Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

Chapter 3

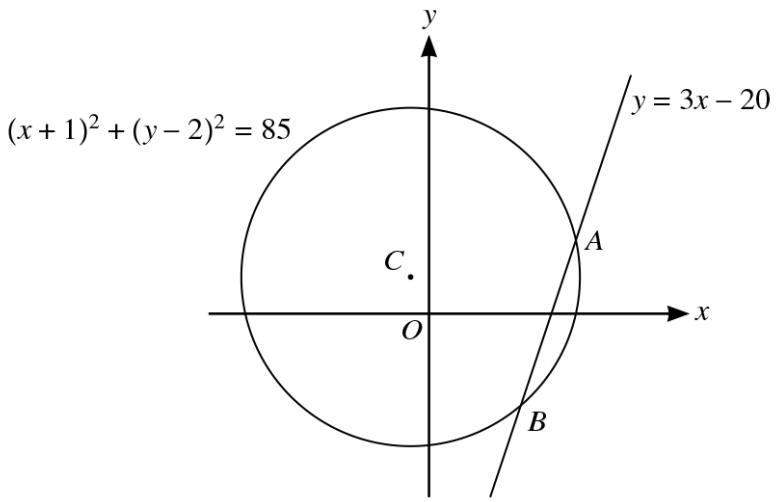
Coordinate geometry

66. 9709_m22_qp_12 Q: 2

A curve has equation $y = x^2 + 2cx + 4$ and a straight line has equation $y = 4x + c$, where c is a constant.

Find the set of values of c for which the curve and line intersect at two distinct points. [5]

67. 9709_m22_qp_12 Q: 6



The circle with equation $(x + 1)^2 + (y - 2)^2 = 85$ and the straight line with equation $y = 3x - 20$ are shown in the diagram. The line intersects the circle at A and B, and the centre of the circle is at C.

(a) Find, by calculation, the coordinates of A and B. [4]

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(b) Find an equation of the circle which has its centre at C and for which the line with equation $y = 3x - 20$ is a tangent to the circle. [4]

68. 9709_m21_qp_12 Q: 4

A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

69. 9709_m21_qp_12 Q: 8

The points $A\,(7,\,1)$, $B\,(7,\,9)$ and $C\,(1,\,9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

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(b) Find an equation of the tangent to the circle at B. [2]

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70. 9709_s21_qp_11 Q: 10

The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

(b) Find the point of intersection of the tangents to the circle at A and B. [6]

71. 9709_s21_qp_12 Q: 6

Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

72. 9709_s21_qp_12 Q: 7

The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

73. 9709_s21_qp_13 Q: 3

A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

74. 9709_s21_qp_13 Q: 10

Points $A\,(-2,\ 3)$, $B\,(3,\ 0)$ and $C\,(6,\ 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

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(b) Hence state the coordinates of D. [1]

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(c) Find an equation of the circle. [2]

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The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

75. 9709_w21_qp_11 Q: 2

A curve has equation $y = kx^2 + 2x - k$ and a line has equation $y = kx - 2$, where k is a constant.

Find the set of values of k for which the curve and line do not intersect. [5]

76. 9709_w21_qp_11 Q: 7

A circle with centre (5, 2) passes through the point (7, 5).

(a) Find an equation of the circle. [2]

The line $y = 5x - 10$ intersects the circle at A and B.

(b) Find the exact length of the chord AB. [7]

77. 9709_w21_qp_13 Q: 9

The line $y = 2x + 5$ intersects the circle with equation $x^2 + y^2 = 20$ at A and B.

(a) Find the coordinates of A and B in surd form and hence find the exact length of the chord AB. [7]

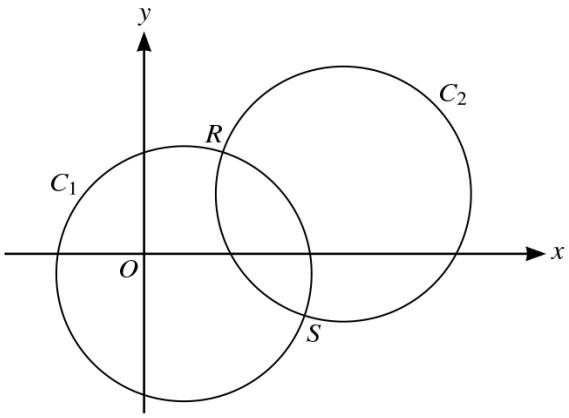
A straight line through the point (10, 0) with gradient m is a tangent to the circle.

