Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

Chapter 7

Differentiation

349. 9709_m22_qp_12 Q: 11

It is given that a curve has equation $y = k(3x - k)^{-1} + 3x$, where k is a constant.

(a) Find, in terms of k, the values of x at which there is a stationary point. [4]

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The function f has a stationary value at $x = a$ and is defined by

$$f(x) = 4(3x - 4)^{-1} + 3x \quad \text{for } x \geqslant \tfrac{3}{2}.$$

(b) Find the value of a and determine the nature of the stationary value. [3]

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(c) The function g is defined by $g(x) = -(3x + 1)^{-1} + 3x$ for $x \geqslant 0$.

Determine, making your reasoning clear, whether g is an increasing function, a decreasing function or neither. [2]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

350. 9709_s21_qp_12 Q: 3

The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2,\ k)$ $B\,(2.9,\ 2.8025)$ $C\,(2.99,\ 2.9800)$ $D\,(2.999,\ 2.9980)$ $E\,(3,\ 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

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(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

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The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

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351. 9709_s21_qp_13 Q: 2

The function f is defined by $f(x) = \frac{1}{3}(2x-1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

352. 9709_w21_qp_12 Q: 9

The volume V m^3 of a large circular mound of iron ore of radius r m is modelled by the equation $V = \frac{3}{2}\left(r - \frac{1}{2}\right)^3 - 1$ for $r \geqslant 2$. Iron ore is added to the mound at a constant rate of 1.5 m^3 per second.

(a) Find the rate at which the radius of the mound is increasing at the instant when the radius is 5.5 m. [3]

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(b) Find the volume of the mound at the instant when the radius is increasing at 0.1 m per second. [3]

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353. 9709_w21_qp_12 Q: 10

The function f is defined by $f(x) = x^2 + \frac{k}{x} + 2$ for $x > 0$.

(a) Given that the curve with equation $y = f(x)$ has a stationary point when $x = 2$, find k. [3]

(b) Determine the nature of the stationary point. [2]

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(c) Given that this is the only stationary point of the curve, find the range of f. [2]

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354. 9709_w21_qp_13 Q: 3

(a) Express $5y^2 - 30y + 50$ in the form $5(y + a)^2 + b$, where a and b are constants. [2]

(b) The function f is defined by $f(x) = x^5 - 10x^3 + 50x$ for $x \in \mathbb{R}$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

355. 9709_m20_qp_12 Q: 1

The function f is defined by $f(x) = \frac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

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356. 9709_m20_qp_12 Q: 4

A curve has equation $y = x^2 - 2x - 3$. A point is moving along the curve in such a way that at P the y-coordinate is increasing at 4 units per second and the x-coordinate is increasing at 6 units per second.

Find the x-coordinate of P. [4]

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357. 9709_s20_qp_11 Q: 9

The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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(b) Find the coordinates of each of the stationary points on the curve. [3]

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(c) Determine the nature of each stationary point. [2]

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358. 9709_s20_qp_12 Q: 3

A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of 600 cm^3 per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

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(b) Find the rate of increase of the radius after 30 seconds. [3]

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359. 9709_s20_qp_12 Q: 10

The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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(b) Find the coordinates of each of the stationary points on the curve. [3]

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(c) Determine the nature of each of the stationary points. [2]

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360. 9709_s20_qp_13 Q: 6

A point P is moving along a curve in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per minute. The equation of the curve is $y = (5x - 1)^{\frac{1}{2}}$.

(a) Find the rate at which the y-coordinate is increasing when $x = 1$. [4]

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(b) Find the value of x when the y-coordinate is increasing at $\frac{5}{8}$ units per minute. [3]

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361. 9709_w20_qp_11 Q: 3

Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50\,\text{cm}^3\,\text{s}^{-1}$.

Find the rate at which the radius of the balloon is increasing when the radius is $10\,\text{cm}$. [3]

362. 9709_w20_qp_11 Q: 6

The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

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363. 9709_w20_qp_13 Q: 8

The equation of a curve is $y = 2x + 1 + \dfrac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\dfrac{dy}{dx}$ and $\dfrac{d^2y}{dx^2}$. [3]

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(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

364. 9709_m19_qp_12 Q: 4

A curve has equation $y = (2x - 1)^{-1} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the x-coordinates of the stationary points and, showing all necessary working, determine the nature of each stationary point. [4]

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365. 9709_m19_qp_12 Q: 5

Two vectors, **u** and **v**, are such that

$$\mathbf{u} = \begin{pmatrix} q \\ 2 \\ 6 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 8 \\ q-1 \\ q^2-7 \end{pmatrix},$$

where q is a constant.

(i) Find the values of q for which **u** is perpendicular to **v**. [3]

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(ii) Find the angle between **u** and **v** when $q = 0$. [4]

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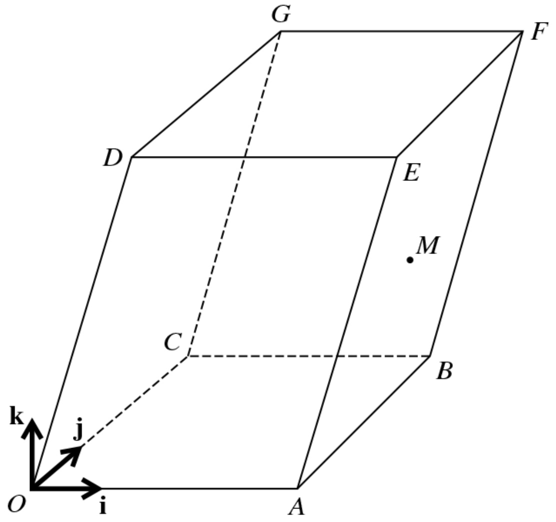
366. 9709_s19_qp_11 Q: 7



The diagram shows a three-dimensional shape in which the base $OABC$ and the upper surface $DEFG$ are identical horizontal squares. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. The point M is the mid-point of AF.

Unit vectors $\mathbf{i}$ and $\mathbf{j}$ are parallel to OA and OC respectively and the unit vector $\mathbf{k}$ is vertically upwards. The position vectors of A and D are given by $\overrightarrow{OA} = 8\mathbf{i}$ and $\overrightarrow{OD} = 3\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{AM}$ and $\overrightarrow{GM}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

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(ii) Use a scalar product to find angle *GMA* correct to the nearest degree. [4]

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367. 9709_s19_qp_12 Q: 8

The position vectors of points A and B, relative to an origin O, are given by

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -2 \\ -6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ k \\ -3 \end{pmatrix},$$

where k is a constant.

