Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 1 [Pure Mathematics 1]

May/June 2015 – February/March 2022

Chapter 5

Trigonometry

180. 9709_m22_qp_12 Q: 7

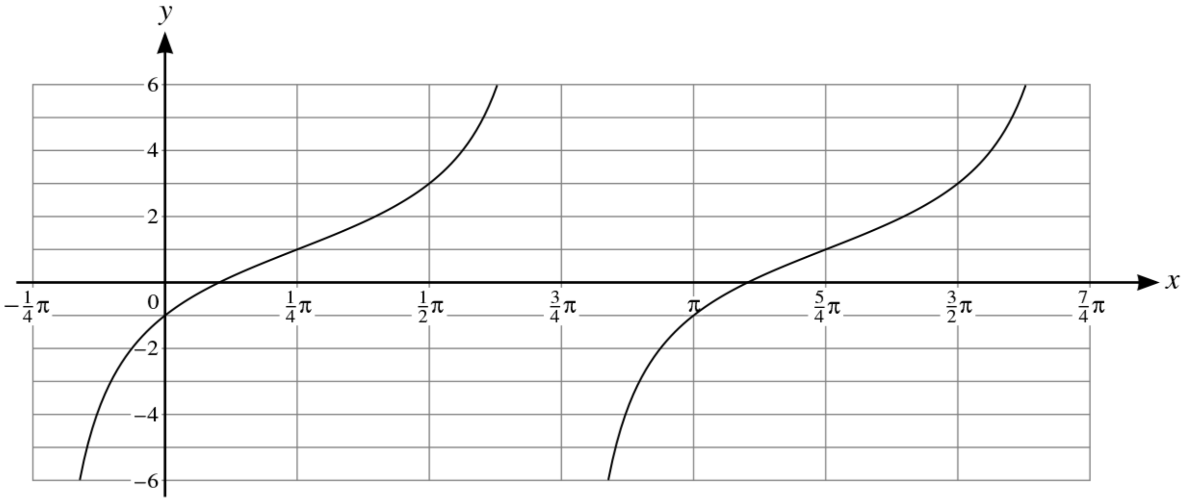
(a) Show that $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} \equiv \dfrac{4}{5\cos^2\theta - 4}$. [4]

(b) Hence solve the equation $\dfrac{\sin\theta + 2\cos\theta}{\cos\theta - 2\sin\theta} - \dfrac{\sin\theta - 2\cos\theta}{\cos\theta + 2\sin\theta} = 5$ for $0° < \theta < 180°$. [3]

181. 9709_m21_qp_12 Q: 3

Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

182. 9709_s21_qp_11 Q: 4



The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a, b and c. [3]

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183. 9709_s21_qp_11 Q: 7

(a) Prove the identity $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} \equiv 1 - \tan^2\theta$. [2]

(b) Hence solve the equation $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

184. 9709_s21_qp_12 Q: 10

(a) Prove the identity $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} \equiv \frac{4\tan x}{\cos x}$. [4]

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(b) Hence solve the equation $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

185. 9709_s21_qp_13 Q: 4

(a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k.$$ [2]

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(b) Hence express $\cos x$ in terms of k. [2]

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(c) Hence solve the equation $\frac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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186. 9709_w21_qp_11 Q: 3

Solve, by factorising, the equation

$$6\cos\theta\tan\theta - 3\cos\theta + 4\tan\theta - 2 = 0,$$

for $0° \leqslant \theta \leqslant 180°$. [4]

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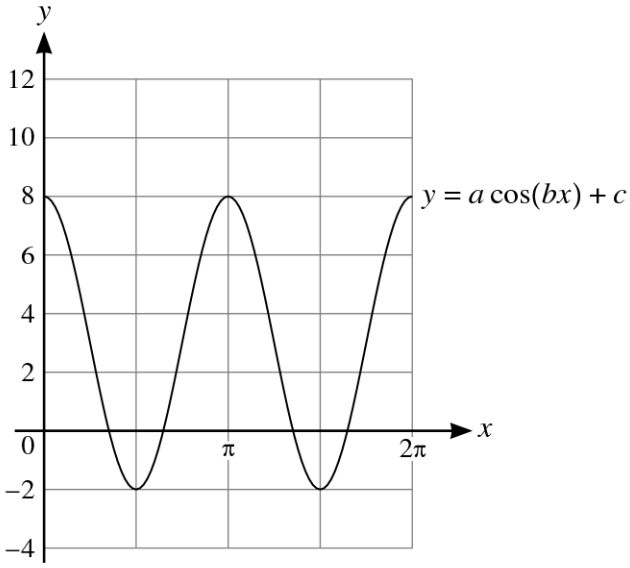
187. 9709_w21_qp_11 Q: 5



The diagram shows part of the graph of $y = a\cos(bx) + c$.

