Topical Past Paper Questions Workbook

AS & A Level Mathematics (9709) Paper 5

[Probability & Statistics 1]

Exam Series: May 2015 – May 2022

Format Type A:
Answers to all questions are provided as an appendix

Chapter 5

The normal distribution

276. 9709_m22_qp_52 Q: 4

The weights of male leopards in a particular region are normally distributed with mean 55 kg and standard deviation 6 kg.

(a) Find the probability that a randomly chosen male leopard from this region weighs between 46 and 62 kg. [4]

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The weights of female leopards in this region are normally distributed with mean 42 kg and standard deviation σ kg. It is known that 25% of female leopards in the region weigh less than 36 kg.

(b) Find the value of σ. [3]

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The distributions of the weights of male and female leopards are independent of each other. A male leopard and a female leopard are each chosen at random.

(c) Find the probability that both the weights of these leopards are less than 46 kg. [4]

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277. 9709_s22_qp_51 Q: 5

The lengths, in cm, of the leaves of a particular type are modelled by the distribution $N(5.2, 1.5^2)$.

(a) Find the probability that a randomly chosen leaf of this type has length less than 6 cm. [2]

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The lengths of the leaves of another type are also modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves of this type and finds that 46 are less than 3 cm long and 95 are more than 8 cm long.

(b) Find estimates for the mean and standard deviation of the lengths of leaves of this type. [5]

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(c) In a random sample of 2000 leaves of this second type, how many would the scientist expect to find with lengths more than 1 standard deviation from the mean? [4]

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278. 9709_s22_qp_52 Q: 4

The weights, in kg, of bags of rice produced by Anders have the distribution $N(2.02, 0.03^2)$.

(a) Find the probability that a randomly chosen bag of rice produced by Anders weighs between 1.98 and 2.03 kg. [3]

The weights of bags of rice produced by Binders are normally distributed with mean 2.55 kg and standard deviation σ kg. In a random sample of 5000 of these bags, 134 weighed more than 2.6 kg.

(b) Find the value of σ. [4]

279. 9709_s22_qp_52 Q: 5

In a large college, 28% of the students do not play any musical instrument, 52% play exactly one musical instrument and the remainder play two or more musical instruments.

A random sample of 12 students from the college is chosen.

(a) Find the probability that more than 9 of these students play at least one musical instrument. [3]

A random sample of 90 students from the college is now chosen.

(b) Use an approximation to find the probability that fewer than 40 of these students play exactly one musical instrument. [5]

280. 9709_s22_qp_53 Q: 5

Farmer Jones grows apples. The weights, in grams, of the apples grown this year are normally distributed with mean 170 and standard deviation 25. Apples that weigh between 142 grams and 205 grams are sold to a supermarket.

(a) Find the probability that a randomly chosen apple grown by Farmer Jones this year is sold to the supermarket. [4]

Farmer Jones sells the apples to the supermarket at \$0.24 each. He sells apples that weigh more than 205 grams to a local shop at \$0.30 each. He does not sell apples that weigh less than 142 grams.

The total number of apples grown by Farmer Jones this year is 20 000.

(b) Calculate an estimate for his total income from this year's apples. [3]

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Farmer Tan also grows apples. The weights, in grams, of the apples grown this year follow the distribution N(182, 20^2). 72% of these apples have a weight more than w grams.

(c) Find the value of w. [3]

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281. 9709_m21_qp_52 Q: 3

The time spent by shoppers in a large shopping centre has a normal distribution with mean 96 minutes and standard deviation 18 minutes.

(a) Find the probability that a shopper chosen at random spends between 85 and 100 minutes in the shopping centre. [3]

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88% of shoppers spend more than t minutes in the shopping centre.

(b) Find the value of t. [3]

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282. 9709_m21_qp_52 Q: 7

There are 400 students at a school in a certain country. Each student was asked whether they preferred swimming, cycling or running and the results are given in the following table.

	Swimming	Cycling	Running
Female	104	50	66
Male	31	57	92

A student is chosen at random.

(a) **(i)** Find the probability that the student prefers swimming. [1]

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(ii) Determine whether the events 'the student is male' and 'the student prefers swimming' are independent, justifying your answer. [2]

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On average at all the schools in this country 30% of the students do not like any sports.

(b) **(i)** 10 of the students from this country are chosen at random.

Find the probability that at least 3 of these students do not like any sports. [3]

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(ii) 90 students from this country are now chosen at random.

Use an approximation to find the probability that fewer than 32 of them do not like any sports. [5]

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283. 9709_s21_qp_51 Q: 2

A company produces a particular type of metal rod. The lengths of these rods are normally distributed with mean 25.2 cm and standard deviation 0.4 cm. A random sample of 500 of these rods is chosen.

How many rods in this sample would you expect to have a length that is within 0.5 cm of the mean length? [5]

284. 9709_s21_qp_51 Q: 6

In Questa, 60% of the adults travel to work by car.

(a) A random sample of 12 adults from Questa is taken.

Find the probability that the number who travel to work by car is less than 10. [3]

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(b) A random sample of 150 adults from Questa is taken.