(i) Find the value of k for which angle AOB is 90°. [2]

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(ii) Find the values of k for which the lengths of OA and OB are equal. [2]

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The point C is such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

(iii) In the case where $k = 4$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]

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368. 9709_s19_qp_12 Q: 9

The curve C_1 has equation $y = x^2 - 4x + 7$. The curve C_2 has equation $y^2 = 4x + k$, where k is a constant. The tangent to C_1 at the point where $x = 3$ is also the tangent to C_2 at the point P. Find the value of k and the coordinates of P. [8]

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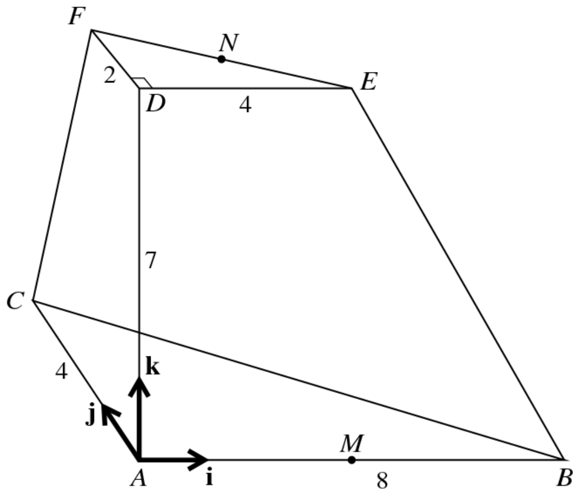
369. 9709_s19_qp_13 Q: 6



The diagram shows a solid figure $ABCDEF$ in which the horizontal base ABC is a triangle right-angled at A. The lengths of AB and AC are 8 units and 4 units respectively and M is the mid-point of AB. The point D is 7 units vertically above A. Triangle DEF lies in a horizontal plane with DE, DF and FE parallel to AB, AC and CB respectively and N is the mid-point of FE. The lengths of DE and DF are 4 units and 2 units respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ respectively.

(i) Find $\overrightarrow{MF}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

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(ii) Find $\overrightarrow{FN}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [1]

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(iii) Find $\overrightarrow{MN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

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(iv) Use a scalar product to find angle FMN. [4]

370. 9709_s19_qp_13 Q: 8

A curve is such that $\frac{dy}{dx} = 3x^2 + ax + b$. The curve has stationary points at $(-1, 2)$ and $(3, k)$. Find the values of the constants a, b and k. [8]

371. 9709_w19_qp_11 Q: 2

An increasing function, f, is defined for $x > n$, where n is an integer. It is given that $f'(x) = x^2 - 6x + 8$. Find the least possible value of n. [3]

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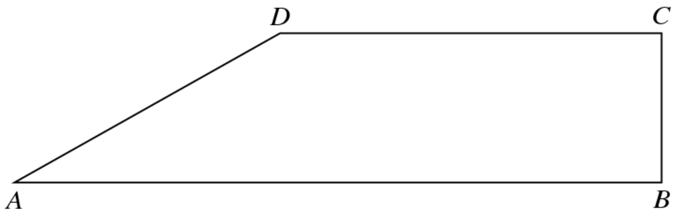
372. 9709_w19_qp_11 Q: 10



Relative to an origin O, the position vectors of the points A, B, C and D, shown in the diagram, are given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}.$$

(i) Show that AB is perpendicular to BC. [3]

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(ii) Show that $ABCD$ is a trapezium. [3]

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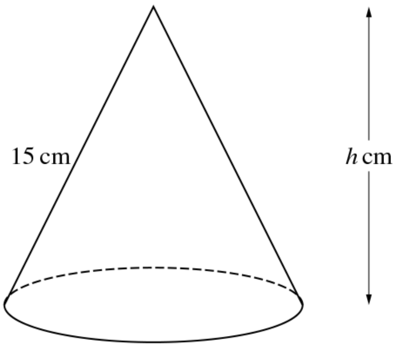
(iii) Find the area of $ABCD$, giving your answer correct to 2 decimal places. [3]

373. 9709_w19_qp_12 Q: 5



The diagram shows a solid cone which has a slant height of 15 cm and a vertical height of h cm.

(i) Show that the volume, V cm^3, of the cone is given by $V = \frac{1}{3}\pi(225h - h^3)$. [2]

[The volume of a cone of radius r and vertical height h is $\frac{1}{3}\pi r^2 h$.]

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(ii) Given that h can vary, find the value of h for which V has a stationary value. Determine, showing all necessary working, the nature of this stationary value. [5]

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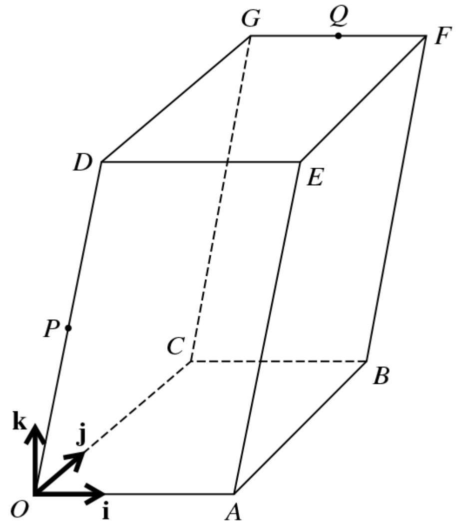
374. 9709_w19_qp_12 Q: 7



The diagram shows a three-dimensional shape $OABCDEFG$. The base $OABC$ and the upper surface $DEFG$ are identical horizontal rectangles. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. Points P and Q are the mid-points of OD and GF respectively. Unit vectors **i** and **j** are parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively and the unit vector **k** is vertically upwards. The position vectors of A, C and D are given by $\overrightarrow{OA} = 6\mathbf{i}$, $\overrightarrow{OC} = 8\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{PB}$ and $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**. [4]

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(ii) Determine whether P is nearer to Q or to B. [2]

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(iii) Use a scalar product to find angle BPQ. [3]

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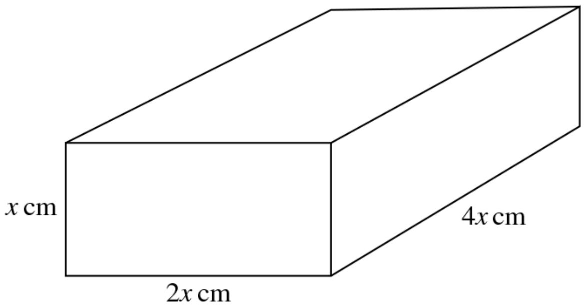
375. 9709_w19_qp_13 Q: 3

The equation of a curve is $y = x^3 + x^2 - 8x + 7$. The curve has no stationary points in the interval $a < x < b$. Find the least possible value of a and the greatest possible value of b. [4]

376. 9709_w19_qp_13 Q: 5



The dimensions of a cuboid are x cm, $2x$ cm and $4x$ cm, as shown in the diagram.