(a) Find the values of the positive integers a, b and c. [3]

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(b) For these values of a, b and c, use the given diagram to determine the number of solutions in the interval $0 \leqslant x \leqslant 2\pi$ for each of the following equations.

(i) $a\cos(bx) + c = \frac{6}{\pi}x$ [1]

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(ii) $a\cos(bx) + c = 6 - \frac{6}{\pi}x$ [1]

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188. 9709_w21_qp_12 Q: 1

Solve the equation $2\cos\theta = 7 - \dfrac{3}{\cos\theta}$ for $-90^\circ < \theta < 90^\circ$. [4]

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189. 9709_w21_qp_13 Q: 7

(a) Show that the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = k$, where k is a constant, can be expressed as

$$(k+1)\sin^2 x + (k-1)\sin x - (k+1) = 0.$$ [4]

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(b) Hence solve the equation $\dfrac{\tan x + \cos x}{\tan x - \cos x} = 4$ for $0° \leqslant x \leqslant 360°$. [4]

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190. 9709_m20_qp_12 Q: 5

Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

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191. 9709_m20_qp_12 Q: 11

(a) Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

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(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$, and find these solutions. [3]

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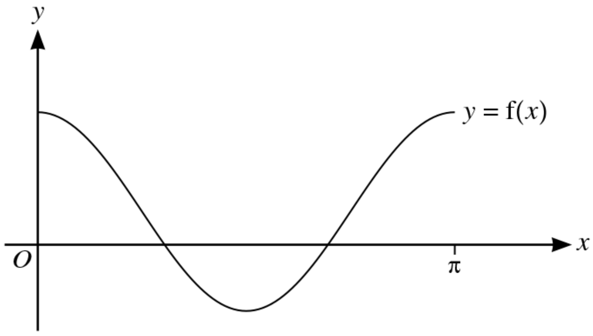
192. 9709_s20_qp_11 Q: 4



The diagram shows the graph of $y = \text{f}(x)$, where $\text{f}(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

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A function g is such that $\text{g}(x) = \text{f}(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = \text{g}(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = \text{f}(x)$ on to $y = \text{g}(x)$. [2]

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(c) State the equation of the curve which is the reflection of $y = \text{f}(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

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193. 9709_s20_qp_11 Q: 7

(a) Prove the identity $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}\equiv\dfrac{2}{\cos\theta}$. [3]

(b) Hence solve the equation $\dfrac{1+\sin\theta}{\cos\theta}+\dfrac{\cos\theta}{1+\sin\theta}=\dfrac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

194. 9709_s20_qp_12 Q: 2

(a) Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

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(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

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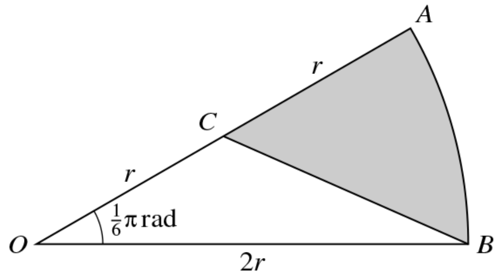
195. 9709_s20_qp_12 Q: 7



In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

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(b) Find the exact perimeter of the shaded region. [2]

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(c) Find the exact area of the shaded region. [3]

