Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

Chapter 2

Functions

13. 9709_m22_qp_12 Q: 5

(a) Express $2x^2 - 8x + 14$ in the form $2[(x - a)^2 + b]$. [2]

The functions f and g are defined by

$$\mathrm{f}(x) = x^2 \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g}(x) = 2x^2 - 8x + 14 \quad \text{for } x \in \mathbb{R}.$$

(b) Describe fully a sequence of transformations that maps the graph of $y = \mathrm{f}(x)$ onto the graph of $y = \mathrm{g}(x)$, making clear the order in which the transformations are applied. [4]

14. 9709_m22_qp_12 Q: 9

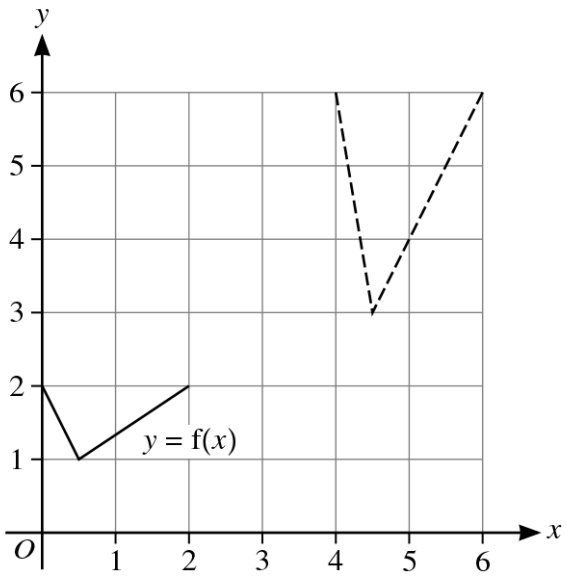
Functions f, g and h are defined as follows:

$$\text{f} : x \mapsto x - 4x^{\frac{1}{2}} + 1 \quad \text{for } x \geqslant 0,$$
$$\text{g} : x \mapsto mx^2 + n \quad \text{for } x \geqslant -2, \text{ where } m \text{ and } n \text{ are constants,}$$
$$\text{h} : x \mapsto x^{\frac{1}{2}} - 2 \quad \text{for } x \geqslant 0.$$

(a) Solve the equation $\text{f}(x) = 0$, giving your solutions in the form $x = a + b\sqrt{c}$, where a, b and c are integers. [4]

(b) Given that $\mathrm{f}(x) \equiv \mathrm{gh}(x)$, find the values of m and n. [4]

15. 9709_m21_qp_12 Q: 5



In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

(a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]

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(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

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16. 9709_m21_qp_12 Q: 7

Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$\mathrm{g} : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$ and state the range of f. [3]

(b) Find an expression for $f^{-1}(x)$. [2]

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(c) Solve the equation $gf(x) = 13$. [3]

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17. 9709_s21_qp_11 Q: 9

Functions f and g are defined as follows:

$$\mathrm{f}(x) = (x-2)^2 - 4 \text{ for } x \geqslant 2,$$
$$\mathrm{g}(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

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(b) Find $\mathrm{f}^{-1}(x)$. [2]

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(c) Given that $a = -\frac{5}{3}$, solve the equation $\mathrm{f}(x) = \mathrm{g}(x)$. [3]

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(d) Given instead that $\mathrm{ggf}^{-1}(12) = 62$, find the possible values of a. [5]

18. 9709_s21_qp_12 Q: 2

(a) The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 2\mathrm{f}(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

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(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

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19. 9709_s21_qp_12 Q: 5

The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

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(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

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20. 9709_s21_qp_13 Q: 6

Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

21. 9709_s21_qp_13 Q: 8

Functions f and g are defined as follows:

$$\text{f} : x \mapsto x^2 - 1 \text{ for } x < 0,$$
$$\text{g} : x \mapsto \frac{1}{2x+1} \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $\text{fg}(x) = 3$. [4]

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$. [3]

22. 9709_w21_qp_11 Q: 8

(a) Express $-3x^2 + 12x + 2$ in the form $-3(x - a)^2 + b$, where a and b are constants. [2]

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The one-one function f is defined by $f : x \mapsto -3x^2 + 12x + 2$ for $x \leqslant k$.

