Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 5

[Probability & Statistics 1]

Exam Series: May 2015 – May 2022

Format Type A:

Answers to all questions are provided as an appendix

Chapter 4

Discrete random variables

193. 9709_m22_qp_52 Q: 1

A fair red spinner has edges numbered 1, 2, 2, 3. A fair blue spinner has edges numbered −3, −2, −1, −1. Each spinner is spun once and the number on the edge on which each spinner lands is noted. The random variable X denotes the sum of the resulting two numbers.

(a) Draw up the probability distribution table for X. [3]

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(b) Given that $E(X) = 0.25$, find the value of $Var(X)$. [2]

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194. 9709_m22_qp_52 Q: 2

In a certain country, the probability of more than 10 cm of rain on any particular day is 0.18, independently of the weather on any other day.

(a) Find the probability that in any randomly chosen 7-day period, more than 2 days have more than 10 cm of rain. [3]

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(b) For 3 randomly chosen 7-day periods, find the probability that exactly two of these periods have at least one day with more than 10 cm of rain. [3]

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195. 9709_m22_qp_52 Q: 6

A factory produces chocolates in three flavours: lemon, orange and strawberry in the ratio 3 : 5 : 7 respectively. Nell checks the chocolates on the production line by choosing chocolates randomly one at a time.

(a) Find the probability that the first chocolate with lemon flavour that Nell chooses is the 7th chocolate that she checks. [1]

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(b) Find the probability that the first chocolate with lemon flavour that Nell chooses is after she has checked at least 6 chocolates. [2]

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'Surprise' boxes of chocolates each contain 15 chocolates: 3 are lemon, 5 are orange and 7 are strawberry.

Petra has a box of Surprise chocolates. She chooses 3 chocolates at random from the box. She eats each chocolate before choosing the next one.

(c) Find the probability that none of Petra's 3 chocolates has orange flavour. [2]

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(d) Find the probability that each of Petra's 3 chocolates has a different flavour. [3]

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(e) Find the probability that at least 2 of Petra's 3 chocolates have strawberry flavour given that none of them has orange flavour. [4]

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196. 9709_s22_qp_51 Q: 4

Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable X. The probability distribution table for X is as follows.

x	0	1	2	3	4
$P(X = x)$	$\frac{3}{80}$	a	b	c	$\frac{7}{80}$

(a) Show that $a = \frac{1}{5}$ and find the values of b and c. [4]

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(b) Find $E(X)$. [1]

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Jacob throws all four coins together 10 times.

(c) Find the probability that he obtains exactly one head on fewer than 3 occasions. [3]

(d) Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. [2]

197. 9709_s22_qp_52 Q: 2

A fair 6-sided die has the numbers 1, 2, 2, 3, 3, 3 on its faces. The die is rolled twice. The random variable X denotes the sum of the two numbers obtained.

(a) Draw up the probability distribution table for X. [3]

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(b) Find $E(X)$ and $Var(X)$. [3]

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198. 9709_s22_qp_53 Q: 3

The random variable X takes the values -2, 1, 2, 3. It is given that $P(X = x) = kx^2$, where k is a constant.

(a) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [3]

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(b) Find $E(X)$ and $Var(X)$. [3]

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199. 9709_s22_qp_53 Q: 4

Ramesh throws an ordinary fair 6-sided die.

(a) Find the probability that he obtains a 4 for the first time on his 8th throw. [1]

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(b) Find the probability that it takes no more than 5 throws for Ramesh to obtain a 4. [2]

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Ramesh now repeatedly throws two ordinary fair 6-sided dice at the same time. Each time he adds the two numbers that he obtains.

(c) For 10 randomly chosen throws of the two dice, find the probability that Ramesh obtains a total of less than 4 on at least three throws. [4]

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200. 9709_m21_qp_52 Q: 1

A fair spinner with 5 sides numbered 1, 2, 3, 4, 5 is spun repeatedly. The score on each spin is the number on the side on which the spinner lands.

(a) Find the probability that a score of 3 is obtained for the first time on the 8th spin. [1]

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(b) Find the probability that fewer than 6 spins are required to obtain a score of 3 for the first time. [2]

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201. 9709_m21_qp_52 Q: 4

The random variable X takes the values 1, 2, 3, 4 only. The probability that X takes the value x is $kx(5-x)$, where k is a constant.

(a) Draw up the probability distribution table for X, in terms of k. [2]

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(b) Show that $\text{Var}(X) = 1.05$. [4]

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202. 9709_s21_qp_51 Q: 7

Sharma knows that she has 3 tins of carrots, 2 tins of peas and 2 tins of sweetcorn in her cupboard. All the tins are the same shape and size, but the labels have all been removed, so Sharma does not know what each tin contains.