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196. 9709_s20_qp_12 Q: 9

Functions f and g are such that

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

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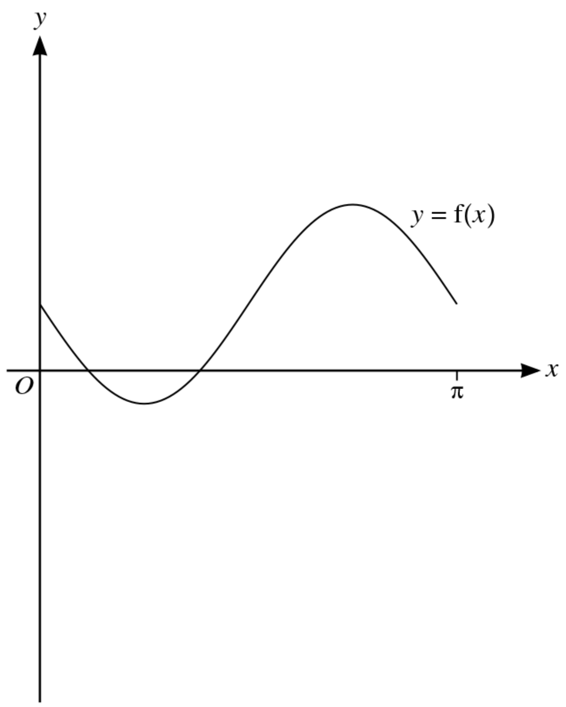
The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

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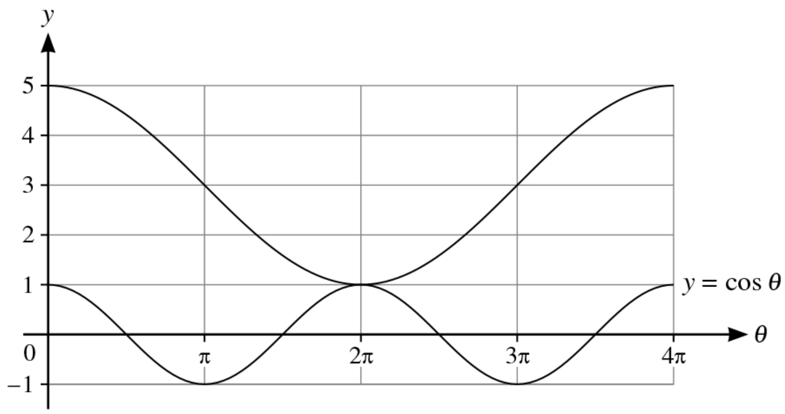
197. 9709_s20_qp_13 Q: 7

(a) Show that $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} \equiv \dfrac{2}{\sin\theta\cos\theta}$. [4]

(b) Hence solve the equation $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

198. 9709_w20_qp_11 Q: 4



In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

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199. 9709_w20_qp_11 Q: 7

(a) Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

(b) Hence solve the equation $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

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200. 9709_w20_qp_12 Q: 6

(a) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

201. 9709_w20_qp_12 Q: 11

A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

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(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

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(ii) $k = 1$ [1]

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(iii) $k = 3$ [1]

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Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

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(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

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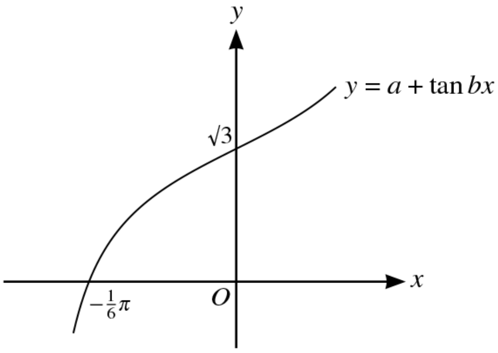
202. 9709_w20_qp_13 Q: 3

Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

203. 9709_m19_qp_12 Q: 7

(a) Solve the equation $3\sin^2 2\theta + 8\cos 2\theta = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

(b)



The diagram shows part of the graph of $y = a + \tan bx$, where x is measured in radians and a and b are constants. The curve intersects the x-axis at $\left(-\frac{1}{6}\pi,\ 0\right)$ and the y-axis at $(0,\ \sqrt{3})$. Find the values of a and b. [3]

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204. 9709_s19_qp_11 Q: 6

(i) Prove the identity $\left(\dfrac{1}{\cos x} - \tan x\right)^2 \equiv \dfrac{1 - \sin x}{1 + \sin x}$. [4]

(ii) Hence solve the equation $\left(\dfrac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leqslant x \leqslant \pi$. [3]

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205. 9709_s19_qp_11 Q: 9

The function f is defined by $f(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant 2\pi$.

(i) State the range of f. [2]

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(ii) Sketch the graph of $y = f(x)$. [2]

The function g is defined by $g(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$, where p is a constant.

(iii) State the largest value of p for which g has an inverse. [1]

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(iv) For this value of p, find an expression for $g^{-1}(x)$. [2]

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206. 9709_s19_qp_12 Q: 4

Angle x is such that $\sin x = a + b$ and $\cos x = a - b$, where a and b are constants.