(b) State the largest possible value of the constant k. [1]

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It is now given that $k = -1$.

(c) State the range of f. [1]

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(d) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

The result of translating the graph of $y = \mathrm{f}(x)$ by $\begin{pmatrix} -3 \\ 1 \end{pmatrix}$ is the graph of $y = \mathrm{g}(x)$.

(e) Express $\mathrm{g}(x)$ in the form $px^2 + qx + r$, where p, q and r are constants. [3]

23. 9709_w21_qp_12 Q: 2

The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = \mathrm{f}(2x) - 3$.

(a) Describe fully the two single transformations that have been combined to give the resulting transformation. [3]

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The point $P(5, 6)$ lies on the transformed curve $y = \mathrm{f}(2x) - 3$.

(b) State the coordinates of the corresponding point on the original curve $y = \mathrm{f}(x)$. [2]

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24. 9709_w21_qp_12 Q: 3

The function f is defined as follows:

$$\mathrm{f}(x) = \frac{x+3}{x-1} \text{ for } x > 1.$$

(a) Find the value of $\mathrm{ff}(5)$. [2]

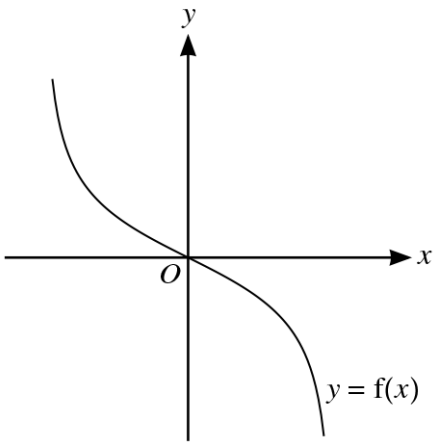
(b) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

25. 9709_w21_qp_13 Q: 1

The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 3 - \mathrm{f}(x)$.

Describe fully, in the correct order, the two transformations that have been combined. [4]

26. 9709_w21_qp_13 Q: 6



The diagram shows the graph of $y = \mathrm{f}(x)$.

(a) On this diagram sketch the graph of $y = \mathrm{f}^{-1}(x)$. [1]

It is now given that $\mathrm{f}(x) = -\dfrac{x}{\sqrt{4 - x^2}}$ where $-2 < x < 2$.

(b) Find an expression for $\mathrm{f}^{-1}(x)$. [4]

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The function g is defined by $g(x) = 2x$ for $-a < x < a$, where a is a constant.

(c) State the maximum possible value of a for which fg can be formed. [1]

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(d) Assuming that fg can be formed, find and simplify an expression for $fg(x)$. [2]

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27. 9709_m20_qp_12 Q: 2

The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 1 + \mathrm{f}\left(\frac{1}{2}x\right)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

28. 9709_m20_qp_12 Q: 9

(a) Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

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The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

(d) State the largest value of k possible for the composition fg to be defined. [1]

29. 9709_s20_qp_11 Q: 6

Functions f and g are defined for $x \in \mathbb{R}$ by

$$\begin{aligned} \mathrm{f} &: x \mapsto \tfrac{1}{2}x - a, \\ \mathrm{g} &: x \mapsto 3x + b, \end{aligned}$$

where a and b are constants.

(a) Given that $\mathrm{gg}(2) = 10$ and $\mathrm{f}^{-1}(2) = 14$, find the values of a and b. [4]

(b) Using these values of a and b, find an expression for $\mathrm{gf}(x)$ in the form $cx + d$, where c and d are constants. [2]

30. 9709_s20_qp_12 Q: 5

The function f is defined for $x \in \mathbb{R}$ by

$$\mathrm{f} : x \mapsto a - 2x,$$

where a is a constant.

