of triangle PQR. [3]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

138. 9709_w21_qp_13 Q: 5

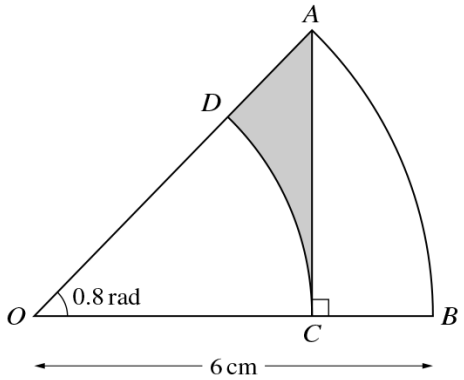


In the diagram, X and Y are points on the line AB such that $BX = 9$ cm and $AY = 11$ cm. Arc BC is part of a circle with centre X and radius 9 cm, where CX is perpendicular to AB. Arc AC is part of a circle with centre Y and radius 11 cm.

(a) Show that angle $XYC = 0.9582$ radians, correct to 4 significant figures. [1]

(b) Find the perimeter of ABC. [6]

139. 9709_m20_qp_12 Q: 7



The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

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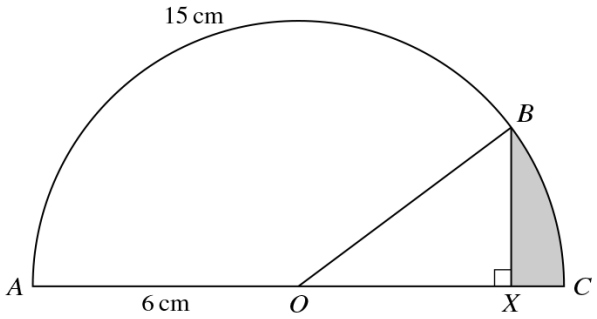
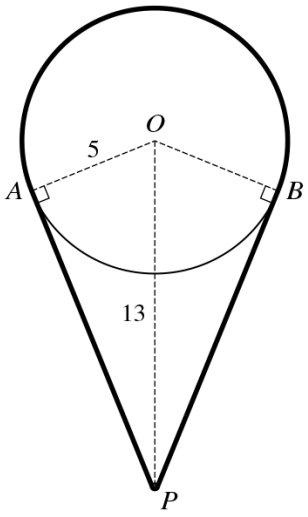
140. 9709_s20_qp_11 Q: 8



In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

141. 9709_s20_qp_13 Q: 5



The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

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142. 9709_s20_qp_13 Q: 10

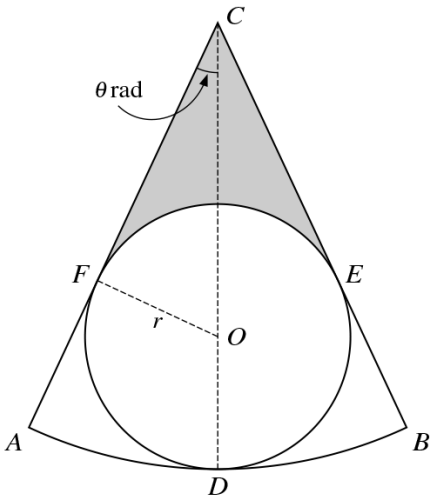
(a) The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

143. 9709_w20_qp_11 Q: 10



The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

(a) Find CD in terms of r and $\sin\theta$. [3]

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It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector CAB in terms of π. [3]

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(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

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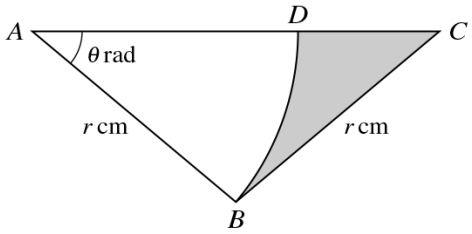
144. 9709_w20_qp_12 Q: 8

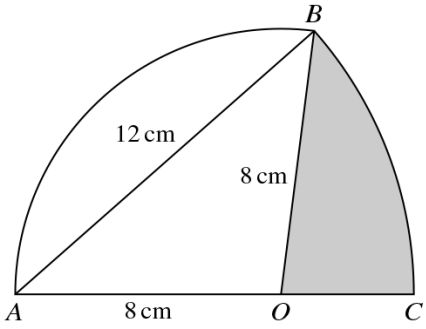


In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

145. 9709_w20_qp_13 Q: 9



In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

(b) Find the area of the shaded region. [4]

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(c) Find the perimeter of the shaded region. [3]

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146. 9709_w20_qp_13 Q: 11

A circle with centre C has equation $(x-8)^2+(y-4)^2=100$.