(b) Find the two possible values of m. [5]

78. 9709_m20_qp_12 Q: 12

A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]



The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

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(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

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79. 9709_s20_qp_11 Q: 10

The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

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(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

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80. 9709_s20_qp_12 Q: 6

The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k. [3]

It is now given that $k = 2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve. [3]

81. 9709_s20_qp_12 Q: 11

The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

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The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

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The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

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The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

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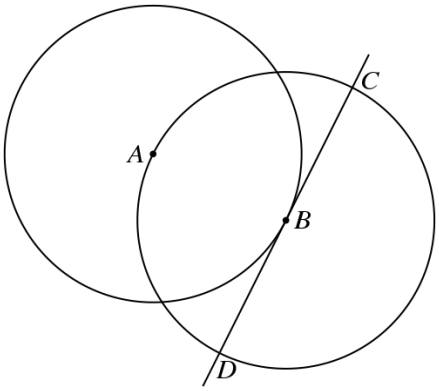
82. 9709_s20_qp_13 Q: 1

Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

83. 9709_w20_qp_11 Q: 1

Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

84. 9709_w20_qp_11 Q: 9



The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

(b) Find an equation of the circle with centre B. [3]

(c) Find, by calculation, the x-coordinates of C and D. [3]

85. 9709_w20_qp_12 Q: 3

The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

86. 9709_w20_qp_12 Q: 9

A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]

The other tangent from D to the circle touches the circle at E.

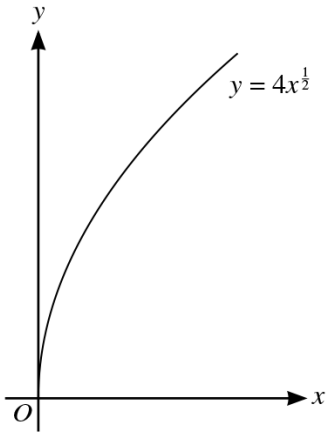
(c) Find the coordinates of E. [2]

87. 9709_w20_qp_13 Q: 4

A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection.
[5]

88. 9709_m19_qp_12 Q: 10



The diagram shows the curve with equation $y = 4x^{\frac{1}{2}}$.

(i) The straight line with equation $y = x + 3$ intersects the curve at points A and B. Find the length of AB. [6]

(ii) The tangent to the curve at a point T is parallel to AB. Find the coordinates of T. [3]

(iii) Find the coordinates of the point of intersection of the normal to the curve at T with the line AB. [3]

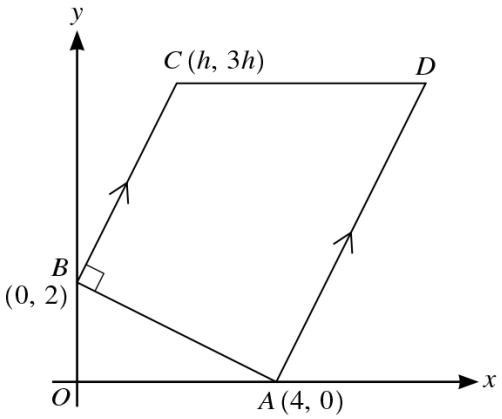
89. 9709_s19_qp_11 Q: 2

The line $4y = x + c$, where c is a constant, is a tangent to the curve $y^2 = x + 3$ at the point P on the curve.

(i) Find the value of c. [3]

(ii) Find the coordinates of P. [2]

90. 9709_s19_qp_11 Q: 4



The diagram shows a trapezium $ABCD$ in which the coordinates of A, B and C are (4, 0), (0, 2) and $(h, 3h)$ respectively. The lines BC and AD are parallel, angle $ABC = 90°$ and CD is parallel to the x-axis.