(i) Show that $a^2 + b^2$ has a constant value for all values of x. [3]

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(ii) In the case where $\tan x = 2$, express a in terms of b. [2]

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207. 9709_s19_qp_12 Q: 6

The equation of a curve is $y = 3\cos 2x$ and the equation of a line is $2y + \frac{3x}{\pi} = 5$.

(i) State the smallest and largest values of y for both the curve and the line for $0 \leqslant x \leqslant 2\pi$. [3]

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(ii) Sketch, on the same diagram, the graphs of $y = 3\cos 2x$ and $2y + \frac{3x}{\pi} = 5$ for $0 \leqslant x \leqslant 2\pi$. [3]

(iii) State the number of solutions of the equation $6\cos 2x = 5 - \frac{3x}{\pi}$ for $0 \leqslant x \leqslant 2\pi$. [1]

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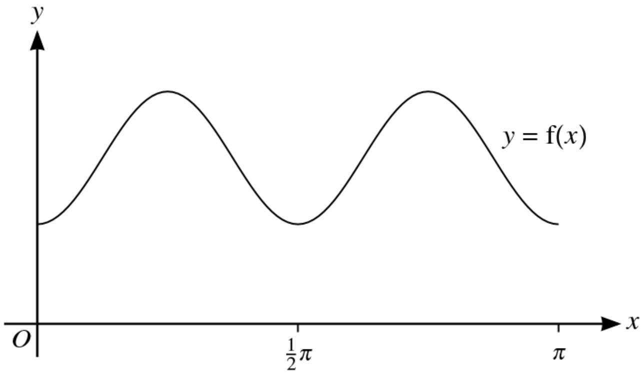
208. 9709_s19_qp_13 Q: 9



The function $f : x \mapsto p\sin^2 2x + q$ is defined for $0 \leqslant x \leqslant \pi$, where p and q are positive constants. The diagram shows the graph of $y = f(x)$.

(i) In terms of p and q, state the range of f. [2]

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(ii) State the number of solutions of the following equations.

(a) $f(x) = p + q$ [1]

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(b) $f(x) = q$ [1]

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(c) $f(x) = \frac{1}{2}p + q$ [1]

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(iii) For the case where $p = 3$ and $q = 2$, solve the equation $f(x) = 4$, showing all necessary working. [5]

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209. 9709_w19_qp_11 Q: 5

(i) Given that $4\tan x + 3\cos x + \dfrac{1}{\cos x} = 0$, show, without using a calculator, that $\sin x = -\frac{2}{3}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$4\tan(2x - 20^\circ) + 3\cos(2x - 20^\circ) + \frac{1}{\cos(2x - 20^\circ)} = 0$$

for $0^\circ \leqslant x \leqslant 180^\circ$. [4]

210. 9709_w19_qp_12 Q: 6

(a) Given that $x > 0$, find the two smallest values of x, in radians, for which $3\tan(2x + 1) = 1$. Show all necessary working. [4]

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(b) The function $f : x \mapsto 3\cos^2 x - 2\sin^2 x$ is defined for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a\cos^2 x + b$, where a and b are constants. [1]

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(ii) Find the range of f. [2]

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211. 9709_w19_qp_13 Q: 7

(i) Show that the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ can be expressed as $3x^2 - 4x + 1 = 0$, where $x = \cos^2\theta$. [2]

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(ii) Hence solve the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

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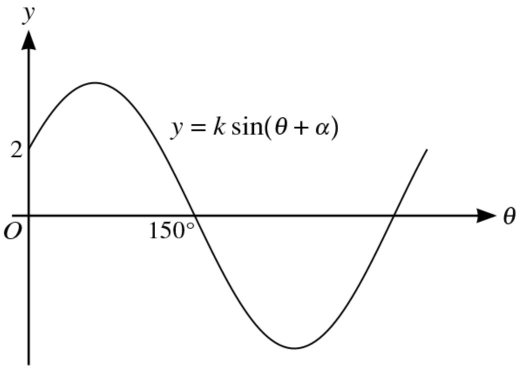
212. 9709_m18_qp_12 Q: 5

(a) Express the equation $\frac{5 + 2\tan x}{3 + 2\tan x} = 1 + \tan x$ as a quadratic equation in $\tan x$ and hence solve the equation for $0 \leqslant x \leqslant \pi$. [4]