(a) Show that the point $T(-6, 6)$ is outside the circle. [3]

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly $45°$. [2]

The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

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(d) Find the x-coordinates of A and B. [3]

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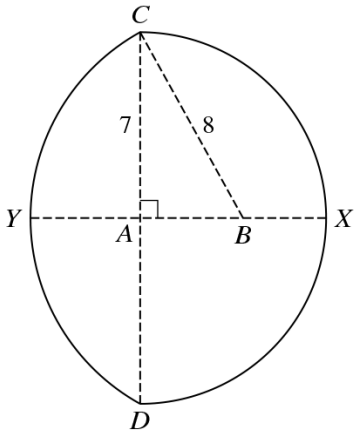
147. 9709_m19_qp_12 Q: 3

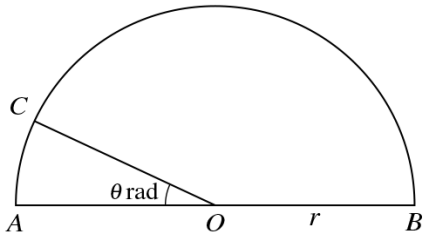


In the diagram, CXD is a semicircle of radius 7 cm with centre A and diameter CD. The straight line $YABX$ is perpendicular to CD, and the arc CYD is part of a circle with centre B and radius 8 cm. Find the total area of the region enclosed by the two arcs. [6]

148. 9709_s19_qp_11 Q: 3

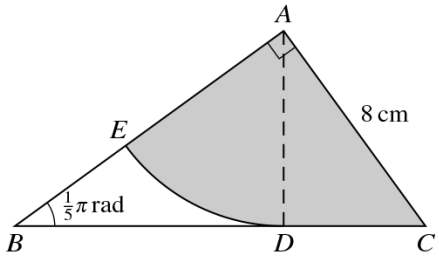
A sector of a circle of radius r cm has an area of A cm^2. Express the perimeter of the sector in terms of r and A. [4]

149. 9709_s19_qp_12 Q: 5



The diagram shows a semicircle with diameter AB, centre O and radius r. The point C lies on the circumference and angle $AOC = \theta$ radians. The perimeter of sector BOC is twice the perimeter of sector AOC. Find the value of θ correct to 2 significant figures. [5]

150. 9709_s19_qp_13 Q: 3



The diagram shows triangle ABC which is right-angled at A. Angle $ABC = \frac{1}{5}\pi$ radians and $AC = 8$ cm. The points D and E lie on BC and BA respectively. The sector ADE is part of a circle with centre A and is such that BDC is the tangent to the arc DE at D.

(i) Find the length of AD. [3]

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(ii) Find the area of the shaded region. [3]

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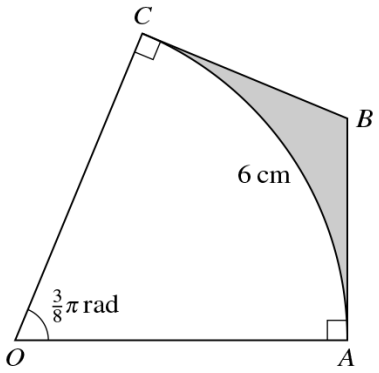
151. 9709_w19_qp_11 Q: 8



The diagram shows a sector OAC of a circle with centre O. Tangents AB and CB to the circle meet at B. The arc AC is of length 6 cm and angle $AOC = \frac{3}{8}\pi$ radians.