(i) Find, by calculation, the value of h. [3]

(ii) Hence find the coordinates of D. [3]

91. 9709_s19_qp_12 Q: 2

Two points A and B have coordinates $(1, 3)$ and $(9, -1)$ respectively. The perpendicular bisector of AB intersects the y-axis at the point C. Find the coordinates of C. [5]

92. 9709_s19_qp_13 Q: 7

The coordinates of two points A and B are (1, 3) and (9, −1) respectively and D is the mid-point of AB. A point C has coordinates (x, y), where x and y are variables.

(i) State the coordinates of D. [1]

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(ii) It is given that $CD^2 = 20$. Write down an equation relating x and y. [1]

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(iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]

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(iv) Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C. [4]

93. 9709_w19_qp_11 Q: 3

The line $y = ax + b$ is a tangent to the curve $y = 2x^3 - 5x^2 - 3x + c$ at the point (2, 6). Find the values of the constants a, b and c. [5]

94. 9709_w19_qp_11 Q: 6

A straight line has gradient m and passes through the point $(0, -2)$. Find the two values of m for which the line is a tangent to the curve $y = x^2 - 2x + 7$ and, for each value of m, find the coordinates of the point where the line touches the curve. [7]

95. 9709_w19_qp_12 Q: 2

The point M is the mid-point of the line joining the points $(3, 7)$ and $(-1, 1)$. Find the equation of the line through M which is parallel to the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

96. 9709_w19_qp_12 Q: 9

Functions f and g are defined by

$$\mathrm{f}(x) = 2x^2 + 8x + 1 \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g}(x) = 2x - k \quad \text{for } x \in \mathbb{R},$$

where k is a constant.

(i) Find the value of k for which the line $y = \mathrm{g}(x)$ is a tangent to the curve $y = \mathrm{f}(x)$. [3]

(ii) In the case where $k = -9$, find the set of values of x for which $\mathrm{f}(x) < \mathrm{g}(x)$. [3]

(iii) In the case where $k = -1$, find $\mathrm{g}^{-1}\mathrm{f}(x)$ and solve the equation $\mathrm{g}^{-1}\mathrm{f}(x) = 0$. [3]

(iv) Express $\mathrm{f}(x)$ in the form $2(x + a)^2 + b$, where a and b are constants, and hence state the least value of $\mathrm{f}(x)$. [3]

97. 9709_w19_qp_13 Q: 6

A line has equation $y = 3kx - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

(i) Find the set of values of k for which the line and curve meet at two distinct points. [4]

(ii) For each of two particular values of k, the line is a tangent to the curve. Show that these two tangents meet on the x-axis. [3]

98. 9709_m18_qp_12 Q: 4

A straight line cuts the positive x-axis at A and the positive y-axis at $B\,(0,\ 2)$. Angle $BAO = \frac{1}{6}\pi$ radians, where O is the origin.

(i) Find the exact value of the x-coordinate of A. [2]

(ii) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$, where m is given exactly and c is an integer. [4]

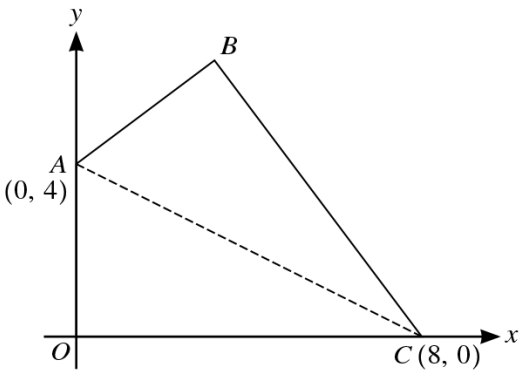
99. 9709_m18_qp_12 Q: 9

A curve has equation $y = \frac{1}{x} + c$ and a line has equation $y = cx - 3$, where c is a constant.

(i) Find the set of values of c for which the curve and the line meet. [4]

(ii) The line is a tangent to the curve for two particular values of c. For each of these values find the x-coordinate of the point at which the tangent touches the curve. [4]

100. 9709_s18_qp_11 Q: 5



The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are $(0, 4)$ and $(8, 0)$ respectively and O is the origin.