(i) Show that the surface area S cm^2 and the volume V cm^3 are connected by the relation

$$S = 7V^{\frac{2}{3}}.$$ [3]

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(ii) When the volume of the cuboid is $1000\,\text{cm}^3$ the surface area is increasing at $2\,\text{cm}^2\,\text{s}^{-1}$. Find the rate of increase of the volume at this instant. [4]

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377. 9709_w19_qp_13 Q: 10

Relative to an origin O, the position vectors of the points A, B and X are given by

$$\overrightarrow{OA} = \begin{pmatrix} -8 \\ -4 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 10 \\ 2 \\ 11 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OX} = \begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}.$$

(i) Find $\overrightarrow{AX}$ and show that AXB is a straight line. [3]

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The position vector of a point C is given by $\overrightarrow{OC} = \begin{pmatrix} 1 \\ -8 \\ 3 \end{pmatrix}$.

(ii) Show that CX is perpendicular to AX. [3]

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(iii) Find the area of triangle ABC. [3]

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378. 9709_m18_qp_12 Q: 7

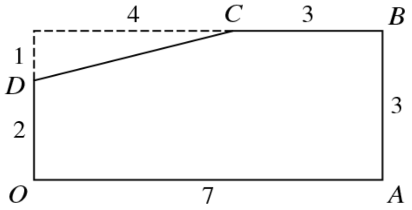

Fig. 1

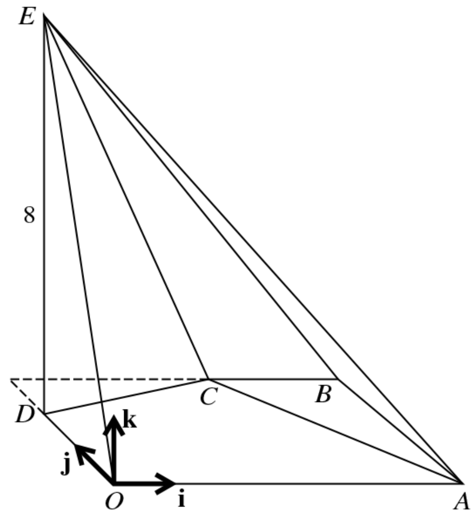

Fig. 2

Fig. 1 shows a rectangle with sides of 7 units and 3 units from which a triangular corner has been removed, leaving a 5-sided polygon $OABCD$. The sides OA, AB, BC and DO have lengths of 7 units, 3 units, 3 units and 2 units respectively. Fig. 2 shows the polygon $OABCD$ forming the horizontal base of a pyramid in which the point E is 8 units vertically above D. Unit vectors **i**, **j** and **k** are parallel to OA, OD and DE respectively.

(i) Find $\overrightarrow{CE}$ and the length of CE. [3]

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(ii) Use a scalar product to find angle ECA, giving your answer in the form $\cos^{-1}\left(\frac{m}{\sqrt{n}}\right)$, where m and n are integers. [5]

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379. 9709_m18_qp_12 Q: 8

A curve has equation $y = \frac{1}{2}x^2 - 4x^{\frac{3}{2}} + 8x$.

(i) Find the x-coordinates of the stationary points. [5]

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(ii) Find $\frac{d^2y}{dx^2}$. [1]

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(iii) Find, showing all necessary working, the nature of each stationary point. [2]

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380. 9709_m18_qp_12 Q: 10

Functions f and g are defined by

$$f(x) = \frac{8}{x-2} + 2 \quad \text{for } x > 2,$$
$$g(x) = \frac{8}{x-2} + 2 \quad \text{for } 2 < x < 4.$$

(i) **(a)** State the range of the function f. [1]

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(b) State the range of the function g. [1]

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(c) State the range of the function fg. [1]

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(ii) Explain why the function gf cannot be formed. [1]

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(iii) Find the set of values of x satisfying the inequality $6f'(x) + 2f^{-1}(x) - 5 < 0$. [6]

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381. 9709_s18_qp_11 Q: 2

A point is moving along the curve $y = 2x + \frac{5}{x}$ in such a way that the x-coordinate is increasing at a constant rate of 0.02 units per second. Find the rate of change of the y-coordinate when $x = 1$. [4]

382. 9709_s18_qp_11 Q: 7

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}.$$

(i) Find $\overrightarrow{AC}$. [1]

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(ii) The point M is the mid-point of AC. Find the unit vector in the direction of $\overrightarrow{OM}$. [3]

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(iii) Evaluate $\overrightarrow{AB}.\overrightarrow{AC}$ and hence find angle BAC. [4]

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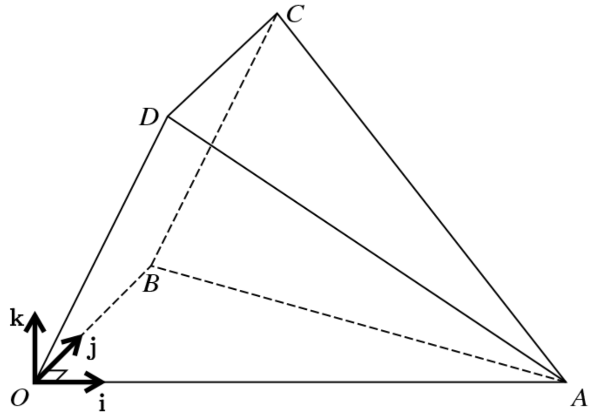
383. 9709_s18_qp_12 Q: 5



The diagram shows a three-dimensional shape. The base OAB is a horizontal triangle in which angle AOB is 90°. The side $OBCD$ is a rectangle and the side OAD lies in a vertical plane. Unit vectors **i** and **j** are parallel to OA and OB respectively and the unit vector **k** is vertical. The position vectors of A, B and D are given by $\overrightarrow{OA} = 8\mathbf{i}$, $\overrightarrow{OB} = 5\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 4\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{DA}$ and $\overrightarrow{CA}$ in terms of **i**, **j** and **k**. [2]

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(ii) Use a scalar product to find angle CAD. [4]

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384. 9709_s18_qp_13 Q: 8

(i) The tangent to the curve $y = x^3 - 9x^2 + 24x - 12$ at a point A is parallel to the line $y = 2 - 3x$. Find the equation of the tangent at A. [6]

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(ii) The function f is defined by $f(x) = x^3 - 9x^2 + 24x - 12$ for $x > k$, where k is a constant. Find the smallest value of k for f to be an increasing function. [2]