(b)



The diagram shows part of the graph of $y = k\sin(\theta + \alpha)$, where k and α are constants and $0° < \alpha < 180°$. Find the value of α and the value of k. [2]

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213. 9709_s18_qp_11 Q: 4

(i) Prove the identity $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$. [3]

(ii) Hence solve the equation $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta$ for $0° \leqslant \theta \leqslant 360°$. [3]

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214. 9709_s18_qp_12 Q: 4

The function f is such that $f(x) = a + b\cos x$ for $0 \leqslant x \leqslant 2\pi$. It is given that $f\left(\frac{1}{3}\pi\right) = 5$ and $f(\pi) = 11$.

(i) Find the values of the constants a and b. [3]

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(ii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [3]

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215. 9709_s18_qp_12 Q: 10

(i) Solve the equation $2\cos x + 3\sin x = 0$, for $0° \leqslant x \leqslant 360°$. [3]

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(ii) Sketch, on the same diagram, the graphs of $y = 2\cos x$ and $y = -3\sin x$ for $0° \leqslant x \leqslant 360°$. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to find the set of values of x for $0° \leqslant x \leqslant 360°$ for which $2\cos x + 3\sin x > 0$. [2]

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216. 9709_s18_qp_13 Q: 7

(a) **(i)** Express $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1}$ in the form $a\sin^2\theta + b$, where a and b are constants to be found. [3]

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(ii) Hence, or otherwise, and showing all necessary working, solve the equation

$$\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{1}{4}$$

for $-90° \leqslant \theta \leqslant 0°$. [2]

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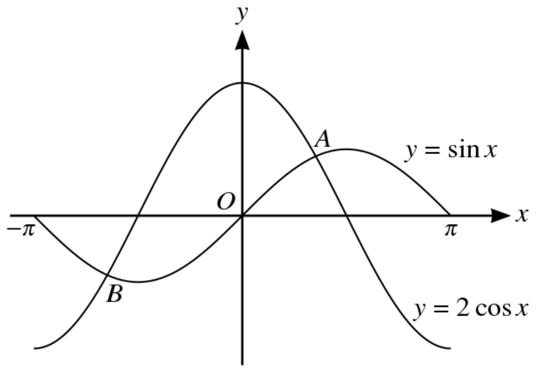
(b)



The diagram shows the graphs of $y = \sin x$ and $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$. The graphs intersect at the points A and B.

(i) Find the x-coordinate of A. [2]

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(ii) Find the y-coordinate of B. [2]

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217. 9709_w18_qp_11 Q: 5

(i) Show that the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

may be expressed as $9\cos^2\theta - 22\cos\theta + 4 = 0$. [3]

(ii) Hence solve the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

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218. 9709_w18_qp_12 Q: 4

Functions f and g are defined by

$$f : x \mapsto 2 - 3\cos x \quad \text{for } 0 \leqslant x \leqslant 2\pi,$$

$$g : x \mapsto \tfrac{1}{2}x \quad \text{for } 0 \leqslant x \leqslant 2\pi.$$

(i) Solve the equation $fg(x) = 1$. [3]

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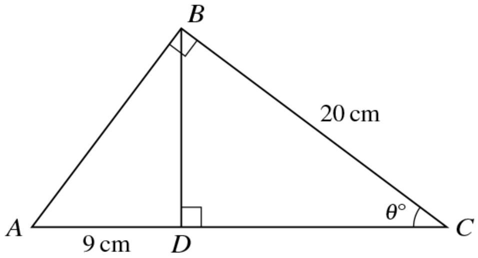
(ii) Sketch the graph of $y = f(x)$. [3]

219. 9709_w18_qp_12 Q: 6



The diagram shows a triangle ABC in which $BC = 20$ cm and angle $ABC = 90°$. The perpendicular from B to AC meets AC at D and $AD = 9$ cm. Angle $BCA = \theta°$.