(i) Find the equations of AC and OB. [4]

(ii) Find, by calculation, the coordinates of B. [3]

101. 9709_s18_qp_11 Q: 9

Functions f and g are defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto \tfrac{1}{2}x - 2,$$
$$\mathrm{g} : x \mapsto 4 + x - \tfrac{1}{2}x^2.$$

(i) Find the points of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. [3]

(ii) Find the set of values of x for which $\mathrm{f}(x) > \mathrm{g}(x)$. [2]

(iii) Find an expression for $\mathrm{fg}(x)$ and deduce the range of fg. [4]

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The function h is defined by $\mathrm{h} : x \mapsto 4 + x - \frac{1}{2}x^2$ for $x \geqslant k$.

(iv) Find the smallest value of k for which h has an inverse. [2]

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102. 9709_s18_qp_12 Q: 2

The equation of a curve is $y = x^2 - 6x + k$, where k is a constant.

(i) Find the set of values of k for which the whole of the curve lies above the x-axis. [2]

(ii) Find the value of k for which the line $y + 2x = 7$ is a tangent to the curve. [3]

103. 9709_s18_qp_12 Q: 8

Points A and B have coordinates (h, h) and $(4h + 6, 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x + 2y = k$. Find the values of the constants h and k. [7]

104. 9709_s18_qp_13 Q: 6

The coordinates of points A and B are $(-3k - 1, k + 3)$ and $(k + 3, 3k + 5)$ respectively, where k is a constant ($k \neq -1$).

(i) Find and simplify the gradient of AB, showing that it is independent of k. [2]

(ii) Find and simplify the equation of the perpendicular bisector of AB. [5]

105. 9709_w18_qp_11 Q: 2

A line has equation $y = x + 1$ and a curve has equation $y = x^2 + bx + 5$. Find the set of values of the constant b for which the line meets the curve. [4]

106. 9709_w18_qp_11 Q: 3

Two points A and B have coordinates $(3a, -a)$ and $(-a, 2a)$ respectively, where a is a positive constant.

(i) Find the equation of the line through the origin parallel to AB. [2]

(ii) The length of the line AB is $3\frac{1}{3}$ units. Find the value of a. [3]

107. 9709_w18_qp_12 Q: 10

The equation of a curve is $y = 2x + \frac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

(i) Find the set of values of k for which the line does not meet the curve. [3]

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In the case where $k = 15$, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B. [3]

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(iii) Find the equation of the perpendicular bisector of the line joining A and B. [3]

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108. 9709_w18_qp_13 Q: 4

Two points A and B have coordinates $(-1, 1)$ and $(3, 4)$ respectively. The line BC is perpendicular to AB and intersects the x-axis at C.

(i) Find the equation of BC and the x-coordinate of C. [4]

(ii) Find the distance AC, giving your answer correct to 3 decimal places. [2]

109. 9709_w18_qp_13 Q: 9

A curve has equation $y = 2x^2 - 3x + 1$ and a line has equation $y = kx + k^2$, where k is a constant.

(i) Show that, for all values of k, the curve and the line meet. [4]

(ii) State the value of k for which the line is a tangent to the curve and find the coordinates of the point where the line touches the curve. [4]

110. 9709_s17_qp_12 Q: 2

The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

(i) Find the equation of AB. [3]

(ii) Find the coordinates of B. [3]

111. 9709_s17_qp_13 Q: 3

Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

112. 9709_s17_qp_13 Q: 8

$A(-1, 1)$ and $P(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

(i) Find an expression for b in terms of a. [2]

(ii) $B(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P. [6]

113. 9709_w17_qp_11 Q: 6

The points $A\,(1,\,1)$ and $B\,(5,\,9)$ lie on the curve $6y = 5x^2 - 18x + 19$.

(i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$. [4]

The perpendicular bisector of AB meets the curve at C and D.

(ii) Find, by calculation, the distance CD, giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$, where p and q are integers. [5]

114. 9709_w17_qp_13 Q: 2

Find the set of values of a for which the curve $y = -\frac{2}{x}$ and the straight line $y = ax + 3a$ meet at two distinct points. [4]

115. 9709_m16_qp_12 Q: 5

Two points have coordinates $A\,(5,\ 7)$ and $B\,(9,\ -1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

The line through $C\,(1,\ 2)$ parallel to AB meets the perpendicular bisector of AB at the point X.