(i) Find the length of OA correct to 4 significant figures. [2]

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(ii) Find the perimeter of the shaded region. [2]

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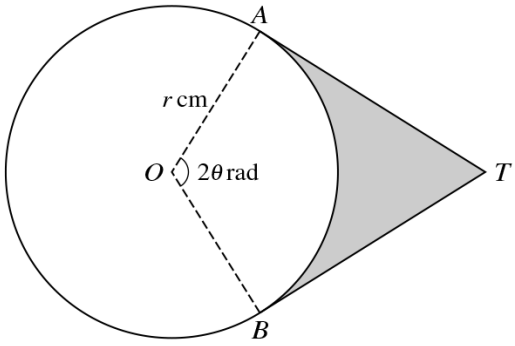
(iii) Find the area of the shaded region. [4]

152. 9709_w19_qp_12 Q: 4

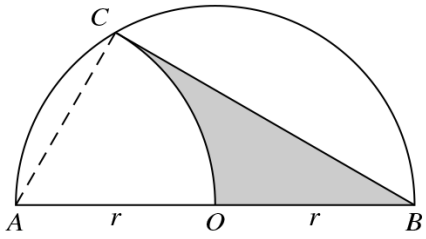


The diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and angle $AOB = 2\theta$ radians. The tangents to the circle at A and B meet at T.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 5$ and $\theta = 1.2$, find the area of the shaded region. [4]

153. 9709_w19_qp_13 Q: 4



The diagram shows a semicircle ACB with centre O and radius r. Arc OC is part of a circle with centre A.

(i) Express angle CAO in radians in terms of π. [1]

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(ii) Find the area of the shaded region in terms of r, π and $\sqrt{3}$, simplifying your answer. [4]

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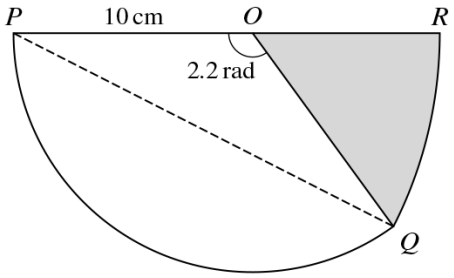
154. 9709_m18_qp_12 Q: 6

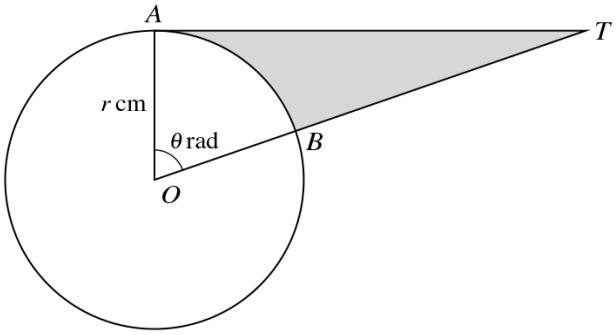


The diagram shows a sector POQ of a circle of radius 10 cm and centre O. Angle POQ is 2.2 radians. QR is an arc of a circle with centre P and POR is a straight line.

(i) Show that the length of PQ is 17.8 cm, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [4]

155. 9709_s18_qp_11 Q: 6

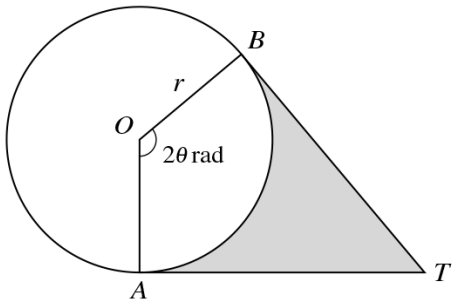


The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle. Angle $AOB = \theta$ radians and OBT is a straight line.

(i) Express the area of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region. [4]

156. 9709_s18_qp_12 Q: 6



The diagram shows points A and B on a circle with centre O and radius r. The tangents to the circle at A and B meet at T. The shaded region is bounded by the minor arc AB and the lines AT and BT. Angle AOB is 2θ radians.