Sharma wants carrots for her meal, and she starts opening the tins one at a time, chosen randomly, until she opens a tin of carrots. The random variable X is the number of tins that she needs to open.

(a) Show that $P(X = 3) = \frac{6}{35}$. [2]

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(b) Draw up the probability distribution table for X. [4]

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(c) Find Var(X). [3]

203. 9709_s21_qp_52 Q: 1

An ordinary fair die is thrown repeatedly until a 5 is obtained. The number of throws taken is denoted by the random variable X.

(a) Write down the mean of X. [1]

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(b) Find the probability that a 5 is first obtained after the 3rd throw but before the 8th throw. [2]

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(c) Find the probability that a 5 is first obtained in fewer than 10 throws. [2]

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204. 9709_s21_qp_52 Q: 4

A fair spinner has sides numbered 1, 2, 2. Another fair spinner has sides numbered −2, 0, 1. Each spinner is spun. The number on the side on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

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(b) Find $E(X)$ and $Var(X)$. [3]

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205. 9709_s21_qp_53 Q: 2

The random variable X can take only the values −2, −1, 0, 1, 2. The probability distribution of X is given in the following table.

x	−2	−1	0	1	2
$P(X = x)$	p	p	0.1	q	q

Given that $P(X \geqslant 0) = 3P(X < 0)$, find the values of p and q. [4]

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206. 9709_s21_qp_53 Q: 4

Three fair six-sided dice, each with faces marked 1, 2, 3, 4, 5, 6, are thrown at the same time, repeatedly. For a single throw of the three dice, the score is the sum of the numbers on the top faces.

(a) Find the probability that the score is 4 on a single throw of the three dice. [3]

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(b) Find the probability that a score of 18 is obtained for the first time on the 5th throw of the three dice. [3]

207. 9709_w21_qp_51 Q: 1

Two fair coins are thrown at the same time. The random variable X is the number of throws of the two coins required to obtain two tails at the same time.

(a) Find the probability that two tails are obtained for the first time on the 7th throw. [2]

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(b) Find the probability that it takes more than 9 throws to obtain two tails for the first time. [2]

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208. 9709_w21_qp_51 Q: 4

A fair spinner has edges numbered 0, 1, 2, 2. Another fair spinner has edges numbered −1, 0, 1. Each spinner is spun. The number on the edge on which a spinner comes to rest is noted. The random variable X is the sum of the numbers for the two spinners.

(a) Draw up the probability distribution table for X. [3]

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(b) Find $\text{Var}(X)$. [3]

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209. 9709_w21_qp_52 Q: 3

A bag contains 5 yellow and 4 green marbles. Three marbles are selected at random from the bag, without replacement.

(a) Show that the probability that exactly one of the marbles is yellow is $\frac{5}{14}$. [3]

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The random variable X is the number of yellow marbles selected.

(b) Draw up the probability distribution table for X. [3]

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(c) Find $E(X)$. [1]

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210. 9709_w21_qp_52 Q: 5

In a certain region, the probability that any given day in October is wet is 0.16, independently of other days.

(a) Find the probability that, in a 10-day period in October, fewer than 3 days will be wet. [3]

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(b) Find the probability that the first wet day in October is 8 October. [2]

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(c) For 4 randomly chosen years, find the probability that in exactly 1 of these years the first wet day in October is 8 October. [2]

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211. 9709_w21_qp_53 Q: 6

In a game, Jim throws three darts at a board. This is called a 'turn'. The centre of the board is called the bull's-eye.

The random variable X is the number of darts in a turn that hit the bull's-eye. The probability distribution of X is given in the following table.

x	0	1	2	3
$P(X = x)$	0.6	p	q	0.05

It is given that $E(X) = 0.55$.

(a) Find the values of p and q. [4]

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(b) Find $\text{Var}(X)$. [2]

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Jim is practising for a competition and he repeatedly throws three darts at the board.

(c) Find the probability that $X = 1$ in at least 3 of 12 randomly chosen turns. [3]

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(d) Find the probability that Jim first succeeds in hitting the bull's-eye with all three darts on his 9th turn. [1]

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212. 9709_m20_qp_52 Q: 2

An ordinary fair die is thrown repeatedly until a 1 or a 6 is obtained.

(a) Find the probability that it takes at least 3 throws but no more than 5 throws to obtain a 1 or a 6. [3]

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On another occasion, this die is thrown 3 times. The random variable X is the number of times that a 1 or a 6 is obtained.