(a) Express $\mathrm{ff}(x)$ and $\mathrm{f}^{-1}(x)$ in terms of a and x. [4]

(b) Given that $\mathrm{ff}(x) = \mathrm{f}^{-1}(x)$, find x in terms of a. [2]

31. 9709_s20_qp_13 Q: 3

In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = \mathrm{f}(x)$. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

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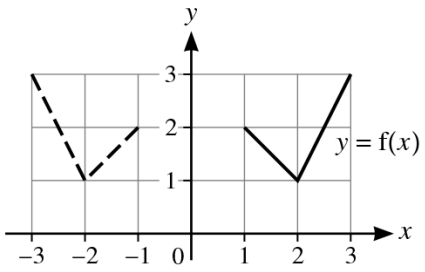

State, in terms of f, the equation of the graph shown with broken lines. [1]

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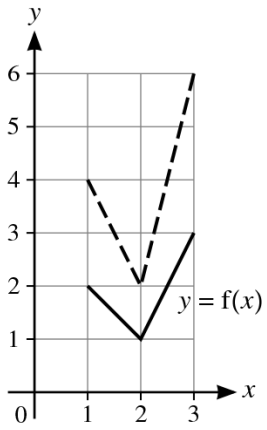
(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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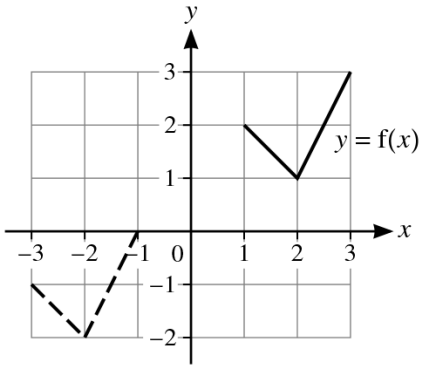
(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

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32. 9709_s20_qp_13 Q: 9

The functions f and g are defined by

$$\mathrm{f}(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$
$$\mathrm{g}(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x-a)^2 + b$. [2]

(b) It is given that f is a one-one function. State the smallest possible value of c. [1]

It is now given that $c = 5$.

(c) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

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(d) Find an expression for $\mathrm{gf}(x)$ and state the range of gf. [3]

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33. 9709_w20_qp_11 Q: 11

The functions f and g are defined by

$$\mathrm{f}(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$\mathrm{g}(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for $\mathrm{fg}(x)$. [1]

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$ and state the domain of $(\mathrm{fg})^{-1}$. [4]

(c) Solve the equation $\mathrm{fg}(x) - 3 = \mathrm{gf}(x)$. [4]

34. 9709_w20_qp_12 Q: 5

Functions f and g are defined by

$$\mathrm{f}(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$
$$\mathrm{g}(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

(b) Find the values of x for which $\mathrm{f}^{-1}(x) = \mathrm{g}^{-1}(x)$. [5]

35. 9709_w20_qp_13 Q: 1

(a) Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

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(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

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36. 9709_w20_qp_13 Q: 6

The function f is defined by $\mathrm{f}(x) = \dfrac{2x}{3x-1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(b) Show that $\dfrac{2}{3} + \dfrac{2}{3(3x-1)}$ can be expressed as $\dfrac{2x}{3x-1}$. [2]

(c) State the range of f. [1]

37. 9709_m19_qp_12 Q: 8

(i) Express $x^2 - 4x + 7$ in the form $(x + a)^2 + b$. [2]

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The function f is defined by $f(x) = x^2 - 4x + 7$ for $x < k$, where k is a constant.

(ii) State the largest value of k for which f is a decreasing function. [1]

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The value of k is now given to be 1.

(iii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(iv) The function g is defined by $g(x) = \dfrac{2}{x-1}$ for $x > 1$. Find an expression for $gf(x)$ and state the range of gf. [4]

38. 9709_s19_qp_11 Q: 5

The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

(i) Express $-2x^2 + 12x - 3$ in the form $-2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the greatest value of $f(x)$. [1]

The function g is defined by $g(x) = 2x + 5$ for $x \in \mathbb{R}$.

(iii) Find the values of x for which $gf(x) + 1 = 0$. [3]

39. 9709_s19_qp_12 Q: 7

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 2, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{2x + 3}{x - 1}, \quad x \in \mathbb{R},\ x \neq 1.$$

(i) Obtain expressions for $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$, stating the value of x for which $\mathrm{g}^{-1}(x)$ is not defined. [4]

(ii) Solve the equation $\mathrm{fg}(x) = \frac{7}{3}$. [3]

40. 9709_s19_qp_13 Q: 4

The function f is defined by $f(x) = \frac{48}{x-1}$ for $3 \leqslant x \leqslant 7$. The function g is defined by $g(x) = 2x - 4$ for $a \leqslant x \leqslant b$, where a and b are constants.