(i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan\theta = 2\theta$. [3]

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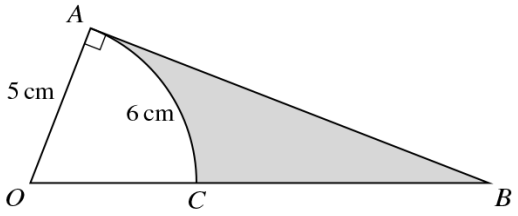
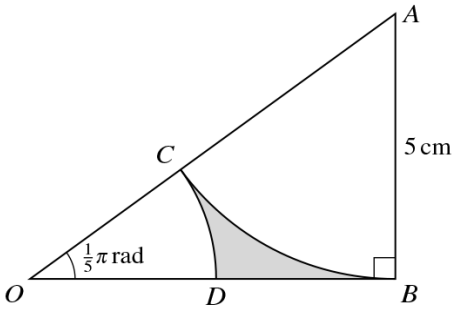
(ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region. [3]

157. 9709_s18_qp_13 Q: 5



The diagram shows a triangle OAB in which angle $OAB = 90°$ and $OA = 5$ cm. The arc AC is part of a circle with centre O. The arc has length 6 cm and it meets OB at C. Find the area of the shaded region. [5]

158. 9709_w18_qp_11 Q: 9



The diagram shows a triangle OAB in which angle ABO is a right angle, angle $AOB = \frac{1}{5}\pi$ radians and $AB = 5$ cm. The arc BC is part of a circle with centre A and meets OA at C. The arc CD is part of a circle with centre O and meets OB at D. Find the area of the shaded region. [8]

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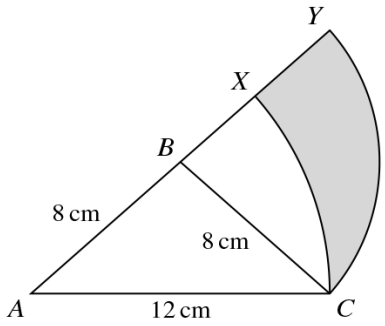
159. 9709_w18_qp_12 Q: 8



The diagram shows an isosceles triangle ACB in which $AB = BC = 8$ cm and $AC = 12$ cm. The arc XC is part of a circle with centre A and radius 12 cm, and the arc YC is part of a circle with centre B and radius 8 cm. The points A, B, X and Y lie on a straight line.

(i) Show that angle $CBY = 1.445$ radians, correct to 4 significant figures. [3]

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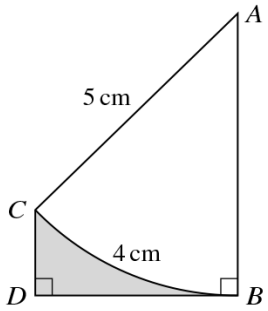
(ii) Find the perimeter of the shaded region. [4]

160. 9709_w18_qp_13 Q: 3



The diagram shows an arc BC of a circle with centre A and radius 5 cm. The length of the arc BC is 4 cm. The point D is such that the line BD is perpendicular to BA and DC is parallel to BA.

(i) Find angle BAC in radians. [1]

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(ii) Find the area of the shaded region BDC. [5]

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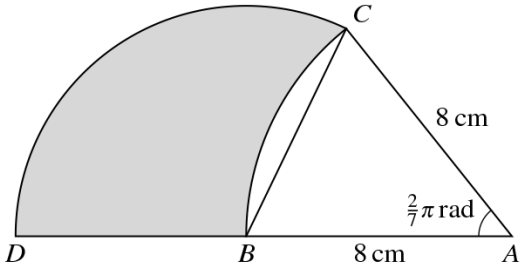
161. 9709_m17_qp_12 Q: 4

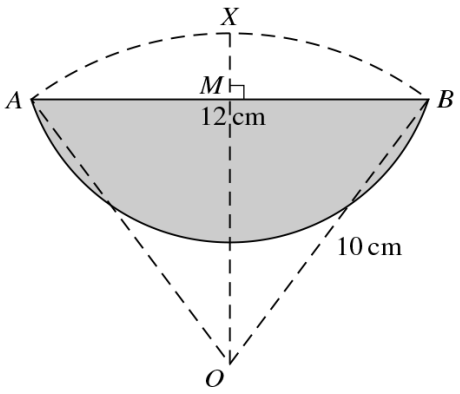


In the diagram, $AB = AC = 8$ cm and angle $CAB = \frac{2}{7}\pi$ radians. The circular arc BC has centre A, the circular arc CD has centre B and ABD is a straight line.