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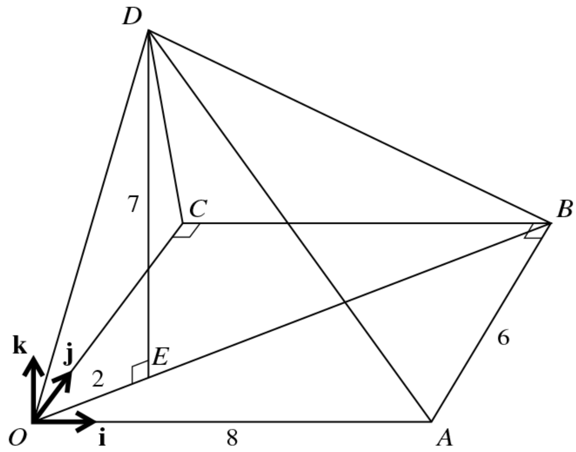
385. 9709_s18_qp_13 Q: 9



The diagram shows a pyramid $OABCD$ with a horizontal rectangular base $OABC$. The sides OA and AB have lengths of 8 units and 6 units respectively. The point E on OB is such that $OE = 2$ units. The point D of the pyramid is 7 units vertically above E. Unit vectors **i**, **j** and **k** are parallel to OA, OC and ED respectively.

(i) Show that $\overrightarrow{OE} = 1.6\mathbf{i} + 1.2\mathbf{j}$. [2]

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(ii) Use a scalar product to find angle BDO. [7]

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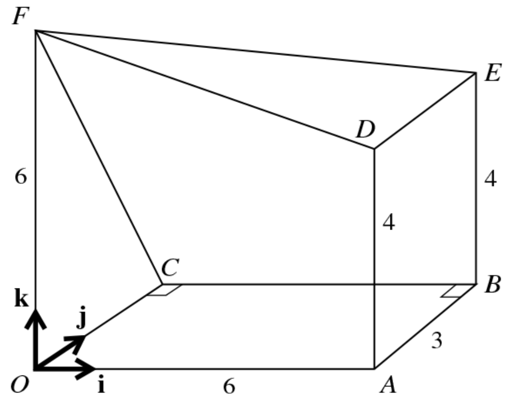
386. 9709_w18_qp_11 Q: 8



The diagram shows a solid figure $OABCDEF$ having a horizontal rectangular base $OABC$ with $OA = 6$ units and $AB = 3$ units. The vertical edges OF, AD and BE have lengths 6 units, 4 units and 4 units respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OF respectively.

(i) Find $\overrightarrow{DF}$. [1]

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(ii) Find the unit vector in the direction of $\overrightarrow{EF}$. [3]

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(iii) Use a scalar product to find angle EFD. [4]

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387. 9709_w18_qp_11 Q: 10

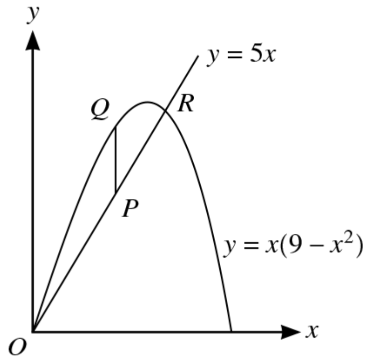
A curve has equation $y = \frac{1}{2}(4x - 3)^{-1}$. The point A on the curve has coordinates $\left(1, \frac{1}{2}\right)$.

(i) **(a)** Find and simplify the equation of the normal through A. [5]

(b) Find the x-coordinate of the point where this normal meets the curve again. [3]

(ii) A point is moving along the curve in such a way that as it passes through A its x-coordinate is decreasing at the rate of 0.3 units per second. Find the rate of change of its y-coordinate at A. [2]

388. 9709_w18_qp_12 Q: 3



The diagram shows part of the curve $y = x(9 - x^2)$ and the line $y = 5x$, intersecting at the origin O and the point R. Point P lies on the line $y = 5x$ between O and R and the x-coordinate of P is t. Point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the length of PQ in terms of t, simplifying your answer. [2]

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(ii) Given that t can vary, find the maximum value of the length of PQ. [3]

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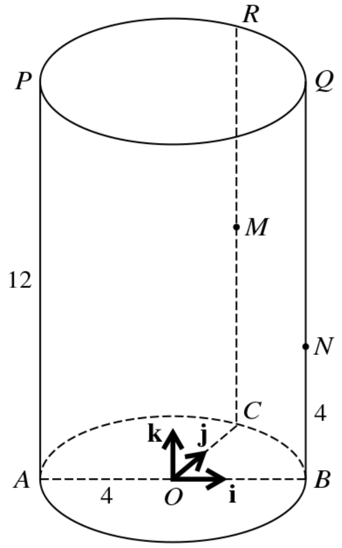
389. 9709_w18_qp_12 Q: 7



The diagram shows a solid cylinder standing on a horizontal circular base with centre O and radius 4 units. Points A, B and C lie on the circumference of the base such that AB is a diameter and angle $BOC = 90°$. Points P, Q and R lie on the upper surface of the cylinder vertically above A, B and C respectively. The height of the cylinder is 12 units. The mid-point of CR is M and N lies on BQ with $BN = 4$ units.

Unit vectors $\mathbf{i}$ and $\mathbf{j}$ are parallel to OB and OC respectively and the unit vector $\mathbf{k}$ is vertically upwards.

Evaluate $\overrightarrow{PN}.\overrightarrow{PM}$ and hence find angle MPN. [7]

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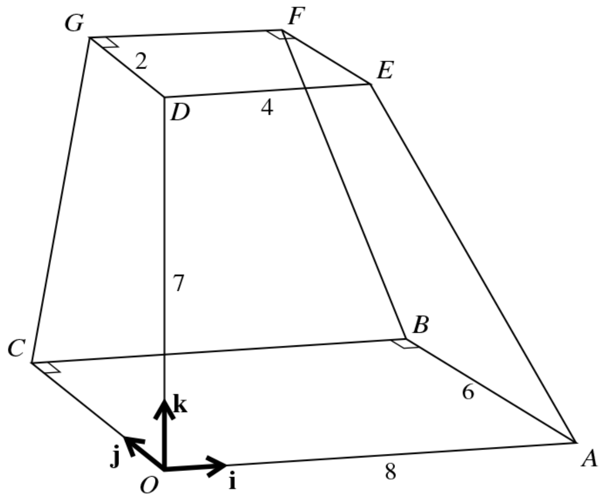
390. 9709_w18_qp_13 Q: 2

The function f is defined by $f(x) = x^3 + 2x^2 - 4x + 7$ for $x \geqslant -2$. Determine, showing all necessary working, whether f is an increasing function, a decreasing function or neither. [4]

391. 9709_w18_qp_13 Q: 6



The diagram shows a solid figure $OABCDEFG$ with a horizontal rectangular base $OABC$ in which $OA = 8$ units and $AB = 6$ units. The rectangle $DEFG$ lies in a horizontal plane and is such that D is 7 units vertically above O and DE is parallel to OA. The sides DE and DG have lengths 4 units and 2 units respectively. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively. Use a scalar product to find angle OBF, giving your answer in the form $\cos^{-1}\left(\frac{a}{b}\right)$, where a and b are integers. [6]

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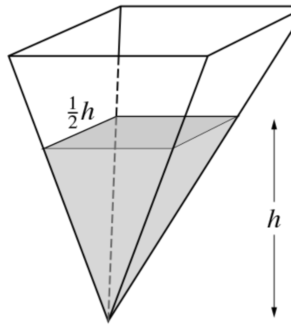
392. 9709_m17_qp_12 Q: 3



The diagram shows a water container in the form of an inverted pyramid, which is such that when the height of the water level is h cm the surface of the water is a square of side $\frac{1}{2}h$ cm.