(i) By expressing the length of BD in terms of θ in each of the triangles ABD and DBC, show that $20\sin^2\theta = 9\cos\theta$. [4]

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(ii) Hence, showing all necessary working, calculate θ. [3]

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220. 9709_w18_qp_13 Q: 7

(i) Show that $\dfrac{\tan\theta + 1}{1 + \cos\theta} + \dfrac{\tan\theta - 1}{1 - \cos\theta} \equiv \dfrac{2(\tan\theta - \cos\theta)}{\sin^2\theta}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$\frac{\tan\theta + 1}{1 + \cos\theta} + \frac{\tan\theta - 1}{1 - \cos\theta} = 0$$

for $0° < \theta < 90°$. [4]

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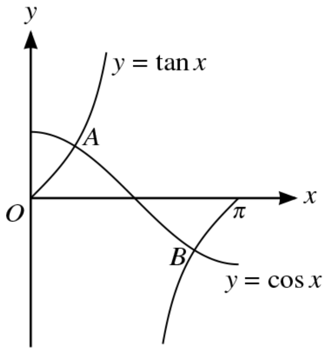
221. 9709_m17_qp_12 Q: 5



The diagram shows the graphs of $y = \tan x$ and $y = \cos x$ for $0 \leqslant x \leqslant \pi$. The graphs intersect at points A and B.

(i) Find by calculation the x-coordinate of A. [4]

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(ii) Find by calculation the coordinates of B. [3]

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222. 9709_s17_qp_11 Q: 3

(i) Prove the identity $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} \equiv \dfrac{2}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{3}{\cos\theta}$ for $0° \leqslant \theta \leqslant 360°$. [3]

223. 9709_s17_qp_11 Q: 5

The equation of a curve is $y = 2\cos x$.

(i) Sketch the graph of $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$, stating the coordinates of the point of intersection with the y-axis. [2]

Points P and Q lie on the curve and have x-coordinates of $\frac{1}{3}\pi$ and π respectively.

(ii) Find the length of PQ correct to 1 decimal place. [2]

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The line through P and Q meets the x-axis at $H(h, 0)$ and the y-axis at $K(0, k)$.

(iii) Show that $h = \frac{5}{9}\pi$ and find the value of k. [3]

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224. 9709_s17_qp_12 Q: 3

(i) Prove the identity $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \frac{1 - \sin\theta}{1 + \sin\theta}$. [3]

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(ii) Hence solve the equation $\left(\dfrac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$, for $0° \leqslant \theta \leqslant 360°$. [3]

225. 9709_s17_qp_12 Q: 10

The function f is defined by $f(x) = 3\tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

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(ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1}. [5]

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(iii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [3]

226. 9709_s17_qp_13 Q: 5

(i) Show that the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ may be expressed as $\cos^2\theta = 2\sin^2\theta$. [3]

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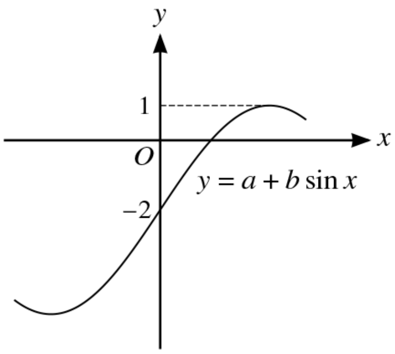
(ii) Hence solve the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ for $0^\circ < \theta < 180^\circ$. [3]

227. 9709_w17_qp_11 Q: 7



The diagram shows part of the graph of $y = a + b\sin x$. Find the values of the constants a and b. [2]

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(b) **(i)** Show that the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

may be expressed as $3\cos^2\theta - 2\cos\theta - 1 = 0$. [3]

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(ii) Hence solve the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

for $-180° \leqslant \theta \leqslant 180°$. [4]

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228. 9709_w17_qp_12 Q: 5

(i) Show that the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ can be expressed as

$$2\cos^2 2x + 3\cos 2x + 1 = 0.$$ [3]

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(ii) Hence solve the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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229. 9709_w17_qp_12 Q: 6

(a) The function f, defined by $f : x \mapsto a + b\sin x$ for $x \in \mathbb{R}$, is such that $f\left(\frac{1}{6}\pi\right) = 4$ and $f\left(\frac{1}{2}\pi\right) = 3$.