(i) Find the greatest value of a and the least value of b which will permit the formation of the composite function gf. [2]

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It is now given that the conditions for the formation of gf are satisfied.

(ii) Find an expression for $gf(x)$. [1]

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(iii) Find an expression for $(gf)^{-1}(x)$. [2]

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41. 9709_w19_qp_11 Q: 7

Functions f and g are defined by

$$\mathrm{f} : x \mapsto \frac{3}{2x+1} \quad \text{for } x > 0,$$
$$\mathrm{g} : x \mapsto \frac{1}{x} + 2 \quad \text{for } x > 0.$$

(i) Find the range of f and the range of g. [3]

(ii) Find an expression for $\mathrm{fg}(x)$, giving your answer in the form $\dfrac{ax}{bx+c}$, where a, b and c are integers. [2]

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(iii) Find an expression for $(\mathrm{fg})^{-1}(x)$, giving your answer in the same form as for part **(ii)**. [3]

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42. 9709_w19_qp_13 Q: 2

The function g is defined by $g(x) = x^2 - 6x + 7$ for $x > 4$. By first completing the square, find an expression for $g^{-1}(x)$ and state the domain of g^{-1}. [5]

43. 9709_s18_qp_12 Q: 7

The function f is defined by $f : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants. [2]

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(ii) State the coordinates of the stationary point on the curve $y = f(x)$. [1]

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The function g is defined by $g : x \mapsto 7 - 2x^2 - 12x$ for $x \geqslant k$.

(iii) State the smallest value of k for which g has an inverse. [1]

(iv) For this value of k, find $g^{-1}(x)$. [3]

44. 9709_s18_qp_13 Q: 10

The one-one function f is defined by $f(x) = (x - 2)^2 + 2$ for $x \geqslant c$, where c is a constant.

(i) State the smallest possible value of c. [1]

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In parts **(ii)** and **(iii)** the value of c is 4.

(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(iii) Solve the equation $\mathrm{ff}(x) = 51$, giving your answer in the form $a + \sqrt{b}$. [5]

45. 9709_w18_qp_11 Q: 11

(a) The one-one function f is defined by $f(x) = (x - 3)^2 - 1$ for $x < a$, where a is a constant.

(i) State the greatest possible value of a. [1]

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(ii) It is given that a takes this greatest possible value. State the range of f and find an expression for $f^{-1}(x)$. [3]

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(b) The function g is defined by $g(x) = (x - 3)^2$ for $x \geqslant 0$.

(i) Show that $gg(2x)$ can be expressed in the form $(2x - 3)^4 + b(2x - 3)^2 + c$, where b and c are constants to be found. [2]

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(ii) Hence expand $gg(2x)$ completely, simplifying your answer. [4]

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46. 9709_w18_qp_12 Q: 9

The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the range of f. [1]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 7$ for $x \leqslant k$.

(iii) State the largest value of k for which g has an inverse. [1]

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(iv) Given that g has an inverse, find an expression for $g^{-1}(x)$. [3]

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47. 9709_w18_qp_13 Q: 11

(i) Express $2x^2 - 12x + 11$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

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The function f is defined by $f(x) = 2x^2 - 12x + 11$ for $x \leqslant k$.

(ii) State the largest value of the constant k for which f is a one-one function. [1]

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(iii) For this value of k find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $g(x) = x + 3$ for $x \leqslant p$.