(b) Draw up the probability distribution table for X. [3]

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(c) Find $E(X)$. [2]

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213. 9709_s20_qp_51 Q: 1

The score when two fair six-sided dice are thrown is the sum of the two numbers on the upper faces.

(a) Show that the probability that the score is 4 is $\frac{1}{12}$. [1]

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The two dice are thrown repeatedly until a score of 4 is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the mean of X. [1]

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(c) Find the probability that a score of 4 is first obtained on the 6th throw. [1]

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(d) Find $P(X < 8)$. [2]

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214. 9709_s20_qp_51 Q: 3

A company produces small boxes of sweets that contain 5 jellies and 3 chocolates. Jemeel chooses 3 sweets at random from a box.

(a) Draw up the probability distribution table for the number of jellies that Jemeel chooses. [4]

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The company also produces large boxes of sweets. For any large box, the probability that it contains more jellies than chocolates is 0.64. 10 large boxes are chosen at random.

(b) Find the probability that no more than 7 of these boxes contain more jellies than chocolates. [3]

215. 9709_s20_qp_52 Q: 5

A fair three-sided spinner has sides numbered 1, 2, 3. A fair five-sided spinner has sides numbered 1, 1, 2, 2, 3. Both spinners are spun once. For each spinner, the number on the side on which it lands is noted. The random variable X is the larger of the two numbers if they are different, and their common value if they are the same.

(a) Show that $P(X = 3) = \frac{7}{15}$. [2]

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(b) Draw up the probability distribution table for X. [3]

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(c) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

216. 9709_s20_qp_53 Q: 2

In a certain large college, 22% of students own a car.

(a) 3 students from the college are chosen at random. Find the probability that all 3 students own a car. [1]

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(b) 16 students from the college are chosen at random. Find the probability that the number of these students who own a car is at least 2 and at most 4. [3]

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217. 9709_s20_qp_53 Q: 4

A fair four-sided spinner has edges numbered 1, 2, 2, 3. A fair three-sided spinner has edges numbered −2, −1, 1. Each spinner is spun and the number on the edge on which it comes to rest is noted. The random variable X is the sum of the two numbers that have been noted.

(a) Draw up the probability distribution table for X. [3]

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(b) Find Var(X). [3]

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218. 9709_w20_qp_51 Q: 3

Kayla is competing in a throwing event. A throw is counted as a success if the distance achieved is greater than 30 metres. The probability that Kayla will achieve a success on any throw is 0.25.

(a) Find the probability that Kayla takes more than 6 throws to achieve a success. [2]

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(b) Find the probability that, for a random sample of 10 throws, Kayla achieves at least 3 successes. [3]

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219. 9709_w20_qp_51 Q: 4

The random variable X takes each of the values 1, 2, 3, 4 with probability $\frac{1}{4}$. Two independent values of X are chosen at random. If the two values of X are the same, the random variable Y takes that value. Otherwise, the value of Y is the larger value of X minus the smaller value of X.

(a) Draw up the probability distribution table for Y. [4]

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(b) Find the probability that $Y = 2$ given that Y is even. [2]

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220. 9709_w20_qp_52 Q: 1

A fair six-sided die, with faces marked 1, 2, 3, 4, 5, 6, is thrown repeatedly until a 4 is obtained.

(a) Find the probability that obtaining a 4 requires fewer than 6 throws. [2]

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On another occasion, the die is thrown 10 times.

(b) Find the probability that a 4 is obtained at least 3 times. [3]

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221. 9709_w20_qp_52 Q: 2

A bag contains 5 red balls and 3 blue balls. Sadie takes 3 balls at random from the bag, without replacement. The random variable X represents the number of red balls that she takes.

(a) Show that the probability that Sadie takes exactly 1 red ball is $\frac{15}{56}$. [2]

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(b) Draw up the probability distribution table for X. [3]

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(c) Given that $E(X) = \frac{15}{8}$, find $\mathrm{Var}(X)$. [2]

222. 9709_w20_qp_53 Q: 2

An ordinary fair die is thrown until a 6 is obtained.

(a) Find the probability that obtaining a 6 takes more than 8 throws. [2]

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Two ordinary fair dice are thrown together until a pair of 6s is obtained. The number of throws taken is denoted by the random variable X.

(b) Find the expected value of X. [1]

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(c) Find the probability that obtaining a pair of 6s takes either 10 or 11 throws. [2]

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223. 9709_w20_qp_53 Q: 6

Three coins A, B and C are each thrown once.