(i) Show that angle $CBD = \frac{9}{14}\pi$ radians. [1]

(ii) Find the perimeter of the shaded region. [5]

162. 9709_s17_qp_11 Q: 8



In the diagram, $OAXB$ is a sector of a circle with centre O and radius 10 cm. The length of the chord AB is 12 cm. The line OX passes through M, the mid-point of AB, and OX is perpendicular to AB. The shaded region is bounded by the chord AB and by the arc of a circle with centre X and radius XA.

(i) Show that angle AXB is 2.498 radians, correct to 3 decimal places. [3]

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(ii) Find the perimeter of the shaded region. [3]

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(iii) Find the area of the shaded region. [3]

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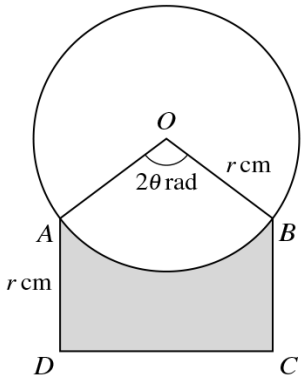
163. 9709_s17_qp_12 Q: 4



The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

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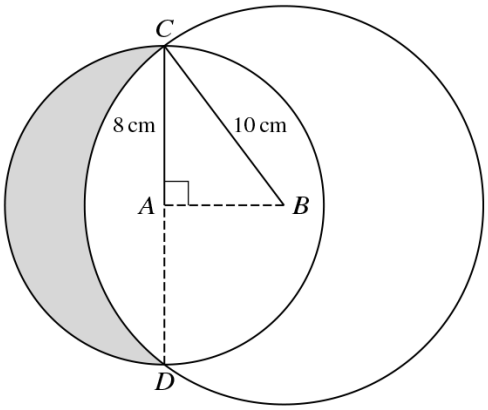
(ii) In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. [4]

164. 9709_s17_qp_13 Q: 7



The diagram shows two circles with centres A and B having radii 8 cm and 10 cm respectively. The two circles intersect at C and D where CAD is a straight line and AB is perpendicular to CD.

(i) Find angle ABC in radians. [1]

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(ii) Find the area of the shaded region. [6]

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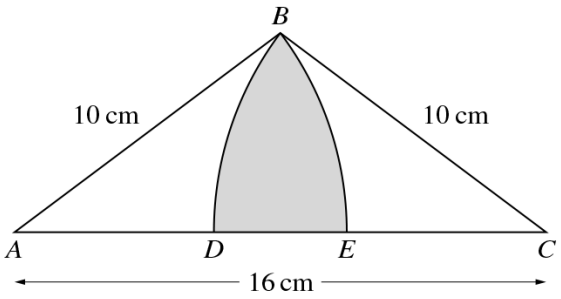
165. 9709_w17_qp_11 Q: 5



The diagram shows an isosceles triangle ABC in which $AC = 16$ cm and $AB = BC = 10$ cm. The circular arcs BE and BD have centres at A and C respectively, where D and E lie on AC.

(i) Show that angle $BAC = 0.6435$ radians, correct to 4 decimal places. [1]

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(ii) Find the area of the shaded region. [5]

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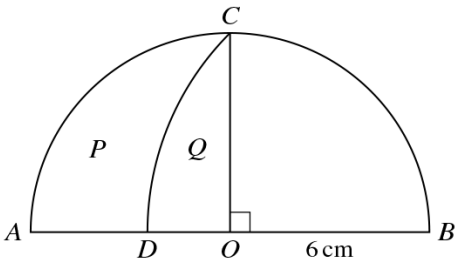
166. 9709_w17_qp_12 Q: 4



The diagram shows a semicircle with centre O and radius 6 cm. The radius OC is perpendicular to the diameter AB. The point D lies on AB, and DC is an arc of a circle with centre B.