(i) Find the values of the constants a and b. [3]

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(ii) Evaluate ff(0). [2]

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(b) The function g is defined by $g : x \mapsto c + d \sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leqslant g(x) \leqslant 10$. Find the values of the constants c and d. [3]

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230. 9709_w17_qp_13 Q: 5

(i) Show that the equation $\dfrac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ may be expressed as $5\cos^2\theta - \cos\theta - 4 = 0$. [3]

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(ii) Hence solve the equation $\dfrac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ for $0^\circ \leqslant \theta \leqslant 360^\circ$. [4]

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231. 9709_m16_qp_12 Q: 4

(a) Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

(b) Solve, by factorising, the equation $2\cos\theta\sin\theta - 2\cos\theta - \sin\theta + 1 = 0$ for $0 \leqslant \theta \leqslant \pi$. [4]

232. 9709_s16_qp_11 Q: 2

Solve the equation $3\sin^2\theta = 4\cos\theta - 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

233. 9709_s16_qp_11 Q: 11

The function f is defined by $\mathrm{f} : x \mapsto 4\sin x - 1$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

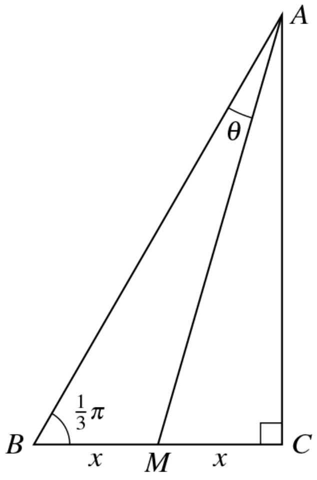
(i) State the range of f. [2]

(ii) Find the coordinates of the points at which the curve $y = \mathrm{f}(x)$ intersects the coordinate axes. [3]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iv) Obtain an expression for $\mathrm{f}^{-1}(x)$, stating both the domain and range of f^{-1}. [4]

234. 9709_s16_qp_12 Q: 5



In the diagram, triangle ABC is right-angled at C and M is the mid-point of BC. It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of BM and MC by x,

(i) find AM in terms of x, [3]

(ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$. [2]

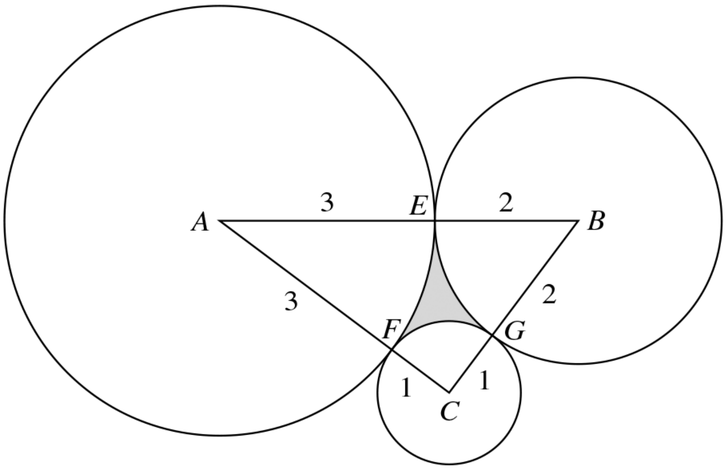
235. 9709_s16_qp_12 Q: 7

(i) Prove the identity $\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta} \equiv \frac{4}{\sin\theta\tan\theta}$. [4]

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1+\cos\theta}{1-\cos\theta} - \frac{1-\cos\theta}{1+\cos\theta}\right) = 3.$$ [3]

236. 9709_s16_qp_13 Q: 6



The diagram shows triangle ABC where $AB = 5$ cm, $AC = 4$ cm and $BC = 3$ cm. Three circles with centres at A, B and C have radii 3 cm, 2 cm and 1 cm respectively. The circles touch each other at points E, F and G, lying on AB, AC and BC respectively. Find the area of the shaded region EFG.

[7]

237. 9709_s16_qp_13 Q: 8

(i) Show that $3\sin x\tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3\sin x\tan x - \cos x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [5]

(ii) Find the solutions to the equation $3\sin 2x\tan 2x - \cos 2x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [3]

238. 9709_w16_qp_11 Q: 6

(i) Show that $\cos^4 x \equiv 1 - 2\sin^2 x + \sin^4 x$. [1]

(ii) Hence, or otherwise, solve the equation $8\sin^4 x + \cos^4 x = 2\cos^2 x$ for $0° \leqslant x \leqslant 360°$. [5]

239. 9709_w16_qp_12 Q: 2

(i) Express the equation $\sin 2x + 3\cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]

(ii) Hence solve the equation for $-90° \leqslant x \leqslant 90°$. [3]