Use an approximation to find the probability that the number who travel to work by car is less than 81. [5]

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(c) Justify the use of your approximation in part **(b)**. [1]

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285. 9709_s21_qp_52 Q: 2

The weights of bags of sugar are normally distributed with mean 1.04 kg and standard deviation σ kg. In a random sample of 2000 bags of sugar, 72 weighed more than 1.10 kg.

Find the value of σ. [4]

286. 9709_s21_qp_52 Q: 5

Every day Richard takes a flight between Astan and Bejin. On any day, the probability that the flight arrives early is 0.15, the probability that it arrives on time is 0.55 and the probability that it arrives late is 0.3.

(a) Find the probability that on each of 3 randomly chosen days, Richard's flight does not arrive late. [1]

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(b) Find the probability that for 9 randomly chosen days, Richard's flight arrives early at least 3 times. [3]

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(c) 60 days are chosen at random.

Use an approximation to find the probability that Richard's flight arrives early at least 12 times. [5]

287. 9709_s21_qp_53 Q: 5

The lengths of the leaves of a particular type of tree are modelled by a normal distribution. A scientist measures the lengths of a random sample of 500 leaves from this type of tree and finds that 42 are less than 4 cm long and 100 are more than 10 cm long.

(a) Find estimates for the mean and standard deviation of the lengths of leaves from this type of tree. [5]

The lengths, in cm, of the leaves of a different type of tree have the distribution $N(\mu, \sigma^2)$. The scientist takes a random sample of 800 leaves from this type of tree.

(b) Find how many of these leaves the scientist would expect to have lengths, in cm, between $\mu - 2\sigma$ and $\mu + 2\sigma$. [4]

288. 9709_s21_qp_53 Q: 7

In the region of Arka, the total number of households in the three villages Reeta, Shan and Teber is 800. Each of the households was asked about the quality of their broadband service. Their responses are summarised in the following table.

		Quality of broadband service		
		Excellent	Good	Poor
Village	Reeta	75	118	32
	Shan	223	177	40
	Teber	12	60	63

(a) **(i)** Find the probability that a randomly chosen household is in Shan and has poor broadband service. [1]

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(ii) Find the probability that a randomly chosen household has good broadband service given that the household is in Shan. [2]

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In the whole of Arka there are a large number of households. A survey showed that 35% of households in Arka have no broadband service.

(b) **(i)** 10 households in Arka are chosen at random.

Find the probability that fewer than 3 of these households have no broadband service. [3]

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(ii) 120 households in Arka are chosen at random.

Use an approximation to find the probability that more than 32 of these households have no broadband service. [5]

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289. 9709_w21_qp_51 Q: 7

The times, in minutes, that Karli spends each day on social media are normally distributed with mean 125 and standard deviation 24.

(a) **(i)** On how many days of the year (365 days) would you expect Karli to spend more than 142 minutes on social media? [5]

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(ii) Find the probability that Karli spends more than 142 minutes on social media on fewer than 2 of 10 randomly chosen days. [3]

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(b) On 90% of days, Karli spends more than t minutes on social media.

Find the value of t. [3]

290. 9709_w21_qp_52 Q: 6

The times taken, in minutes, to complete a particular task by employees at a large company are normally distributed with mean 32.2 and standard deviation 9.6.

(a) Find the probability that a randomly chosen employee takes more than 28.6 minutes to complete the task. [3]

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(b) 20% of employees take longer than t minutes to complete the task.

Find the value of t. [3]

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(c) Find the probability that the time taken to complete the task by a randomly chosen employee differs from the mean by less than 15.0 minutes. [4]

291. 9709_w21_qp_53 Q: 4

Raj wants to improve his fitness, so every day he goes for a run. The times, in minutes, of his runs have a normal distribution with mean 41.2 and standard deviation 3.6.

(a) Find the probability that on a randomly chosen day Raj runs for more than 43.2 minutes. [3]

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(b) Find an estimate for the number of days in a year (365 days) on which Raj runs for less than 43.2 minutes. [2]

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(c) On 95% of days, Raj runs for more than t minutes.

Find the value of t. [3]

292. 9709_m20_qp_52 Q: 3

The weights of apples of a certain variety are normally distributed with mean 82 grams. 22% of these apples have a weight greater than 87 grams.

(a) Find the standard deviation of the weights of these apples. [3]

(b) Find the probability that the weight of a randomly chosen apple of this variety differs from the mean weight by less than 4 grams. [4]

293. 9709_m20_qp_52 Q: 5

In Greenton, 70% of the adults own a car. A random sample of 8 adults from Greenton is chosen.

(a) Find the probability that the number of adults in this sample who own a car is less than 6. [3]

A random sample of 120 adults from Greenton is now chosen.

(b) Use an approximation to find the probability that more than 75 of them own a car. [5]

294. 9709_s20_qp_51 Q: 6

The lengths of female snakes of a particular species are normally distributed with mean 54 cm and standard deviation 6.1 cm.