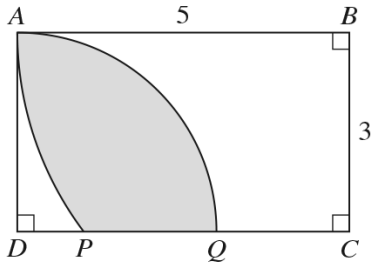
(i) Calculate the length of the arc DC. [3]

(ii) Find the value of

$$\frac{\text{area of region } P}{\text{area of region } Q},$$

giving your answer correct to 3 significant figures. [4]

167. 9709_w17_qp_13 Q: 7



The diagram shows a rectangle $ABCD$ in which $AB = 5$ units and $BC = 3$ units. Point P lies on DC and AP is an arc of a circle with centre B. Point Q lies on DC and AQ is an arc of a circle with centre D.

(i) Show that angle $ABP = 0.6435$ radians, correct to 4 decimal places. [1]

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(ii) Calculate the areas of the sectors BAP and DAQ. [3]

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(iii) Calculate the area of the shaded region. [3]

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168. 9709_m16_qp_12 Q: 9

(a)

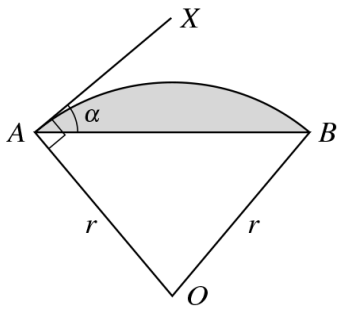


Fig. 1

In Fig. 1, OAB is a sector of a circle with centre O and radius r. AX is the tangent at A to the arc AB and angle $BAX = \alpha$.

(i) Show that angle $AOB = 2\alpha$. [2]

(ii) Find the area of the shaded segment in terms of r and α. [2]

(b)

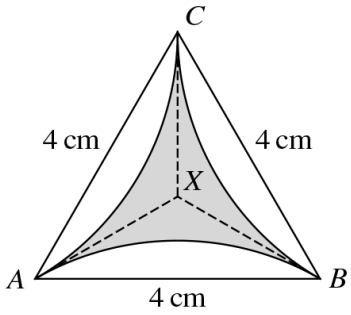
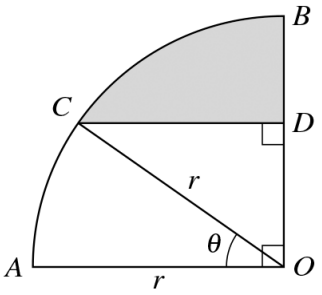


Fig. 2

In Fig. 2, ABC is an equilateral triangle of side 4 cm. The lines AX, BX and CX are tangents to the equal circular arcs AB, BC and CA. Use the results in part **(a)** to find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [6]

169. 9709_s16_qp_11 Q: 7

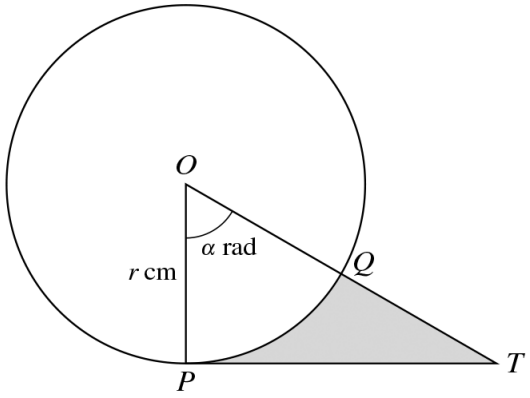


In the diagram, AOB is a quarter circle with centre O and radius r. The point C lies on the arc AB and the point D lies on OB. The line CD is parallel to AO and angle $AOC = \theta$ radians.

(i) Express the perimeter of the shaded region in terms of r, θ and π. [4]

(ii) For the case where $r = 5$ cm and $\theta = 0.6$, find the area of the shaded region. [3]

170. 9709_s16_qp_12 Q: 6

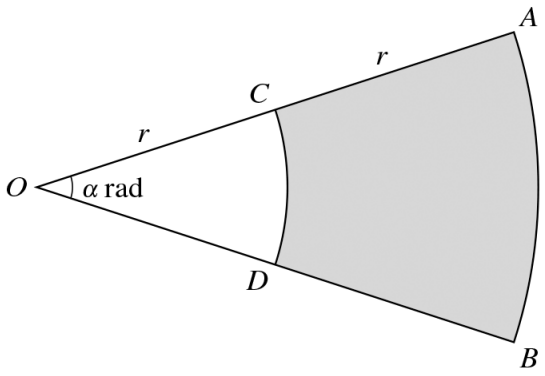


The diagram shows a circle with radius r cm and centre O. The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q.