- Coins A and B are each biased so that the probability of obtaining a head is $\frac{2}{3}$.
- Coin C is biased so that the probability of obtaining a head is $\frac{4}{5}$.

(a) Show that the probability of obtaining exactly 2 heads and 1 tail is $\frac{4}{9}$. [3]

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The random variable X is the number of heads obtained when the three coins are thrown.

(b) Draw up the probability distribution table for X. [3]

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(c) Given that $E(X) = \frac{32}{15}$, find $Var(X)$. [2]

224. 9709_m19_qp_62 Q: 4

The random variable X takes the values -1, 1, 2, 3 only. The probability that X takes the value x is kx^2, where k is a constant.

(i) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]

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(ii) Find E(X) and Var(X). [3]

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225. 9709_s19_qp_61 Q: 6

At a funfair, Amy pays \$1 for two attempts to make a bell ring by shooting at it with a water pistol.

- If she makes the bell ring on her first attempt, she receives \$3 and stops playing. This means that overall she has gained \$2.
- If she makes the bell ring on her second attempt, she receives \$1.50 and stops playing. This means that overall she has gained \$0.50.
- If she does not make the bell ring in the two attempts, she has lost her original \$1.

The probability that Amy makes the bell ring on any attempt is 0.2, independently of other attempts.

(i) Show that the probability that Amy loses her original \$1 is 0.64. [2]

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(ii) Complete the probability distribution table for the amount that Amy gains. [4]

Amy's gain ($)			
Probability	0.64		

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(iii) Calculate Amy's expected gain. [1]

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226. 9709_s19_qp_62 Q: 3

The probability that Janice will buy an item online in any week is 0.35. Janice does not buy more than one item online in any week.

(i) Find the probability that, in a 10-week period, Janice buys at most 7 items online. [3]

(ii) The probability that Janice buys at least one item online in a period of n weeks is greater than 0.99. Find the smallest possible value of n. [3]

227. 9709_s19_qp_62 Q: 5

Maryam has 7 sweets in a tin; 6 are toffees and 1 is a chocolate. She chooses one sweet at random and takes it out. Her friend adds 3 chocolates to the tin. Then Maryam takes another sweet at random out of the tin.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Draw up the probability distribution table for the number of toffees taken. [3]

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(iii) Find the mean number of toffees taken. [1]

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(iv) Find the probability that the first sweet taken is a chocolate, given that the second sweet taken is a toffee. [4]

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228. 9709_s19_qp_63 Q: 6

A fair five-sided spinner has sides numbered 1, 1, 1, 2, 3. A fair three-sided spinner has sides numbered 1, 2, 3. Both spinners are spun once and the score is the product of the numbers on the sides the spinners land on.

(i) Draw up the probability distribution table for the score. [4]

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(ii) Find the mean and the variance of the score. [3]

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(iii) Find the probability that the score is greater than the mean score. [2]

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229. 9709_w19_qp_61 Q: 2

Annan has designed a new logo for a sportswear company. A survey of a large number of customers found that 42% of customers rated the logo as good.

(i) A random sample of 10 customers is chosen. Find the probability that fewer than 8 of them rate the logo as good. [3]

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(ii) On another occasion, a random sample of n customers of the company is chosen. Find the smallest value of n for which the probability that at least one person rates the logo as good is greater than 0.995. [3]

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230. 9709_w19_qp_61 Q: 4

In a probability distribution the random variable X takes the values -1, 0, 1, 2, 4. The probability distribution table for X is as follows.

x	-1	0	1	2	4
$P(X = x)$	$\frac{1}{4}$	p	p	$\frac{3}{8}$	$4p$

(i) Find the value of p. [2]

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(ii) Find $E(X)$ and $Var(X)$. [3]

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(iii) Given that X is greater than zero, find the probability that X is equal to 2. [2]

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231. 9709_w19_qp_62 Q: 5

A fair red spinner has four sides, numbered 1, 2, 3, 3. A fair blue spinner has three sides, numbered −1, 0, 2. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [4]

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(ii) Find Var(X). [3]

232. 9709_w19_qp_63 Q: 6

A box contains 3 red balls and 5 white balls. One ball is chosen at random from the box and is not returned to the box. A second ball is now chosen at random from the box.

(i) Find the probability that both balls chosen are red. [1]

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(ii) Show that the probability that the balls chosen are of different colours is $\frac{15}{28}$. [2]

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(iii) Given that the second ball chosen is red, find the probability that the first ball chosen is red. [2]

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The random variable X denotes the number of red balls chosen.