(a) Find the probability that a randomly chosen female snake of this species has length between 50 cm and 60 cm. [4]

The lengths of male snakes of this species also have a normal distribution. A scientist measures the lengths of a random sample of 200 male snakes of this species. He finds that 32 have lengths less than 45 cm and 17 have lengths more than 56 cm.

(b) Find estimates for the mean and standard deviation of the lengths of male snakes of this species. [5]

295. 9709_s20_qp_52 Q: 4

Trees in the Redian forest are classified as tall, medium or short, according to their height. The heights can be modelled by a normal distribution with mean 40 m and standard deviation 12 m. Trees with a height of less than 25 m are classified as short.

(a) Find the probability that a randomly chosen tree is classified as short. [3]

Of the trees that are classified as tall or medium, one third are tall and two thirds are medium.

(b) Show that the probability that a randomly chosen tree is classified as tall is 0.298, correct to 3 decimal places. [2]

(c) Find the height above which trees are classified as tall. [3]

296. 9709_s20_qp_52 Q: 7

On any given day, the probability that Moena messages her friend Pasha is 0.72.

(a) Find the probability that for a random sample of 12 days Moena messages Pasha on no more than 9 days. [3]

(b) Moena messages Pasha on 1 January. Find the probability that the next day on which she messages Pasha is 5 January. [1]

(**c**) Use an approximation to find the probability that in any period of 100 days Moena messages Pasha on fewer than 64 days. [5]

297. 9709_s20_qp_53 Q: 3

In a certain town, the time, X hours, for which people watch television in a week has a normal distribution with mean 15.8 hours and standard deviation 4.2 hours.

(a) Find the probability that a randomly chosen person from this town watches television for less than 21 hours in a week. [2]

(b) Find the value of k such that $P(X < k) = 0.75$. [3]

298. 9709_s20_qp_53 Q: 5

A pair of fair coins is thrown repeatedly until a pair of tails is obtained. The random variable X denotes the number of throws required to obtain a pair of tails.

(a) Find the expected value of X. [1]

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(b) Find the probability that exactly 3 throws are required to obtain a pair of tails. [1]

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(c) Find the probability that fewer than 6 throws are required to obtain a pair of tails. [2]

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On a different occasion, a pair of fair coins is thrown 80 times.

(d) Use an approximation to find the probability that a pair of tails is obtained more than 25 times. [5]

299. 9709_w20_qp_51 Q: 5

The time in hours that Davin plays on his games machine each day is normally distributed with mean 3.5 and standard deviation 0.9.

(a) Find the probability that on a randomly chosen day Davin plays on his games machine for more than 4.2 hours. [3]

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(b) On 90% of days Davin plays on his games machine for more than t hours. Find the value of t. [3]

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(c) Calculate an estimate for the number of days in a year (365 days) on which Davin plays on his games machine for between 2.8 and 4.2 hours. [3]

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300. 9709_w20_qp_52 Q: 3

Pia runs 2 km every day and her times in minutes are normally distributed with mean 10.1 and standard deviation 1.3.

(a) Find the probability that on a randomly chosen day Pia takes longer than 11.3 minutes to run 2 km. [3]

(b) On 75% of days, Pia takes longer than t minutes to run 2 km. Find the value of t. [3]

(c) On how many days in a period of 90 days would you expect Pia to take between 8.9 and 11.3 minutes to run 2 km? [3]

301. 9709_w20_qp_53 Q: 1

The times taken to swim 100 metres by members of a large swimming club have a normal distribution with mean 62 seconds and standard deviation 5 seconds.

(a) Find the probability that a randomly chosen member of the club takes between 56 and 66 seconds to swim 100 metres. [3]

(b) 13% of the members of the club take more than t minutes to swim 100 metres. Find the value of t. [3]

302. 9709_w20_qp_53 Q: 4

The 13 00 train from Jahor to Keman runs every day. The probability that the train arrives late in Keman is 0.35.

(a) For a random sample of 7 days, find the probability that the train arrives late on fewer than 3 days. [3]

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A random sample of 142 days is taken.

(b) Use an approximation to find the probability that the train arrives late on more than 40 days. [5]

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303. 9709_m19_qp_62 Q: 3

The times taken, in minutes, for trains to travel between Alphaton and Beeton are normally distributed with mean 140 and standard deviation 12.

(i) Find the probability that a randomly chosen train will take less than 132 minutes to travel between Alphaton and Beeton. [3]

(ii) The probability that a randomly chosen train takes more than k minutes to travel between Alphaton and Beeton is 0.675. Find the value of k. [3]

304. 9709_m19_qp_62 Q: 6

The results of a survey by a large supermarket show that 35% of its customers shop online.

(i) Six customers are chosen at random. Find the probability that more than three of them shop online. [3]

(ii) For a random sample of n customers, the probability that at least one of them shops online is greater than 0.95. Find the least possible value of n. [3]

(iii) For a random sample of 100 customers, use a suitable approximating distribution to find the probability that more than 39 shop online. [5]

305. 9709_s19_qp_61 Q: 5

In a certain country the probability that a child owns a bicycle is 0.65.