240. 9709_w16_qp_12 Q: 10

A function f is defined by $f : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Solve the equation $f(x) = 6$, giving answers in terms of π. [3]

The function g is defined by $g : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $g^{-1}(x)$. [3]

241. 9709_w16_qp_13 Q: 3

Showing all necessary working, solve the equation $6\sin^2 x - 5\cos^2 x = 2\sin^2 x + \cos^2 x$ for $0° \leqslant x \leqslant 360°$. [4]

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242. 9709_s15_qp_11 Q: 1

Given that θ is an obtuse angle measured in radians and that $\sin\theta = k$, find, in terms of k, an expression for

(i) $\cos\theta$, [1]

(ii) $\tan\theta$, [2]

(iii) $\sin(\theta + \pi)$. [1]

243. 9709_s15_qp_11 Q: 8

The function $f : x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ is defined for $0 \leqslant x \leqslant 2\pi$.

(i) Solve the equation $f(x) = 7$, giving your answer correct to 2 decimal places. [3]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Explain why f has an inverse. [1]

(iv) Obtain an expression for $f^{-1}(x)$. [3]

244. 9709_s15_qp_12 Q: 1

The function f is such that $f'(x) = 5 - 2x^2$ and $(3, 5)$ is a point on the curve $y = f(x)$. Find $f(x)$. [3]

245. 9709_s15_qp_12 Q: 8

(a) The first, second and last terms in an arithmetic progression are 56, 53 and −22 respectively. Find the sum of all the terms in the progression. [4]

(b) The first, second and third terms of a geometric progression are $2k + 6$, $2k$ and $k + 2$ respectively, where k is a positive constant.

(i) Find the value of k. [3]

(ii) Find the sum to infinity of the progression. [2]

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246. 9709_s15_qp_13 Q: 4

(i) Express the equation $3 \sin \theta = \cos \theta$ in the form $\tan \theta = k$ and solve the equation for $0° < \theta < 180°$. [2]

(ii) Solve the equation $3 \sin^2 2x = \cos^2 2x$ for $0° < x < 180°$. [4]

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247. 9709_w15_qp_11 Q: 3

Solve the equation $\sin^{-1}(4x^4 + x^2) = \frac{1}{6}\pi$. [4]

248. 9709_w15_qp_11 Q: 4

(i) Show that the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ can be expressed as

$$4\sin^2\theta - 15\sin\theta - 4 = 0.$$ [3]

(ii) Hence solve the equation $\frac{4\cos\theta}{\tan\theta} + 15 = 0$ for $0° \leqslant \theta \leqslant 360°$. [3]

249. 9709_w15_qp_12 Q: 4

(i) Prove the identity $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 \equiv \frac{1 - \cos x}{1 + \cos x}$. [4]

(ii) Hence solve the equation $\left(\frac{1}{\sin x} - \frac{1}{\tan x}\right)^2 = \frac{2}{5}$ for $0 \leqslant x \leqslant 2\pi$. [3]

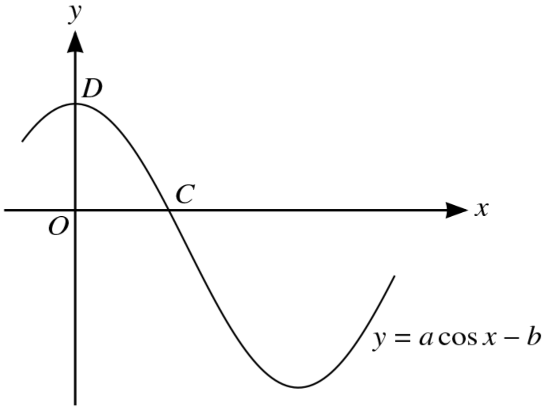
250. 9709_w15_qp_13 Q: 7

(a) Show that the equation $\dfrac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ can be expressed as

$$3\cos^2\theta - 4\cos\theta - 4 = 0,$$

and hence solve the equation $\dfrac{1}{\cos\theta} + 3\sin\theta\tan\theta + 4 = 0$ for $0° \leqslant \theta \leqslant 360°$. [6]

(b)



The diagram shows part of the graph of $y = a\cos x - b$, where a and b are constants. The graph crosses the x-axis at the point $C\,(\cos^{-1} c,\ 0)$ and the y-axis at the point $D\,(0,\ d)$. Find c and d in terms of a and b. [2]