(iv) Draw up the probability distribution table for X. [2]

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(v) Find Var(X). [3]

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233. 9709_m18_qp_62 Q: 4

The discrete random variable X has the following probability distribution.

x	-2	0	1	3	4
$P(X = x)$	0.2	0.1	p	0.1	q

(i) Given that $E(X) = 1.7$, find the values of p and q. [4]

(ii) Find $Var(X)$. [2]

234. 9709_s18_qp_61 Q: 3

Andy has 4 red socks and 8 black socks in his drawer. He takes 2 socks at random from his drawer.

(i) Find the probability that the socks taken are of different colours. [2]

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The random variable X is the number of red socks taken.

(ii) Draw up the probability distribution table for X. [3]

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(iii) Find $E(X)$. [1]

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235. 9709_s18_qp_61 Q: 6

Vehicles approaching a certain road junction from town A can either turn left, turn right or go straight on. Over time it has been noted that of the vehicles approaching this particular junction from town A, 55% turn left, 15% turn right and 30% go straight on. The direction a vehicle takes at the junction is independent of the direction any other vehicle takes at the junction.

(i) Find the probability that, of the next three vehicles approaching the junction from town A, one goes straight on and the other two either both turn left or both turn right. [4]

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(ii) Three vehicles approach the junction from town A. Given that all three drivers choose the same direction at the junction, find the probability that they all go straight on. [4]

236. 9709_s18_qp_62 Q: 4

Mrs Rupal chooses 3 animals at random from 5 dogs and 2 cats. The random variable X is the number of cats chosen.

(i) Draw up the probability distribution table for X. [4]

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(ii) You are given that $E(X) = \frac{6}{7}$. Find the value of $\text{Var}(X)$. [2]

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237. 9709_s18_qp_63 Q: 2

The random variable X has the distribution $N(-3, \sigma^2)$. The probability that a randomly chosen value of X is positive is 0.25.

(i) Find the value of σ. [3]

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(ii) Find the probability that, of 8 random values of X, fewer than 2 will be positive. [3]

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238. 9709_s18_qp_63 Q: 5

A game is played with 3 coins, A, B and C. Coins A and B are biased so that the probability of obtaining a head is 0.4 for coin A and 0.75 for coin B. Coin C is not biased. The 3 coins are thrown once.

(i) Draw up the probability distribution table for the number of heads obtained. [5]

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(ii) Hence calculate the mean and variance of the number of heads obtained. [3]

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239. 9709_w18_qp_61 Q: 2

A random variable X has the probability distribution shown in the following table, where p is a constant.

x	-1	0	1	2	4
$P(X = x)$	p	p	$2p$	$2p$	0.1

(i) Find the value of p. [1]

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(ii) Given that $E(X) = 1.15$, find $Var(X)$. [2]

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240. 9709_w18_qp_62 Q: 3

Jake attempts the crossword puzzle in his daily newspaper every day. The probability that he will complete the puzzle on any given day is 0.75, independently of all other days.

(i) Find the probability that he will complete the puzzle at least three times over a period of five days. [3]

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Kenny also attempts the puzzle every day. The probability that he will complete the puzzle on a Monday is 0.8. The probability that he will complete it on a Tuesday is 0.9 if he completed it on the previous day and 0.6 if he did not complete it on the previous day.

(ii) Find the probability that Kenny will complete the puzzle on at least one of the two days Monday and Tuesday in a randomly chosen week. [3]

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241. 9709_w18_qp_62 Q: 6

A fair red spinner has 4 sides, numbered 1, 2, 3, 4. A fair blue spinner has 3 sides, numbered 1, 2, 3. When a spinner is spun, the score is the number on the side on which it lands. The spinners are spun at the same time. The random variable X denotes the score on the red spinner minus the score on the blue spinner.

(i) Draw up the probability distribution table for X. [3]

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(ii) Find Var(X). [3]

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(iii) Find the probability that X is equal to 1, given that X is non-zero. [3]

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242. 9709_w18_qp_63 Q: 2

A fair 6-sided die has the numbers −1, −1, 0, 0, 1, 2 on its faces. A fair 3-sided spinner has edges numbered −1, 0, 1. The die is thrown and the spinner is spun. The number on the uppermost face of the die and the number on the edge on which the spinner comes to rest are noted. The sum of these two numbers is denoted by X.

(i) Draw up a table showing the probability distribution of X. [3]

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(ii) Find Var(X). [3]

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243. 9709_m17_qp_62 Q: 2

A bag contains 10 pink balloons, 9 yellow balloons, 12 green balloons and 9 white balloons. 7 balloons are selected at random without replacement. Find the probability that exactly 3 of them are green. [3]

244. 9709_m17_qp_62 Q: 6

Pack A consists of ten cards numbered 0, 0, 1, 1, 1, 1, 1, 3, 3, 3. Pack B consists of six cards numbered 0, 0, 2, 2, 2, 2. One card is chosen at random from each pack. The random variable X is defined as the sum of the two numbers on the cards.