(i) A random sample of 15 children from this country is chosen. Find the probability that more than 12 own a bicycle. [3]

(ii) A random sample of 250 children from this country is chosen. Use a suitable approximation to find the probability that fewer than 179 own a bicycle. [4]

306. 9709_s19_qp_61 Q: 7

The weight of adult female giraffes has a normal distribution with mean 830 kg and standard deviation 120 kg.

(i) There are 430 adult female giraffes in a particular game reserve. Find the number of these adult female giraffes which can be expected to weigh less than 700 kg. [4]

(ii) Given that 90% of adult female giraffes weigh between $(830 - w)$ kg and $(830 + w)$ kg, find the value of w. [3]

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The weight of adult male giraffes has a normal distribution with mean 1190 kg and standard deviation σ kg.

(iii) Given that 83.4% of adult male giraffes weigh more than 950 kg, find the value of σ. [3]

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307. 9709_s19_qp_62 Q: 2

The volume of ink in a certain type of ink cartridge has a normal distribution with mean 30 ml and standard deviation 1.5 ml. People in an office use a total of 8 cartridges of this ink per month. Find the expected number of cartridges per month that contain less than 28.9 ml of this ink. [4]

308. 9709_s19_qp_62 Q: 4

It is known that 20% of male giant pandas in a certain area weigh more than 121 kg and 71.9% weigh more than 102 kg. Weights of male giant pandas in this area have a normal distribution. Find the mean and standard deviation of the weights of male giant pandas in this area. [5]

309. 9709_s19_qp_63 Q: 1

The time taken, in minutes, by a ferry to cross a lake has a normal distribution with mean 85 and standard deviation 6.8.

(i) Find the probability that, on a randomly chosen occasion, the time taken by the ferry to cross the lake is between 79 and 91 minutes. [3]

(ii) Over a long period it is found that 96% of ferry crossings take longer than a certain time t minutes. Find the value of t. [3]

310. 9709_s19_qp_63 Q: 5

On average, 34% of the people who go to a particular theatre are men.

(i) A random sample of 14 people who go to the theatre is chosen. Find the probability that at most 2 people are men. [3]

(ii) Use an approximation to find the probability that, in a random sample of 600 people who go to the theatre, fewer than 190 are men. [5]

311. 9709_w19_qp_61 Q: 7

The shortest time recorded by an athlete in a 400 m race is called their personal best (PB). The PBs of the athletes in a large athletics club are normally distributed with mean 49.2 seconds and standard deviation 2.8 seconds.

(i) Find the probability that a randomly chosen athlete from this club has a PB between 46 and 53 seconds. [4]

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(ii) It is found that 92% of athletes from this club have PBs of more than t seconds. Find the value of t. [3]

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Three athletes from the club are chosen at random.

(iii) Find the probability that exactly 2 have PBs of less than 46 seconds. [3]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

312. 9709_w19_qp_62 Q: 4

In Quarendon, 66% of households are satisfied with the speed of their wifi connection.

(i) Find the probability that, out of 10 households chosen at random in Quarendon, at least 8 are satisfied with the speed of their wifi connection. [3]

(ii) A random sample of 150 households in Quarendon is chosen. Use a suitable approximation to find the probability that more than 84 are satisfied with the speed of their wifi connection. [5]

313. 9709_w19_qp_62 Q: 6

The heights, in metres, of fir trees in a large forest have a normal distribution with mean 40 and standard deviation 8.

(i) Find the probability that a fir tree chosen at random in this forest has a height less than 45 metres. [2]

(ii) Find the probability that a fir tree chosen at random in this forest has a height within 5 metres of the mean. [2]

In another forest, the heights of another type of fir tree are modelled by a normal distribution. A scientist measures the heights of 500 randomly chosen trees of this type. He finds that 48 trees are less than 10 m high and 76 trees are more than 24 m high.

(iii) Find the mean and standard deviation of the heights of trees of this type. [5]

314. 9709_w19_qp_63 Q: 4

The heights of students at the Mainland college are normally distributed with mean 148 cm and standard deviation 8 cm.

(i) The probability that a Mainland student chosen at random has a height less than h cm is 0.67. Find the value of h. [3]

120 Mainland students are chosen at random.

(ii) Find the number of these students that would be expected to have a height within half a standard deviation of the mean. [4]

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315. 9709_w19_qp_63 Q: 7

A competition is taking place between two choirs, the Notes and the Classics. There is a large audience for the competition.

- 30% of the audience are Notes supporters.
- 45% of the audience are Classics supporters.
- The rest of the audience are not supporters of either of these choirs.
- No one in the audience supports both of these choirs.

(i) A random sample of 6 people is chosen from the audience.

(a) Find the probability that no more than 2 of the 6 people are Notes supporters. [3]

(b) Find the probability that none of the 6 people support either of these choirs. [2]

(ii) A random sample of 240 people is chosen from the audience. Use a suitable approximation to find the probability that fewer than 50 do not support either of the choirs. [5]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

316. 9709_m18_qp_62 Q: 7

The weights of packets of a certain type of biscuit are normally distributed with mean 400 grams and standard deviation σ grams.