(iv) With k now taking the value 1, find the largest value of the constant p which allows the composite function fg to be formed, and find an expression for $fg(x)$ whenever this composite function exists. [3]

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48. 9709_m17_qp_12 Q: 8

The functions f and g are defined for $x \geqslant 0$ by

$$\text{f} : x \mapsto 2x^2 + 3,$$
$$\text{g} : x \mapsto 3x + 2.$$

(i) Show that $\text{gf}(x) = 6x^2 + 11$ and obtain an unsimplified expression for $\text{fg}(x)$. [2]

(ii) Find an expression for $(\text{fg})^{-1}(x)$ and determine the domain of $(\text{fg})^{-1}$. [5]

(iii) Solve the equation $\mathrm{gf}(2x) = \mathrm{fg}(x)$. [3]

49. 9709_s17_qp_11 Q: 9

The function f is defined by $\mathrm{f} : x \mapsto \dfrac{2}{3-2x}$ for $x \in \mathbb{R},\ x \neq \frac{3}{2}$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

(ii) Find the value of a for which $gf(-1) = 3$. [3]

(iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

50. 9709_s17_qp_13 Q: 9

(i) Express $9x^2 - 6x + 6$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

The function f is defined by $f(x) = 9x^2 - 6x + 6$ for $x \geqslant p$, where p is a constant.

(ii) State the smallest value of p for which f is a one-one function. [1]

(iii) For this value of p, obtain an expression for $\mathrm{f}^{-1}(x)$, and state the domain of f^{-1}. [4]

(iv) State the set of values of q for which the equation $\mathrm{f}(x) = q$ has no solution. [1]

51. 9709_w17_qp_11 Q: 9

Functions f and g are defined for $x > 3$ by

$$\mathrm{f} : x \mapsto \frac{1}{x^2 - 9},$$
$$\mathrm{g} : x \mapsto 2x - 3.$$

(i) Find and simplify an expression for $\mathrm{gg}(x)$. [2]

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(ii) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

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(iii) Solve the equation $\mathrm{fg}(x) = \frac{1}{7}$. [4]

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52. 9709_w17_qp_12 Q: 2

A function f is defined by $f : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

(i) Find an expression for $f^{-1}(x)$ and find the point of intersection of the graphs of $y = f(x)$ and $y = f^{-1}(x)$. [3]

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(ii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

53. 9709_w17_qp_13 Q: 6

The functions f and g are defined by

$$\mathrm{f}(x) = \frac{2}{x^2 - 1} \text{ for } x < -1,$$
$$\mathrm{g}(x) = x^2 + 1 \text{ for } x > 0.$$

(i) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(ii) Solve the equation $\mathrm{gf}(x) = 5$. [4]

54. 9709_m16_qp_12 Q: 8

The function f is such that $f(x) = a^2x^2 - ax + 3b$ for $x \leqslant \frac{1}{2a}$, where a and b are constants.

(i) For the case where $f(-2) = 4a^2 - b + 8$ and $f(-3) = 7a^2 - b + 14$, find the possible values of a and b. [5]

(ii) For the case where $a = 1$ and $b = -1$, find an expression for $f^{-1}(x)$ and give the domain of f^{-1}. [5]

55. 9709_s16_qp_12 Q: 1

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R},\ x \neq \tfrac{3}{2}.$$

Solve the equation $\mathrm{ff}(x) = \mathrm{gf}(2)$. [3]

56. 9709_s16_qp_12 Q: 11

The function f is defined by $\mathrm{f} : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $\mathrm{f}(x) \leqslant 3$. [3]

(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = \mathrm{f}(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $\mathrm{g} : x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

(iv) State the smallest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{g}^{-1}(x)$. [2]

57. 9709_s16_qp_13 Q: 10

The function f is such that $f(x) = 2x + 3$ for $x \geqslant 0$. The function g is such that $g(x) = ax^2 + b$ for $x \leqslant q$, where a, b and q are constants. The function fg is such that $fg(x) = 6x^2 - 21$ for $x \leqslant q$.

(i) Find the values of a and b. [3]

(ii) Find the greatest possible value of q. [2]

It is now given that $q = -3$.