(i) Express the perimeter of the shaded region PQT in terms of r and α. [3]

(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. [3]

171. 9709_w16_qp_11 Q: 3

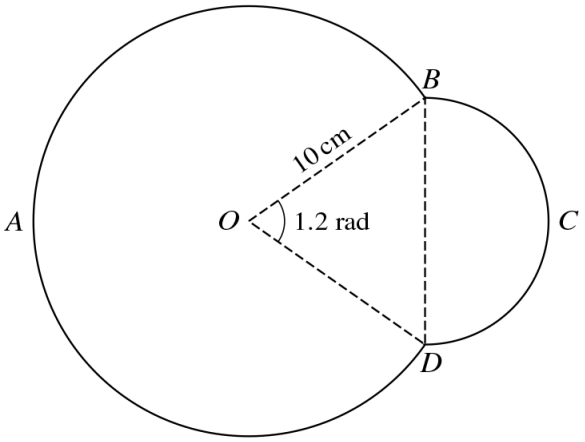


In the diagram OCA and ODB are radii of a circle with centre O and radius $2r$ cm. Angle $AOB = \alpha$ radians. CD and AB are arcs of circles with centre O and radii r cm and $2r$ cm respectively. The perimeter of the shaded region $ABDC$ is $4.4r$ cm.

(i) Find the value of α. [2]

(ii) It is given that the area of the shaded region is $30\,\text{cm}^2$. Find the value of r. [3]

172. 9709_w16_qp_12 Q: 6



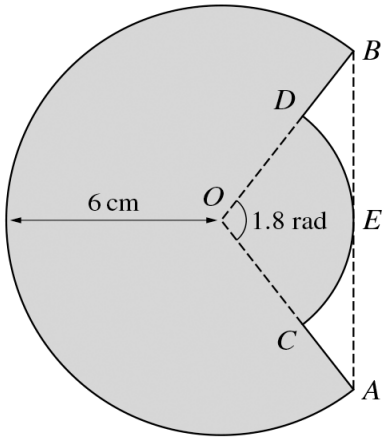
The diagram shows a metal plate $ABCD$ made from two parts. The part BCD is a semicircle. The part DAB is a segment of a circle with centre O and radius 10 cm. Angle BOD is 1.2 radians.

(i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

173. 9709_w16_qp_13 Q: 5



The diagram shows a major arc AB of a circle with centre O and radius 6 cm. Points C and D on OA and OB respectively are such that the line AB is a tangent at E to the arc CED of a smaller circle also with centre O. Angle COD = 1.8 radians.

(i) Show that the radius of the arc CED is 3.73 cm, correct to 3 significant figures. [2]

(ii) Find the area of the shaded region. [4]

174. 9709_s15_qp_11 Q: 5

A piece of wire of length 24 cm is bent to form the perimeter of a sector of a circle of radius r cm.

(i) Show that the area of the sector, A cm^2, is given by $A = 12r - r^2$. [3]

(ii) Express A in the form $a - (r - b)^2$, where a and b are constants. [2]

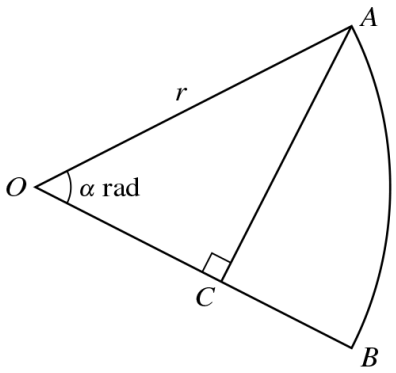
(iii) Given that r can vary, state the greatest value of A and find the corresponding angle of the sector. [2]

175. 9709_s15_qp_12 Q: 5

(i) Prove the identity $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} \equiv \dfrac{\tan\theta - 1}{\tan\theta + 1}$. [1]

(ii) Hence solve the equation $\dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \dfrac{\tan\theta}{6}$, for $0° \leqslant \theta \leqslant 180°$. [4]

176. 9709_s15_qp_13 Q: 11



In the diagram, OAB is a sector of a circle with centre O and radius r. The point C on OB is such that angle ACO is a right angle. Angle AOB is α radians and is such that AC divides the sector into two regions of equal area.