(i) Show that $P(X = 2) = \frac{2}{15}$. [2]

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(ii) Draw up the probability distribution table for X. [4]

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(iii) Given that $X = 3$, find the probability that the card chosen from pack A is a 1. [3]

245. 9709_s17_qp_61 Q: 5

Eggs are sold in boxes of 20. Cracked eggs occur independently and the mean number of cracked eggs in a box is 1.4.

(i) Calculate the probability that a randomly chosen box contains exactly 2 cracked eggs. [3]

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(ii) Calculate the probability that a randomly chosen box contains at least 1 cracked egg. [2]

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iii) A shop sells n of these boxes of eggs. Find the smallest value of n such that the probability of there being at least 1 cracked egg in each box sold is less than 0.01. [2]

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246. 9709_s17_qp_62 Q: 3

In a probability distribution the random variable X takes the value x with probability kx^2, where k is a constant and x takes values $-2, -1, 2, 4$ only.

(i) Show that $P(X = -2)$ has the same value as $P(X = 2)$. [1]

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(ii) Draw up the probability distribution table for X, in terms of k, and find the value of k. [3]

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(iii) Find $E(X)$. [2]

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247. 9709_s17_qp_62 Q: 4

Two identical biased triangular spinners with sides marked 1, 2 and 3 are spun. For each spinner, the probabilities of landing on the sides marked 1, 2 and 3 are p, q and r respectively. The score is the sum of the numbers on the sides on which the spinners land. You are given that P(score is 6) = $\frac{1}{36}$ and P(score is 5) = $\frac{1}{9}$. Find the values of p, q and r. [6]

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248. 9709_s17_qp_62 Q: 7

During the school holidays, each day Khalid either rides on his bicycle with probability 0.6, or on his skateboard with probability 0.4. Khalid does not ride on both on the same day. If he rides on his bicycle then the probability that he hurts himself is 0.05. If he rides on his skateboard the probability that he hurts himself is 0.75.

(i) Find the probability that Khalid hurts himself on any particular day. [2]

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(ii) Given that Khalid hurts himself on a particular day, find the probability that he is riding on his skateboard. [2]

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(iii) There are 45 days of school holidays. Show that the variance of the number of days Khalid rides on his skateboard is the same as the variance of the number of days that Khalid rides on his bicycle. [2]

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(iv) Find the probability that Khalid rides on his skateboard on at least 2 of 10 randomly chosen days in the school holidays. [3]

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249. 9709_s17_qp_63 Q: 5

Hebe attempts a crossword puzzle every day. The number of puzzles she completes in a week (7 days) is denoted by X.

(i) State two conditions that are required for X to have a binomial distribution. [2]

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On average, Hebe completes 7 out of 10 of these puzzles.

(ii) Use a binomial distribution to find the probability that Hebe completes at least 5 puzzles in a week. [3]

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(iii) Use a binomial distribution to find the probability that, over the next 10 weeks, Hebe completes 4 or fewer puzzles in exactly 3 of the 10 weeks. [3]

250. 9709_s17_qp_63 Q: 6

Find how many numbers between 3000 and 5000 can be formed from the digits 1, 2, 3, 4 and 5,

(i) if digits are not repeated, [2]

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(ii) if digits can be repeated and the number formed is odd. [3]

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(b) A box of 20 biscuits contains 4 different chocolate biscuits, 2 different oatmeal biscuits and 14 different ginger biscuits. 6 biscuits are selected from the box at random.

(i) Find the number of different selections that include the 2 oatmeal biscuits. [2]

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(ii) Find the probability that fewer than 3 chocolate biscuits are selected. [4]

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251. 9709_w17_qp_61 Q: 1

The discrete random variable X has the following probability distribution.

x	1	2	3	6
$P(X = x)$	0.15	p	0.4	q

Given that $E(X) = 3.05$, find the values of p and q. [4]

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252. 9709_w17_qp_61 Q: 3

An experiment consists of throwing a biased die 30 times and noting the number of 4s obtained. This experiment was repeated many times and the average number of 4s obtained in 30 throws was found to be 6.21.