(iii) Find the range of fg. [1]

(iv) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [3]

58. 9709_w16_qp_11 Q: 8

The functions f and g are defined by

$$\mathrm{f}(x) = \frac{4}{x} - 2 \quad \text{for } x > 0,$$

$$\mathrm{g}(x) = \frac{4}{5x+2} \quad \text{for } x \geqslant 0.$$

(i) Find and simplify an expression for $\mathrm{fg}(x)$ and state the range of fg. [3]

(ii) Find an expression for $\mathrm{g}^{-1}(x)$ and find the domain of g^{-1}. [5]

59. 9709_w16_qp_13 Q: 8

(i) Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

(ii) Functions f and g are both defined for $x > 0$. It is given that $\mathrm{f}(x) = x^2 + 1$ and $\mathrm{fg}(x) = 4x^2 + 12x + 10$. Find $\mathrm{g}(x)$. [1]

(iii) Find $(\mathrm{fg})^{-1}(x)$ and give the domain of $(\mathrm{fg})^{-1}$. [4]

60. 9709_s15_qp_12 Q: 11

The function f is defined by $\mathrm{f} : x \mapsto 2x^2 - 6x + 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of p for which the equation $\mathrm{f}(x) = p$ has no real roots. [3]

The function g is defined by $\mathrm{g} : x \mapsto 2x^2 - 6x + 5$ for $0 \leqslant x \leqslant 4$.

(ii) Express $\mathrm{g}(x)$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

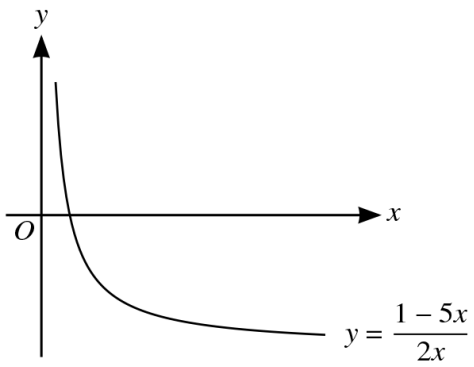
(iii) Find the range of g. [2]

The function h is defined by $\mathrm{h} : x \mapsto 2x^2 - 6x + 5$ for $k \leqslant x \leqslant 4$, where k is a constant.

(iv) State the smallest value of k for which h has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{h}^{-1}(x)$. [3]

61. 9709_s15_qp_13 Q: 6



The diagram shows the graph of $y = \mathrm{f}^{-1}(x)$, where f^{-1} is defined by $\mathrm{f}^{-1}(x) = \frac{1 - 5x}{2x}$ for $0 < x \leqslant 2$.

(i) Find an expression for $\mathrm{f}(x)$ and state the domain of f. [5]

(ii) The function g is defined by $\mathrm{g}(x) = \frac{1}{x}$ for $x \geqslant 1$. Find an expression for $\mathrm{f}^{-1}\mathrm{g}(x)$, giving your answer in the form $ax + b$, where a and b are constants to be found. [2]

62. 9709_w15_qp_11 Q: 9

(i) Express $-x^2 + 6x - 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

The function $\mathrm{f} : x \mapsto -x^2 + 6x - 5$ is defined for $x \geqslant m$, where m is a constant.

(ii) State the smallest value of m for which f is one-one. [1]

(iii) For the case where $m = 5$, find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

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63. 9709_w15_qp_12 Q: 1

Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x + 2, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 4x - 12, \quad x \in \mathbb{R}.$$

Solve the equation $\mathrm{f}^{-1}(x) = \mathrm{gf}(x)$. [4]

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64. 9709_w15_qp_12 Q: 8

The function f is defined, for $x \in \mathbb{R}$, by $f : x \mapsto x^2 + ax + b$, where a and b are constants.

(i) In the case where $a = 6$ and $b = -8$, find the range of f. [3]

(ii) In the case where $a = 5$, the roots of the equation $f(x) = 0$ are k and $-2k$, where k is a constant. Find the values of b and k. [3]

(iii) Show that if the equation $f(x + a) = a$ has no real roots, then $a^2 < 4(b - a)$. [3]

65. 9709_w15_qp_13 Q: 8

The function f is defined by $f(x) = 3x + 1$ for $x \leqslant a$, where a is a constant. The function g is defined by $g(x) = -1 - x^2$ for $x \leqslant -1$.

(i) Find the largest value of a for which the composite function gf can be formed. [2]

For the case where $a = -1$,

(ii) solve the equation $fg(x) + 14 = 0$, [3]

(iii) find the set of values of x which satisfy the inequality $gf(x) \leqslant -50$. [4]