(i) Show that $\sin\alpha\cos\alpha = \frac{1}{2}\alpha$. [4]

It is given that the solution of the equation in part **(i)** is $\alpha = 0.9477$, correct to 4 decimal places.

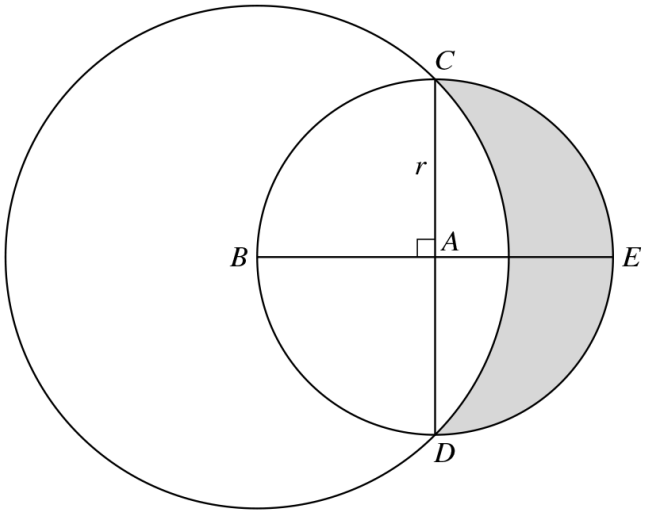
(ii) Find the ratio

perimeter of region OAC : perimeter of region ACB,

giving your answer in the form k : 1, where k is given correct to 1 decimal place. [5]

(iii) Find angle AOB in degrees. [1]

177. 9709_w15_qp_11 Q: 7

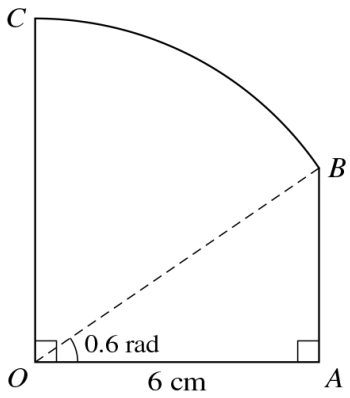


The diagram shows a circle with centre A and radius r. Diameters CAD and BAE are perpendicular to each other. A larger circle has centre B and passes through C and D.

(i) Show that the radius of the larger circle is $r\sqrt{2}$. [1]

(ii) Find the area of the shaded region in terms of r. [6]

178. 9709_w15_qp_12 Q: 5



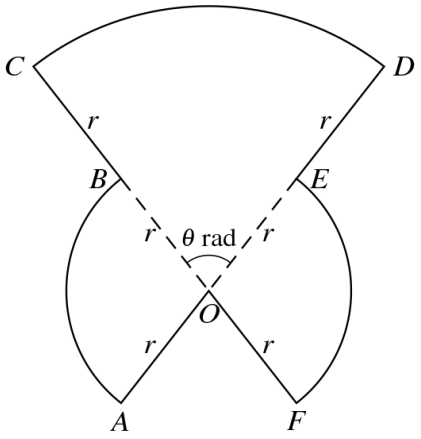
The diagram shows a metal plate $OABC$, consisting of a right-angled triangle OAB and a sector OBC of a circle with centre O. Angle $AOB = 0.6$ radians, $OA = 6$ cm and OA is perpendicular to OC.

(i) Show that the length of OB is 7.270 cm, correct to 3 decimal places. [1]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

179. 9709_w15_qp_13 Q: 4



The diagram shows a metal plate $OABCDEF$ consisting of 3 sectors, each with centre O. The radius of sector COD is $2r$ and angle COD is θ radians. The radius of each of the sectors BOA and FOE is r, and $AOED$ and $CBOF$ are straight lines.

(i) Show that the area of the metal plate is $r^2(\pi + \theta)$. [3]

(ii) Show that the perimeter of the metal plate is independent of θ. [4]