(i) In a random sample of 6000 packets of this type of biscuit, 225 packets weighed more than 410 grams. Find the value of σ. [4]

(ii) In a random sample of 500 packets of this type of biscuit, how many packets would you expect to find with weights that are more than 1.5 standard deviations from the mean? [4]

317. 9709_m18_qp_62 Q: 8

The results of a survey at a certain large college show that the proportion of students who own a car is $\frac{1}{4}$.

(i) Five students at the college are chosen at random. Find the probability that at least four of these students own a car. [3]

(ii) For a random sample of n students at the college, the probability that at least one of the students owns a car is greater than 0.995. Find the least possible value of n. [3]

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(iii) For a random sample of 160 students at the college, use a suitable approximate distribution to find the probability that fewer than 50 own a car. [4]

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If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

318. 9709_s18_qp_61 Q: 4

(a) The distance that car tyres of a certain make can travel before they need to be replaced has a normal distribution. A survey of a large number of these tyres found that the probability of this distance being more than 36 800 km is 0.0082 and the probability of this distance being more than 31 000 km is 0.6915. Find the mean and standard deviation of the distribution. [5]

(b) The random variable X has the distribution $N(\mu, \sigma^2)$, where $3\sigma = 4\mu$ and $\mu \neq 0$. Find $P(X < 3\mu)$. [3]

319. 9709_s18_qp_61 Q: 5

In Pelmerdon 22% of families own a dishwasher.

(i) Find the probability that, of 15 families chosen at random from Pelmerdon, between 4 and 6 inclusive own a dishwasher. [3]

(ii) A random sample of 145 families from Pelmerdon is chosen. Use a suitable approximation to find the probability that more than 26 families own a dishwasher. [5]

320. 9709_s18_qp_62 Q: 3

(i) The volume of soup in Super Soup cartons has a normal distribution with mean μ millilitres and standard deviation 9 millilitres. Tests have shown that 10% of cartons contain less than 440 millilitres of soup. Find the value of μ. [3]

ii) A food retailer orders 150 Super Soup cartons. Calculate the number of these cartons for which you would expect the volume of soup to be more than 1.8 standard deviations above the mean. [3]

321. 9709_s18_qp_62 Q: 7

In a certain country, 60% of mobile phones sold are made by Company A, 35% are made by Company B and 5% are made by other companies.

(i) Find the probability that, out of a random sample of 13 people who buy a mobile phone, fewer than 11 choose a mobile phone made by Company A. [3]

(ii) Use a suitable approximation to find the probability that, out of a random sample of 130 people who buy a mobile phone, at least 50 choose a mobile phone made by Company B. [5]

(iii) A random sample of n mobile phones sold is chosen. The probability that at least one of these phones is made by Company B is more than 0.98. Find the least possible value of n. [3]

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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322. 9709_s18_qp_63 Q: 6

The diameters of apples in an orchard have a normal distribution with mean 5.7 cm and standard deviation 0.8 cm. Apples with diameters between 4.1 cm and 5 cm can be used as toffee apples.

(i) Find the probability that an apple selected at random can be used as a toffee apple. [3]

(ii) 250 apples are chosen at random. Use a suitable approximation to find the probability that fewer than 50 can be used as toffee apples. [5]

323. 9709_w18_qp_61 Q: 4

(a) It is given that $X \sim \mathrm{N}(31.4, 3.6)$. Find the probability that a randomly chosen value of X is less than 29.4. [3]

(b) The lengths of fish of a particular species are modelled by a normal distribution. A scientist measures the lengths of 400 randomly chosen fish of this species. He finds that 42 fish are less than 12 cm long and 58 are more than 19 cm long. Find estimates for the mean and standard deviation of the lengths of fish of this species. [5]

324. 9709_w18_qp_61 Q: 5

At the Nonland Business College, all students sit an accountancy examination at the end of their first year of study. On average, 80% of the students pass this examination.

(i) A random sample of 9 students who will take this examination is chosen. Find the probability that at most 6 of these students will pass the examination. [3]

(ii) A random sample of 200 students who will take this examination is chosen. Use a suitable approximate distribution to find the probability that more than 166 of them will pass the examination. [5]

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(iii) Justify the use of your approximate distribution in part **(ii)**. [1]

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325. 9709_w18_qp_62 Q: 7

(a) The time, X hours, for which students use a games machine in any given day has a normal distribution with mean 3.24 hours and standard deviation 0.96 hours.

(i) On how many days of the year (365 days) would you expect a randomly chosen student to use a games machine for less than 4 hours? [3]

(ii) Find the value of k such that $P(X > k) = 0.2$. [3]

(iii) Find the probability that the number of hours for which a randomly chosen student uses a games machine in a day is within 1.5 standard deviations of the mean. [3]

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(b) The variable Y is normally distributed with mean μ and standard deviation σ, where $4\sigma = 3\mu$ and $\mu \neq 0$. Find the probability that a randomly chosen value of Y is positive. [3]

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326. 9709_w18_qp_63 Q: 5

The weights of apples sold by a store can be modelled by a normal distribution with mean 120 grams and standard deviation 24 grams. Apples weighing less than 90 grams are graded as 'small'; apples weighing more than 140 grams are graded as 'large'; the remainder are graded as 'medium'.