(i) Express the volume of water in the container in terms of h. [1]

[The volume of a pyramid having a base area A and vertical height h is $\frac{1}{3}Ah$.]

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Water is steadily dripping into the container at a constant rate of $20\,\text{cm}^3$ per minute.

(ii) Find the rate, in cm per minute, at which the water level is rising when the height of the water level is $10\,\text{cm}$. [4]

393. 9709_m17_qp_12 Q: 6

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 7\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

(i) Use a scalar product to find angle OAB. [5]

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(ii) Find the area of triangle OAB. [2]

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394. 9709_m17_qp_12 Q: 7

The function f is defined for $x \geqslant 0$ by $\mathrm{f}(x) = (4x + 1)^{\frac{3}{2}}$.

(i) Find $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$. [3]

The first, second and third terms of a geometric progression are respectively $\mathrm{f}(2)$, $\mathrm{f}'(2)$ and $k\mathrm{f}''(2)$.

(ii) Find the value of the constant k. [5]

395. 9709_m17_qp_12 Q: 9

The point A (2, 2) lies on the curve $y = x^2 - 2x + 2$.

(i) Find the equation of the tangent to the curve at A. [3]

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The normal to the curve at A intersects the curve again at B.

(ii) Find the coordinates of B. [4]

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The tangents at A and B intersect each other at C.

(iii) Find the coordinates of C. [4]

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396. 9709_s17_qp_11 Q: 2

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix},$$

and angle $AOB = 90°$.

(i) Find the value of p. [2]

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The point C is such that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$.

(ii) Find the unit vector in the direction of $\overrightarrow{BC}$. [4]

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397. 9709_s17_qp_11 Q: 6

The horizontal base of a solid prism is an equilateral triangle of side x cm. The sides of the prism are vertical. The height of the prism is h cm and the volume of the prism is 2000 cm^3.

(i) Express h in terms of x and show that the total surface area of the prism, A cm^2, is given by

$$A = \frac{\sqrt{3}}{2}x^2 + \frac{24\,000}{\sqrt{3}}x^{-1}.$$ [3]

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(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

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(iii) Determine, showing all necessary working, the nature of this stationary value. [2]

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398. 9709_s17_qp_12 Q: 5

A curve has equation $y = 3 + \dfrac{12}{2 - x}$.

(i) Find the equation of the tangent to the curve at the point where the curve crosses the x-axis. [5]

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(ii) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.04 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

399. 9709_s17_qp_12 Q: 8

Relative to an origin O, the position vectors of three points A, B and C are given by

$$\overrightarrow{OA} = 3\mathbf{i} + p\mathbf{j} - 2p\mathbf{k}, \quad \overrightarrow{OB} = 6\mathbf{i} + (p+4)\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = (p-1)\mathbf{i} + 2\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) In the case where $p = 2$, use a scalar product to find angle AOB. [4]

(ii) In the case where $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$, find the values of p and q. [4]

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400. 9709_s17_qp_12 Q: 9

The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve. [3]

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(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

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(iii) Find the values of x at which the line $y = 6$ meets the curve. [3]

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(iv) State the set of values of k for which the line $y = k$ does not meet the curve. [1]

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401. 9709_s17_qp_13 Q: 4

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}.$$

The point P lies on AB and is such that $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$.

(i) Find the position vector of P. [3]

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(ii) Find the distance OP. [1]

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(iii) Determine whether OP is perpendicular to AB. Justify your answer. [2]

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402. 9709_s17_qp_13 Q: 6

The line $3y + x = 25$ is a normal to the curve $y = x^2 - 5x + k$. Find the value of the constant k. [6]

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403. 9709_w17_qp_11 Q: 1

A curve has equation $y = 2x^{\frac{3}{2}} - 3x - 4x^{\frac{1}{2}} + 4$. Find the equation of the tangent to the curve at the point (4, 0). [4]

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404. 9709_w17_qp_11 Q: 2

A function f is defined by $f : x \mapsto x^3 - x^2 - 8x + 5$ for $x < a$. It is given that f is an increasing function. Find the largest possible value of the constant a. [4]

405. 9709_w17_qp_11 Q: 4

Machines in a factory make cardboard cones of base radius r cm and vertical height h cm. The volume, V cm^3, of such a cone is given by $V = \frac{1}{3}\pi r^2 h$. The machines produce cones for which $h + r = 18$.

(i) Show that $V = 6\pi r^2 - \frac{1}{3}\pi r^3$. [1]

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(ii) Given that r can vary, find the non-zero value of r for which V has a stationary value and show that the stationary value is a maximum. [4]

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(iii) Find the maximum volume of a cone that can be made by these machines. [1]

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406. 9709_w17_qp_11 Q: 8

(a) Relative to an origin O, the position vectors of two points P and Q are $\mathbf{p}$ and $\mathbf{q}$ respectively. The point R is such that PQR is a straight line with Q the mid-point of PR. Find the position vector of R in terms of $\mathbf{p}$ and $\mathbf{q}$, simplifying your answer. [3]

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(b) The vector $6\mathbf{i} + a\mathbf{j} + b\mathbf{k}$ has magnitude 21 and is perpendicular to $3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Find the possible values of a and b, showing all necessary working. [6]

407. 9709_w17_qp_12 Q: 7

Points A and B lie on the curve $y = x^2 - 4x + 7$. Point A has coordinates (4, 7) and B is the stationary point of the curve. The equation of a line L is $y = mx - 2$, where m is a constant.