(i) Estimate the probability of throwing a 4. [1]

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(ii) find the variance of the number of 4s obtained in 30 throws, [1]

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(iii) find the probability that in 15 throws the number of 4s obtained is 2 or more. [3]

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253. 9709_w17_qp_62 Q: 3

A box contains 6 identical-sized discs, of which 4 are blue and 2 are red. Discs are taken at random from the box in turn and not replaced. Let X be the number of discs taken, up to and including the first blue one.

(i) Show that $P(X = 3) = \frac{1}{15}$. [2]

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(ii) Draw up the probability distribution table for X. [3]

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254. 9709_w17_qp_62 Q: 4

A fair tetrahedral die has faces numbered 1, 2, 3, 4. A coin is biased so that the probability of showing a head when thrown is $\frac{1}{3}$. The die is thrown once and the number n that it lands on is noted. The biased coin is then thrown n times. So, for example, if the die lands on 3, the coin is thrown 3 times.

(i) Find the probability that the die lands on 4 and the number of times the coin shows heads is 2. [3]

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(ii) Find the probability that the die lands on 3 and the number of times the coin shows heads is 3. [1]

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(iii) Find the probability that the number the die lands on is the same as the number of times the coin shows heads. [3]

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255. 9709_w17_qp_63 Q: 1

A statistics student asks people to complete a survey. The probability that a randomly chosen person agrees to complete the survey is 0.2. Find the probability that at least one of the first three people asked agrees to complete the survey. [2]

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256. 9709_w17_qp_63 Q: 4

A fair die with faces numbered 1, 2, 2, 2, 3, 6 is thrown. The score, X, is found by squaring the number on the face the die shows and then subtracting 4.

(i) Draw up a table to show the probability distribution of X. [3]

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(ii) Find $E(X)$ and $Var(X)$. [3]

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257. 9709_m16_qp_62 Q: 2

A flower shop has 5 yellow roses, 3 red roses and 2 white roses. Martin chooses 3 roses at random. Draw up the probability distribution table for the number of white roses Martin chooses. [4]

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258. 9709_s16_qp_61 Q: 2

The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The random variable X is the score when the die is thrown. The following is the probability distribution table for X.

x	1	2	3	4	5	6
$P(X = x)$	p	p	p	p	0.2	0.2

The die is thrown 3 times. Find the probability that the score is 4 on not more than 1 of the 3 throws. [5]

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259. 9709_s16_qp_61 Q: 4

A box contains 2 green sweets and 5 blue sweets. Two sweets are taken at random from the box, without replacement. The random variable X is the number of green sweets taken. Find $E(X)$ and $Var(X)$. [6]

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260. 9709_s16_qp_62 Q: 3

A particular type of bird lays 1, 2, 3 or 4 eggs in a nest each year. The probability of x eggs is equal to kx, where k is a constant.

(i) Draw up a probability distribution table, in terms of k, for the number of eggs laid in a year and find the value of k. [3]

(ii) Find the mean and variance of the number of eggs laid in a year by this type of bird. [3]

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261. 9709_s16_qp_62 Q: 4

When people visit a certain large shop, on average 34% of them do not buy anything, 53% spend less than \$50 and 13% spend at least \$50.

(i) 15 people visiting the shop are chosen at random. Calculate the probability that at least 14 of them buy something. [3]

(ii) n people visiting the shop are chosen at random. The probability that none of them spends at least \$50 is less than 0.04. Find the smallest possible value of n. [3]

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262. 9709_s16_qp_63 Q: 3

Two ordinary fair dice are thrown. The resulting score is found as follows.

- If the two dice show different numbers, the score is the smaller of the two numbers.
- If the two dice show equal numbers, the score is 0.

(i) Draw up the probability distribution table for the score. [4]

(ii) Calculate the expected score. [2]

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263. 9709_w16_qp_61 Q: 1

The random variable X is such that $X \sim N(20, 49)$. Given that $P(X > k) = 0.25$, find the value of k. [3]

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264. 9709_w16_qp_61 Q: 2

Two fair six-sided dice with faces numbered 1, 2, 3, 4, 5, 6 are thrown and the two scores are noted. The difference between the two scores is defined as follows.

- If the scores are equal the difference is zero.
- If the scores are not equal the difference is the larger score minus the smaller score.

Find the expectation of the difference between the two scores. [5]

265. 9709_w16_qp_61 Q: 3

Visitors to a Wildlife Park in Africa have independent probabilities of 0.9 of seeing giraffes, 0.95 of seeing elephants, 0.85 of seeing zebras and 0.1 of seeing lions.