(i) Show that the probability that an apple chosen at random is graded as medium is 0.692, correct to 3 significant figures. [4]

(ii) Four apples are chosen at random. Find the probability that at least two are graded as medium. [4]

327. 9709_w18_qp_63 Q: 6

The lifetimes, in hours, of a particular type of light bulb are normally distributed with mean 2000 hours and standard deviation σ hours. The probability that a randomly chosen light bulb of this type has a lifetime of more than 1800 hours is 0.96.

(i) Find the value of σ. [3]

New technology has resulted in a new type of light bulb. It is found that on average one in five of these new light bulbs has a lifetime of more than 2500 hours.

(ii) For a random selection of 300 of these new light bulbs, use a suitable approximate distribution to find the probability that fewer than 70 have a lifetime of more than 2500 hours. [4]

(iii) Justify the use of your approximate distribution in part **(ii)**. [1]

328. 9709_m17_qp_62 Q: 3

It is found that 10% of the population enjoy watching Historical Drama on television. Use an appropriate approximation to find the probability that, out of 160 people chosen randomly, more than 17 people enjoy watching Historical Drama on television. [5]

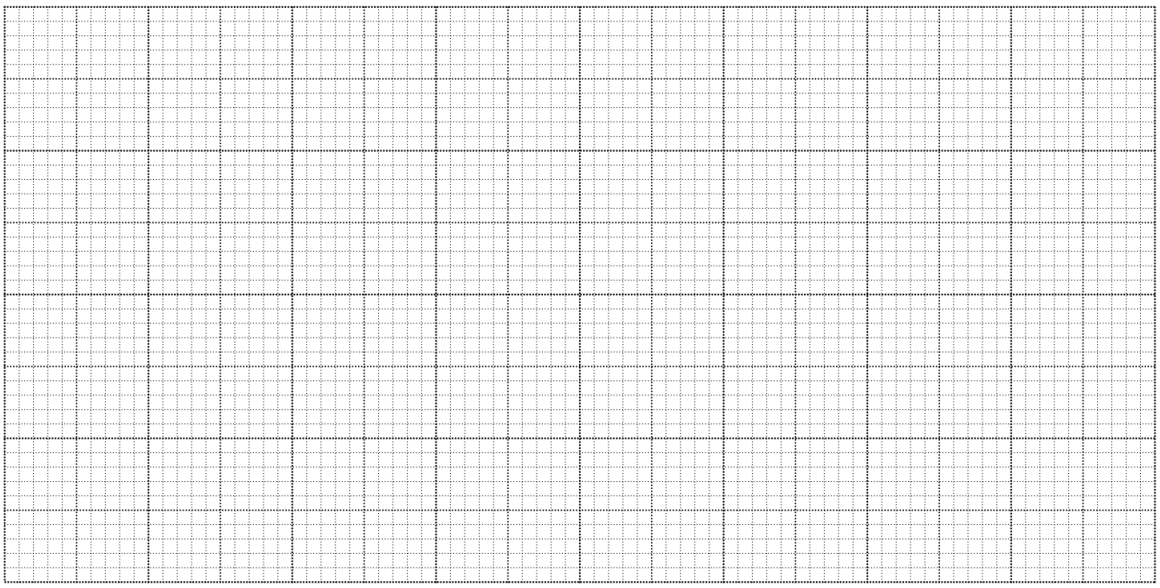
329. 9709_m17_qp_62 Q: 4

The weights in kilograms of packets of cereal were noted correct to 4 significant figures. The following stem-and-leaf diagram shows the data.

747	3	(1)
748	1 2 5 7 7 9	(6)
749	0 2 2 2 3 5 5 5 6 7 8 9	(12)
750	1 1 2 2 2 3 4 4 5 6 7 7 8 8 9	(15)
751	0 0 2 3 3 4 4 4 5 5 7 7 9	(13)
752	0 0 0 1 1 2 2 3 4 4 4	(11)
753	2	(1)

Key: 748 | 5 represents 0.7485 kg.

(i) On the grid, draw a box-and-whisker plot to represent the data. [5]



(ii) Name a distribution that might be a suitable model for the weights of this type of cereal packet. Justify your answer. [2]

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330. 9709_m17_qp_62 Q: 7

The lengths, in centimetres, of middle fingers of women in Raneland have a normal distribution with mean μ and standard deviation σ. It is found that 25% of these women have fingers longer than 8.8 cm and 17.5% have fingers shorter than 7.7 cm.

(i) Find the values of μ and σ. [5]

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The lengths, in centimetres, of middle fingers of women in Snoland have a normal distribution with mean 7.9 and standard deviation 0.44. A random sample of 5 women from Snoland is chosen.

(ii) Find the probability that exactly 3 of these women have middle fingers shorter than 8.2 cm. [5]

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(b) The random variable X has a normal distribution with mean equal to the standard deviation. Find the probability that a particular value of X is less than 1.5 times the mean. [3]

331. 9709_s17_qp_61 Q: 6

The random variable X has a normal distribution with mean μ and standard deviation σ. You are given that $\sigma = 0.25\mu$ and $P(X < 6.8) = 0.75$.