(ii) Find, by calculation, the distance BX. [5]

116. 9709_s16_qp_12 Q: 8

Three points have coordinates $A\,(0,\ 7)$, $B\,(8,\ 3)$ and $C\,(3k,\ k)$. Find the value of the constant k for which

(i) C lies on the line that passes through A and B, [4]

(ii) C lies on the perpendicular bisector of AB. [4]

117. 9709_s16_qp_13 Q: 11

Triangle ABC has vertices at $A(-2, -1)$, $B(4, 6)$ and $C(6, -3)$.

(i) Show that triangle ABC is isosceles and find the exact area of this triangle. [6]

(ii) The point D is the point on AB such that CD is perpendicular to AB. Calculate the x-coordinate of D. [6]

118. 9709_w16_qp_11 Q: 4

C is the mid-point of the line joining $A\,(14, -7)$ to $B\,(-6, 3)$. The line through C perpendicular to AB crosses the y-axis at D.

(i) Find the equation of the line CD, giving your answer in the form $y = mx + c$. [4]

(ii) Find the distance AD. [2]

119. 9709_w16_qp_12 Q: 3

A curve has equation $y = 2x^2 - 6x + 5$.

(i) Find the set of values of x for which $y > 13$. [3]

(ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [3]

120. 9709_w16_qp_12 Q: 5

The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x- and y-axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance $AB = 10$. Find the values of a and b. [6]

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121. 9709_w16_qp_13 Q: 1

Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]

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122. 9709_w16_qp_13 Q: 6

Three points, A, B and C, are such that B is the mid-point of AC. The coordinates of A are $(2, m)$ and the coordinates of B are $(n, -6)$, where m and n are constants.

(i) Find the coordinates of C in terms of m and n. [2]

The line $y = x + 1$ passes through C and is perpendicular to AB.

(ii) Find the values of m and n. [5]

123. 9709_s15_qp_11 Q: 6

The line with gradient -2 passing through the point $P(3t, 2t)$ intersects the x-axis at A and the y-axis at B.

(i) Find the area of triangle AOB in terms of t. [3]

The line through P perpendicular to AB intersects the x-axis at C.

(ii) Show that the mid-point of PC lies on the line $y = x$. [4]

124. 9709_s15_qp_12 Q: 6

A tourist attraction in a city centre is a big vertical wheel on which passengers can ride. The wheel turns in such a way that the height, h m, of a passenger above the ground is given by the formula $h = 60(1 - \cos kt)$. In this formula, k is a constant, t is the time in minutes that has elapsed since the passenger started the ride at ground level and kt is measured in radians.

(i) Find the greatest height of the passenger above the ground. [1]

One complete revolution of the wheel takes 30 minutes.

(ii) Show that $k = \frac{1}{15}\pi$. [2]

(iii) Find the time for which the passenger is above a height of 90 m. [3]

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125. 9709_s15_qp_13 Q: 7

The point A has coordinates $(p, 1)$ and the point B has coordinates $(9, 3p + 1)$, where p is a constant.

(i) For the case where the distance AB is 13 units, find the possible values of p. [3]

(ii) For the case in which the line with equation $2x + 3y = 9$ is perpendicular to AB, find the value of p. [4]

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126. 9709_w15_qp_11 Q: 6

A curve has equation $y = x^2 - x + 3$ and a line has equation $y = 3x + a$, where a is a constant.

(i) Show that the x-coordinates of the points of intersection of the line and the curve are given by the equation $x^2 - 4x + (3 - a) = 0$. [1]

(ii) For the case where the line intersects the curve at two points, it is given that the x-coordinate of one of the points of intersection is -1. Find the x-coordinate of the other point of intersection. [2]

(iii) For the case where the line is a tangent to the curve at a point P, find the value of a and the coordinates of P. [4]

127. 9709_w15_qp_12 Q: 6

Points A, B and C have coordinates $A\,(-3,\ 7)$, $B\,(5,\ 1)$ and $C\,(-1,\ k)$, where k is a constant.

(i) Given that $AB = BC$, calculate the possible values of k. [3]

The perpendicular bisector of AB intersects the x-axis at D.

(ii) Calculate the coordinates of D. [5]

128. 9709_w15_qp_13 Q: 1

A line has equation $y = 2x - 7$ and a curve has equation $y = x^2 - 4x + c$, where c is a constant. Find the set of possible values of c for which the line does not intersect the curve. [3]