(i) In the case where L passes through the mid-point of AB, find the value of m. [4]

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(ii) Find the set of values of m for which L does not meet the curve. [4]

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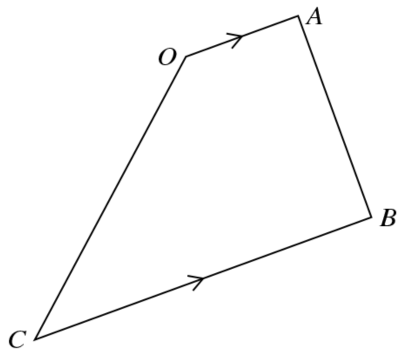
408. 9709_w17_qp_12 Q: 9



The diagram shows a trapezium $OABC$ in which OA is parallel to CB. The position vectors of A and B relative to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$.

(i) Show that angle OAB is 90°. [3]

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The magnitude of $\overrightarrow{CB}$ is three times the magnitude of $\overrightarrow{OA}$.

(ii) Find the position vector of C. [3]

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(iii) Find the exact area of the trapezium $OABC$, giving your answer in the form $a\sqrt{b}$, where a and b are integers. [3]

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409. 9709_w17_qp_13 Q: 4

The function f is such that $f(x) = (2x - 1)^{\frac{3}{2}} - 6x$ for $\frac{1}{2} < x < k$, where k is a constant. Find the largest value of k for which f is a decreasing function. [5]

410. 9709_w17_qp_13 Q: 9

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 8 \\ -6 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -10 \\ 3 \\ -13 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}.$$

A fourth point, D, is such that the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$ are the first, second and third terms respectively of a geometric progression.

(i) Find the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$. [5]

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(ii) Given that D is a point lying on the line through B and C, find the two possible position vectors of the point D. [4]

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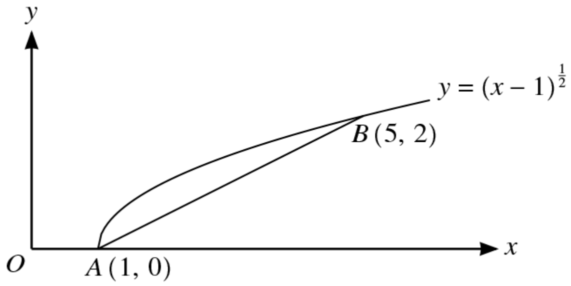
411. 9709_w17_qp_13 Q: 11



The diagram shows the curve $y = (x - 1)^{\frac{1}{2}}$ and points $A\ (1, 0)$ and $B\ (5, 2)$ lying on the curve.

(i) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [2]

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(ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to AB. [5]

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(iii) Find the perpendicular distance between the line AB and the tangent parallel to AB. Give your answer correct to 2 decimal places. [3]

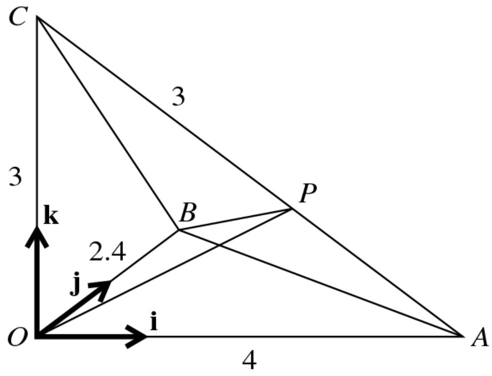
412. 9709_m16_qp_12 Q: 6

A vacuum flask (for keeping drinks hot) is modelled as a closed cylinder in which the internal radius is r cm and the internal height is h cm. The volume of the flask is $1000\,\text{cm}^3$. A flask is most efficient when the total internal surface area, $A\,\text{cm}^2$, is a minimum.

(i) Show that $A = 2\pi r^2 + \dfrac{2000}{r}$. [3]

(ii) Given that r can vary, find the value of r, correct to 1 decimal place, for which A has a stationary value and verify that the flask is most efficient when r takes this value. [5]

413. 9709_m16_qp_12 Q: 7



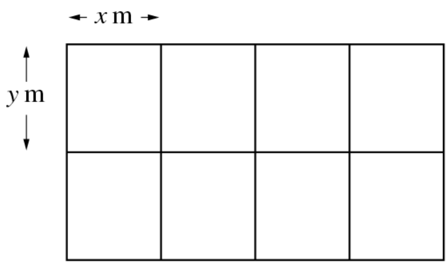
The diagram shows a pyramid $OABC$ with a horizontal triangular base OAB and vertical height OC. Angles AOB, BOC and AOC are each right angles. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OC respectively, with $OA = 4$ units, $OB = 2.4$ units and $OC = 3$ units. The point P on CA is such that $CP = 3$ units.

(i) Show that $\overrightarrow{CP} = 2.4\mathbf{i} - 1.8\mathbf{k}$. [2]

(ii) Express $\overrightarrow{OP}$ and $\overrightarrow{BP}$ in terms of **i**, **j** and **k**. [2]

(iii) Use a scalar product to find angle BPC. [4]

414. 9709_s16_qp_11 Q: 5



A farmer divides a rectangular piece of land into 8 equal-sized rectangular sheep pens as shown in the diagram. Each sheep pen measures x m by y m and is fully enclosed by metal fencing. The farmer uses 480 m of fencing.

(i) Show that the total area of land used for the sheep pens, A m^2, is given by

$$A = 384x - 9.6x^2. \quad [3]$$

(ii) Given that x and y can vary, find the dimensions of each sheep pen for which the value of A is a maximum. (There is no need to verify that the value of A is a maximum.) [3]

415. 9709_s16_qp_11 Q: 8

A curve has equation $y = 3x - \frac{4}{x}$ and passes through the points $A\,(1, -1)$ and $B\,(4, 11)$. At each of the points C and D on the curve, the tangent is parallel to AB. Find the equation of the perpendicular bisector of CD. [7]

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416. 9709_s16_qp_11 Q: 10

Relative to an origin O, the position vectors of points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$$

respectively, where k is a constant.

(i) Find the value of k in the case where angle $AOB = 90°$. [2]

(ii) Find the possible values of k for which the lengths of AB and OC are equal. [4]

The point D is such that $\overrightarrow{OD}$ is in the same direction as $\overrightarrow{OA}$ and has magnitude 9 units. The point E is such that $\overrightarrow{OE}$ is in the same direction as $\overrightarrow{OC}$ and has magnitude 14 units.