(i) Find the value of μ. [4]

(ii) Find $P(X < 4.7)$. [3]

(b) The lengths of metal rods have a normal distribution with mean 16 cm and standard deviation 0.2 cm. Rods which are shorter than 15.75 cm or longer than 16.25 cm are not usable. Find the expected number of usable rods in a batch of 1000 rods. [4]

332. 9709_s17_qp_62 Q: 5

The lengths of videos of a certain popular song have a normal distribution with mean 3.9 minutes. 18% of these videos last for longer than 4.2 minutes.

(i) Find the standard deviation of the lengths of these videos. [3]

(ii) Find the probability that the length of a randomly chosen video differs from the mean by less than half a minute. [4]

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The lengths of videos of another popular song have a normal distribution with the same mean of 3.9 minutes but the standard deviation is twice the standard deviation in part **(i)**. The probability that the length of a randomly chosen video of this song differs from the mean by less than half a minute is denoted by p.

(iii) Without any further calculation, determine whether p is more than, equal to, or less than your answer to part **(ii)**. You must explain your reasoning. [2]

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333. 9709_s17_qp_63 Q: 2

The probability that George goes swimming on any day is $\frac{1}{3}$. Use an approximation to calculate the probability that in 270 days George goes swimming at least 100 times. [5]

334. 9709_s17_qp_63 Q: 4

(a) The random variable X has the distribution $N(\mu, \sigma^2)$, where $\mu = 1.5\sigma$. A random value of X is chosen. Find the probability that this value of X is greater than 0. [3]

(b) The life of a particular type of torch battery is normally distributed with mean 120 hours and standard deviation s hours. It is known that 87.5% of these batteries last longer than 70 hours. Find the value of s. [3]

335. 9709_w17_qp_61 Q: 7

The weight, in grams, of pineapples is denoted by the random variable X which has a normal distribution with mean 500 and standard deviation 91.5. Pineapples weighing over 570 grams are classified as 'large'. Those weighing under 390 grams are classified as 'small' and the rest are classified as 'medium'.

(i) Find the proportions of large, small and medium pineapples. [5]

(ii) Find the weight exceeded by the heaviest 5% of pineapples. [3]

(iii) Find the value of k such that $P(k < X < 610) = 0.3$. [5]

336. 9709_w17_qp_62 Q: 5

Blank CDs are packed in boxes of 30. The probability that a blank CD is faulty is 0.04. A box is rejected if more than 2 of the blank CDs are faulty.

(i) Find the probability that a box is rejected. [3]

(ii) 280 boxes are chosen randomly. Use an approximation to find the probability that at least 30 of these boxes are rejected. [5]

337. 9709_w17_qp_62 Q: 7

In Jimpuri the weights, in kilograms, of boys aged 16 years have a normal distribution with mean 61.4 and standard deviation 12.3.

(i) Find the probability that a randomly chosen boy aged 16 years in Jimpuri weighs more than 65 kilograms. [3]

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(ii) For boys aged 16 years in Jimpuri, 25% have a weight between 65 kilograms and k kilograms, where k is greater than 65. Find k. [4]

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In Brigville the weights, in kilograms, of boys aged 16 years have a normal distribution. 99% of the boys weigh less than 97.2 kilograms and 33% of the boys weigh less than 55.2 kilograms.

(iii) Find the mean and standard deviation of the weights of boys aged 16 years in Brigville. [5]

338. 9709_w17_qp_63 Q: 7

Josie aims to catch a bus which departs at a fixed time every day. Josie arrives at the bus stop T minutes before the bus departs, where $T \sim \mathrm{N}(5.3, 2.1^2)$.

(i) Find the probability that Josie has to wait longer than 6 minutes at the bus stop. [3]

On 5% of days Josie has to wait longer than x minutes at the bus stop.

(ii) Find the value of x. [3]

(iii) Find the probability that Josie waits longer than x minutes on fewer than 3 days in 10 days. [3]

(iv) Find the probability that Josie misses the bus. [3]

339. 9709_m16_qp_62 Q: 7

The times taken by a garage to fit a tow bar onto a car have a normal distribution with mean m hours and standard deviation 0.35 hours. It is found that 95% of times taken are longer than 0.9 hours.

(i) Find the value of m. [3]

(ii) On one day 4 cars have a tow bar fitted. Find the probability that none of them takes more than 2 hours to fit. [5]

The times in hours taken by another garage to fit a tow bar onto a car have the distribution $N(\mu, \sigma^2)$ where $\mu = 3\sigma$.

(iii) Find the probability that it takes more than 0.6μ hours to fit a tow bar onto a randomly chosen car at this garage. [3]

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340. 9709_s16_qp_61 Q: 1

The height of maize plants in Mpapwa is normally distributed with mean 1.62 m and standard deviation σ m. The probability that a randomly chosen plant has a height greater than 1.8 m is 0.15. Find the value of σ. [3]

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341. 9709_s16_qp_61 Q: 5

Plastic drinking straws are manufactured to fit into drinks cartons which have a hole in the top. A straw fits into the hole if the diameter of the straw is less than 3 mm. The diameters of the straws have a normal distribution with mean 2.6 mm and standard deviation 0.25 mm.