(i) Find the probability that a visitor to the Wildlife Park sees all these animals. [1]

(ii) Find the probability that, out of 12 randomly chosen visitors, fewer than 3 see lions. [3]

(iii) 50 people independently visit the Wildlife Park. Find the mean and variance of the number of these people who see zebras. [2]

266. 9709_w16_qp_62 Q: 2

Noor has 3 T-shirts, 4 blouses and 5 jumpers. She chooses 3 items at random. The random variable X is the number of T-shirts chosen.

(i) Show that the probability that Noor chooses exactly one T-shirt is $\frac{27}{55}$. [3]

(ii) Draw up the probability distribution table for X. [4]

267. 9709_w16_qp_63 Q: 2

A fair triangular spinner has three sides numbered 1, 2, 3. When the spinner is spun, the score is the number of the side on which it lands. The spinner is spun four times.

(i) Find the probability that at least two of the scores are 3. [3]

(ii) Find the probability that the sum of the four scores is 5. [3]

268. 9709_s15_qp_62 Q: 1

A fair die is thrown 10 times. Find the probability that the number of sixes obtained is between 3 and 5 inclusive. [3]

269. 9709_s15_qp_62 Q: 5

A box contains 5 discs, numbered 1, 2, 4, 6, 7. William takes 3 discs at random, without replacement, and notes the numbers on the discs.

(i) Find the probability that the numbers on the 3 discs are two even numbers and one odd number. [3]

The smallest of the numbers on the 3 discs taken is denoted by the random variable S.

(ii) By listing all possible selections (126, 246 and so on) draw up the probability distribution table for S. [5]

270. 9709_s15_qp_63 Q: 2

When Joanna cooks, the probability that the meal is served on time is $\frac{1}{5}$. The probability that the kitchen is left in a mess is $\frac{3}{5}$. The probability that the meal is not served on time and the kitchen is not left in a mess is $\frac{3}{10}$. Some of this information is shown in the following table.

	Kitchen left in a mess	Kitchen not left in a mess	Total
Meal served on time			$\frac{1}{5}$
Meal not served on time		$\frac{3}{10}$	
Total			1

(i) Copy and complete the table. [3]

(ii) Given that the kitchen is left in a mess, find the probability that the meal is not served on time. [2]

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271. 9709_s15_qp_63 Q: 4

A pet shop has 9 rabbits for sale, 6 of which are white. A random sample of two rabbits is chosen without replacement.

(i) Show that the probability that exactly one of the two rabbits in the sample is white is $\frac{1}{2}$. [2]

(ii) Construct the probability distribution table for the number of white rabbits in the sample. [3]

(iii) Find the expected value of the number of white rabbits in the sample. [1]

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272. 9709_w15_qp_61 Q: 1

In a certain town, 76% of cars are fitted with satellite navigation equipment. A random sample of 11 cars from this town is chosen. Find the probability that fewer than 10 of these cars are fitted with this equipment. [4]

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273. 9709_w15_qp_61 Q: 6

Nadia is very forgetful. Every time she logs in to her online bank she only has a 40% chance of remembering her password correctly. She is allowed 3 unsuccessful attempts on any one day and then the bank will not let her try again until the next day.

(i) Draw a fully labelled tree diagram to illustrate this situation. [3]

(ii) Let X be the number of unsuccessful attempts Nadia makes on any day that she tries to log in to her bank. Copy and complete the following table to show the probability distribution of X. [4]

x	0	1	2	3
$P(X = x)$		0.24		

(iii) Calculate the expected number of unsuccessful attempts made by Nadia on any day that she tries to log in. [2]

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274. 9709_w15_qp_62 Q: 3

One plastic robot is given away free inside each packet of a certain brand of biscuits. There are four colours of plastic robot (red, yellow, blue and green) and each colour is equally likely to occur. Nick buys some packets of these biscuits. Find the probability that

(i) he gets a green robot on opening his first packet, [1]

(ii) he gets his first green robot on opening his fifth packet. [2]

Nick's friend Amos is also collecting robots.

(iii) Find the probability that the first four packets Amos opens all contain different coloured robots. [3]

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275. 9709_w15_qp_62 Q: 6

A fair spinner A has edges numbered 1, 2, 3, 3. A fair spinner B has edges numbered −3, −2, −1, 1. Each spinner is spun. The number on the edge that the spinner comes to rest on is noted. Let X be the sum of the numbers for the two spinners.

(i) Copy and complete the table showing the possible values of X. [1]

		Spinner A			
		1	2	3	3
Spinner B	−3	−2			
	−2			1	
	−1				
	1				

(ii) Draw up a table showing the probability distribution of X. [3]

(iii) Find Var(X). [3]

(iv) Find the probability that X is even, given that X is positive. [2]