(iii) Find the magnitude of $\overrightarrow{DE}$ in the form $\sqrt{n}$ where n is an integer. [4]

417. 9709_s16_qp_12 Q: 3

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$$

The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

418. 9709_s16_qp_13 Q: 5

A curve has equation $y = 8x + (2x - 1)^{-1}$. Find the values of x at which the curve has a stationary point and determine the nature of each stationary point, justifying your answers. [7]

419. 9709_s16_qp_13 Q: 7

The point $P(x, y)$ is moving along the curve $y = x^2 - \frac{10}{3}x^{\frac{3}{2}} + 5x$ in such a way that the rate of change of y is constant. Find the values of x at the points at which the rate of change of x is equal to half the rate of change of y. [7]

420. 9709_s16_qp_13 Q: 9

The position vectors of A, B and C relative to an origin O are given by

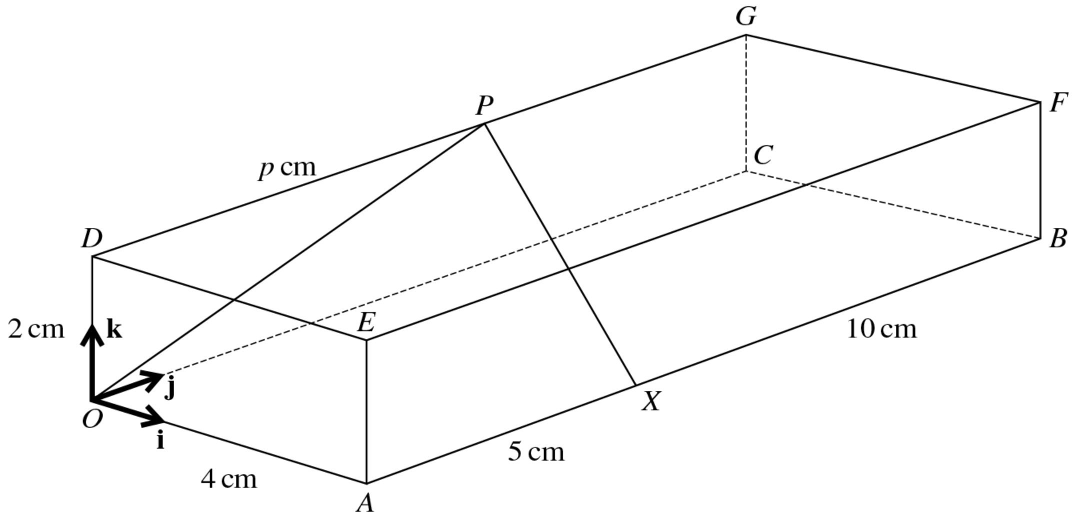
$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix},$$

where p is a constant.

(i) Find the value of p for which the lengths of AB and CB are equal. [4]

(ii) For the case where $p = 1$, use a scalar product to find angle ABC. [4]

421. 9709_w16_qp_11 Q: 9



The diagram shows a cuboid $OABCDEFG$ with a horizontal base $OABC$ in which $OA = 4$ cm and $AB = 15$ cm. The height OD of the cuboid is 2 cm. The point X on AB is such that $AX = 5$ cm and the point P on DG is such that $DP = p$ cm, where p is a constant. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OD respectively.

(i) Find the possible values of p such that angle $OPX = 90°$. [4]

(ii) For the case where $p = 9$, find the unit vector in the direction of $\overrightarrow{XP}$. [2]

(iii) A point Q lies on the face $CBFG$ and is such that XQ is parallel to AG. Find $\overrightarrow{XQ}$. [3]

422. 9709_w16_qp_11 Q: 11

The point $P(3,\ 5)$ lies on the curve $y = \dfrac{1}{x-1} - \dfrac{9}{x-5}$.

(i) Find the x-coordinate of the point where the normal to the curve at P intersects the x-axis. [5]

(ii) Find the x-coordinate of each of the stationary points on the curve and determine the nature of each stationary point, justifying your answers. [6]

423. 9709_w16_qp_12 Q: 7

The equation of a curve is $y = 2 + \dfrac{3}{2x - 1}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x-coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

424. 9709_w16_qp_12 Q: 9

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}.$$

(i) Use a scalar product to find angle AOB. [4]

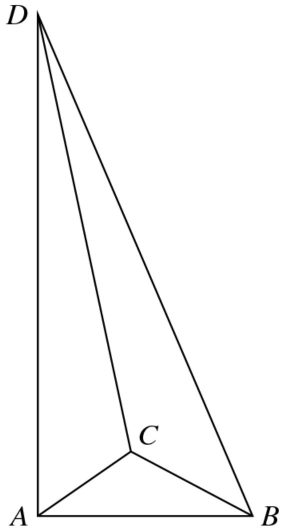
(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and of magnitude 15 units. [3]

(iii) Find the value of the constant p for which $p\overrightarrow{OA} + \overrightarrow{OC}$ is perpendicular to $\overrightarrow{OB}$. [3]

425. 9709_w16_qp_13 Q: 4

The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n. [4]

426. 9709_w16_qp_13 Q: 7



The diagram shows a triangular pyramid $ABCD$. It is given that

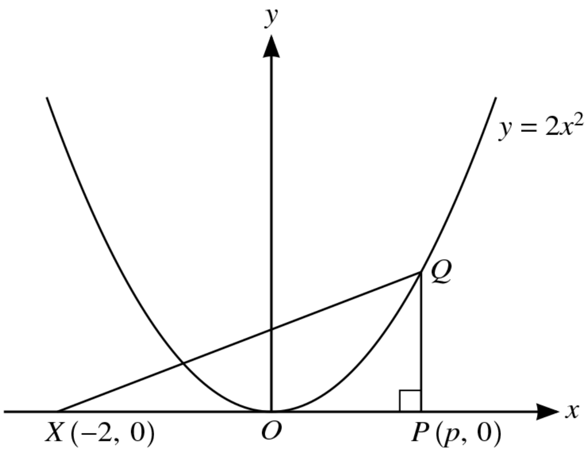
$$\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{AD} = \mathbf{i} + 4\mathbf{j} - 7\mathbf{k}.$$

(i) Verify, showing all necessary working, that each of the angles DAB, DAC and CAB is 90°. [3]

(ii) Find the exact value of the area of the triangle ABC, and hence find the exact value of the volume of the pyramid. [4]

[The volume V of a pyramid of base area A and vertical height h is given by $V = \frac{1}{3}Ah$.]

427. 9709_s15_qp_11 Q: 2



The diagram shows the curve $y = 2x^2$ and the points $X(-2, 0)$ and $P(p, 0)$. The point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the area, A, of triangle XPQ in terms of p. [2]

The point P moves along the x-axis at a constant rate of 0.02 units per second and Q moves along the curve so that PQ remains parallel to the y-axis.

(ii) Find the rate at which A is increasing when $p = 2$. [3]

428. 9709_s15_qp_11 Q: 4

Relative to the origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}.$$

(i) Find the cosine of angle AOB. [3]

The position vector of C is given by $\overrightarrow{OC} = \begin{pmatrix} k \\ -2k \\ 2k-3 \end{pmatrix}$.

(ii) Given that AB and OC have the same length, find the possible values of k. [4]

429. 9709_s15_qp_11 Q: 9

The equation of a curve is $y = x^3 + px^2$, where p is a positive constant.

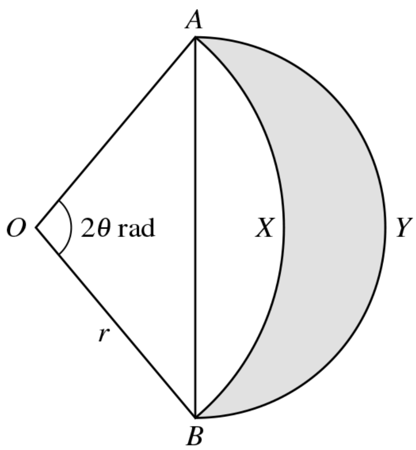
(i) Show that the origin is a stationary point on the curve and find the coordinates of the other stationary point in terms of p. [4]

(ii) Find the nature of each of the stationary points. [3]

Another curve has equation $y = x^3 + px^2 + px$.

(iii) Find the set of values of p for which this curve has no stationary points. [3]

430. 9709_s15_qp_12 Q: 2



In the diagram, AYB is a semicircle with AB as diameter and $OAXB$ is a sector of a circle with centre O and radius r. Angle $AOB = 2\theta$ radians. Find an expression, in terms of r and θ, for the area of the shaded region. [4]

431. 9709_s15_qp_12 Q: 4

Variables u, x and y are such that $u = 2x(y - x)$ and $x + 3y = 12$. Express u in terms of x and hence find the stationary value of u. [5]

432. 9709_s15_qp_12 Q: 9

Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}.$$

(i) Use a vector method to find angle AOB. [4]

The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$.

(ii) Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

(iii) Show that triangle OAC is isosceles. [1]

433. 9709_s15_qp_13 Q: 5

Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix}.$$

(i) Show that angle ABC is $90°$. [4]

(ii) Find the area of triangle ABC, giving your answer correct to 1 decimal place. [3]

434. 9709_s15_qp_13 Q: 8

The function f is defined by $f(x) = \frac{1}{x+1} + \frac{1}{(x+1)^2}$ for $x > -1$.

(i) Find $f'(x)$. [3]

(ii) State, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

The function g is defined by $g(x) = \frac{1}{x+1} + \frac{1}{(x+1)^2}$ for $x < -1$.

(iii) Find the coordinates of the stationary point on the curve $y = g(x)$. [4]

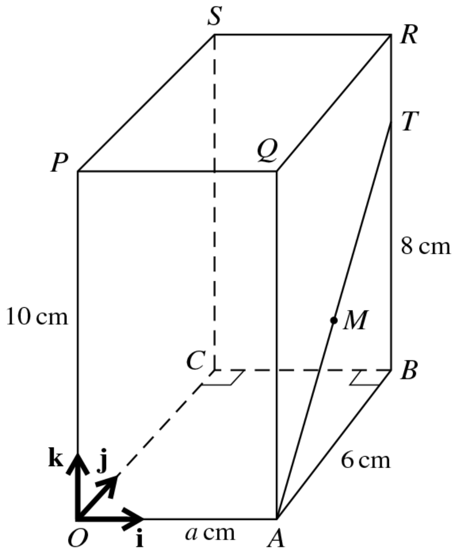
435. 9709_w15_qp_11 Q: 5

A curve has equation $y = \frac{8}{x} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the coordinates of the stationary points and state, with a reason, the nature of each stationary point. [5]

436. 9709_w15_qp_11 Q: 10



The diagram shows a cuboid $OABCPQRS$ with a horizontal base $OABC$ in which $AB = 6$ cm and $OA = a$ cm, where a is a constant. The height OP of the cuboid is 10 cm. The point T on BR is such that $BT = 8$ cm, and M is the mid-point of AT. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OP respectively.

(i) For the case where $a = 2$, find the unit vector in the direction of $\overrightarrow{PM}$. [4]

(ii) For the case where angle $ATP = \cos^{-1}\left(\frac{2}{7}\right)$, find the value of a. [5]

437. 9709_w15_qp_12 Q: 3

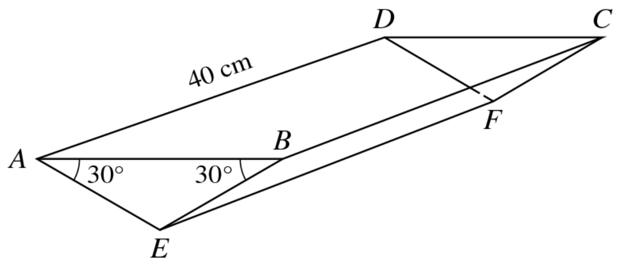


Fig. 1

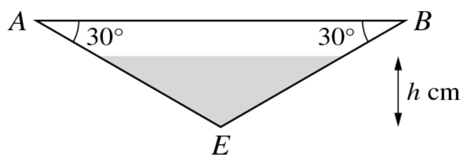


Fig. 2

Fig. 1 shows an open tank in the shape of a triangular prism. The vertical ends ABE and DCF are identical isosceles triangles. Angle ABE = angle $BAE = 30°$. The length of AD is 40 cm. The tank is fixed in position with the open top $ABCD$ horizontal. Water is poured into the tank at a constant rate of 200 $\text{cm}^3\,\text{s}^{-1}$. The depth of water, t seconds after filling starts, is h cm (see Fig. 2).

(i) Show that, when the depth of water in the tank is h cm, the volume, $V\,\text{cm}^3$, of water in the tank is given by $V = (40\sqrt{3})h^2$. [3]

(ii) Find the rate at which h is increasing when $h = 5$. [3]

438. 9709_w15_qp_12 Q: 7

Relative to an origin O, the position vectors of points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 2 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ 5 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ p \\ q \end{pmatrix}.$$

(i) In the case where ABC is a straight line, find the values of p and q. [4]

(ii) In the case where angle BAC is 90°, express q in terms of p. [2]

(iii) In the case where $p = 3$ and the lengths of AB and AC are equal, find the possible values of q. [3]

439. 9709_w15_qp_12 Q: 9

The curve $y = \mathrm{f}(x)$ has a stationary point at $(2, 10)$ and it is given that $\mathrm{f}''(x) = \dfrac{12}{x^3}$.

(i) Find $\mathrm{f}(x)$. [6]

(ii) Find the coordinates of the other stationary point. [2]

(iii) Find the nature of each of the stationary points. [2]

440. 9709_w15_qp_13 Q: 5

Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} p-6 \\ 2p-6 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4-2p \\ p \\ 2 \end{pmatrix},$$

where p is a constant.

(i) For the case where OA is perpendicular to OB, find the value of p. [3]

(ii) For the case where OAB is a straight line, find the vectors $\overrightarrow{OA}$ and $\overrightarrow{OB}$. Find also the length of the line OA. [4]