(i) A straw is chosen at random. Find the probability that it fits into the hole in a drinks carton. [3]

(ii) 500 straws are chosen at random. Use a suitable approximation to find the probability that at least 480 straws fit into the holes in drinks cartons. [5]

(iii) Justify the use of your approximation. [1]

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342. 9709_s16_qp_62 Q: 2

When visiting the dentist the probability of waiting less than 5 minutes is 0.16, and the probability of waiting less than 10 minutes is 0.88.

(i) Find the probability of waiting between 5 and 10 minutes. [1]

A random sample of 180 people who visit the dentist is chosen.

(ii) Use a suitable approximation to find the probability that more than 115 of these people wait between 5 and 10 minutes. [5]

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343. 9709_s16_qp_62 Q: 6

The time in minutes taken by Peter to walk to the shop and buy a newspaper is normally distributed with mean 9.5 and standard deviation 1.3.

(i) Find the probability that on a randomly chosen day Peter takes longer than 10.2 minutes. [3]

(ii) On 90% of days he takes longer than t minutes. Find the value of t. [3]

(iii) Calculate an estimate of the number of days in a year (365 days) on which Peter takes less than 8.8 minutes to walk to the shop and buy a newspaper. [3]

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344. 9709_s16_qp_63 Q: 5

The heights of school desks have a normal distribution with mean 69 cm and standard deviation σ cm. It is known that 15.5% of these desks have a height greater than 70 cm.

(i) Find the value of σ. [3]

When Jodu sits at a desk, his knees are at a height of 58 cm above the floor. A desk is comfortable for Jodu if his knees are at least 9 cm below the top of the desk. Jodu's school has 300 desks.

(ii) Calculate an estimate of the number of these desks that are comfortable for Jodu. [5]

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345. 9709_s16_qp_63 Q: 7

Passengers are travelling to Picton by minibus. The probability that each passenger carries a backpack is 0.65, independently of other passengers. Each minibus has seats for 12 passengers.

(i) Find the probability that, in a full minibus travelling to Picton, between 8 passengers and 10 passengers inclusive carry a backpack. [3]

(ii) Passengers get on to an empty minibus. Find the probability that the fourth passenger who gets on to the minibus will be the first to be carrying a backpack. [2]

(iii) Find the probability that, of a random sample of 250 full minibuses travelling to Picton, more than 54 will contain exactly 7 passengers carrying backpacks. [6]

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346. 9709_w16_qp_61 Q: 4

Packets of rice are filled by a machine and have weights which are normally distributed with mean 1.04 kg and standard deviation 0.017 kg.

(i) Find the probability that a randomly chosen packet weighs less than 1 kg. [3]

(ii) How many packets of rice, on average, would the machine fill from 1000 kg of rice? [1]

The factory manager wants to produce more packets of rice. He changes the settings on the machine so that the standard deviation is the same but the mean is reduced to μ kg. With this mean the probability that a packet weighs less than 1 kg is 0.0388.

(iii) Find the value of μ. [3]

(iv) How many packets of rice, on average, would the machine now fill from 1000 kg of rice? [1]

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347. 9709_w16_qp_62 Q: 3

On any day at noon, the probabilities that Kersley is asleep or studying are 0.2 and 0.6 respectively.

(i) Find the probability that, in any 7-day period, Kersley is either asleep or studying at noon on at least 6 days. [3]

(ii) Use an approximation to find the probability that, in any period of 100 days, Kersley is asleep at noon on at most 30 days. [5]

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348. 9709_w16_qp_62 Q: 4

The time taken to cook an egg by people living in a certain town has a normal distribution with mean 4.2 minutes and standard deviation 0.6 minutes.

(i) Find the probability that a person chosen at random takes between 3.5 and 4.5 minutes to cook an egg. [3]

12% of people take more than t minutes to cook an egg.

(ii) Find the value of t. [3]

(iii) A random sample of n people is taken. Find the smallest possible value of n if the probability that none of these people takes more than t minutes to cook an egg is less than 0.003. [3]

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349. 9709_w16_qp_63 Q: 6

The weights of bananas in a fruit shop have a normal distribution with mean 150 grams and standard deviation 50 grams. Three sizes of banana are sold.

Small: under 95 grams
Medium: between 95 grams and 205 grams
Large: over 205 grams

(i) Find the proportion of bananas that are small. [3]

(ii) Find the weight exceeded by 10% of bananas. [3]

The prices of bananas are 10 cents for a small banana, 20 cents for a medium banana and 25 cents for a large banana.

(iii) **(a)** Show that the probability that a randomly chosen banana costs 20 cents is 0.7286. [1]

(b) Calculate the expected total cost of 100 randomly chosen bananas. [3]

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350. 9709_w16_qp_63 Q: 7

Each day Annabel eats rice, potato or pasta. Independently of each other, the probability that she eats rice is 0.75, the probability that she eats potato is 0.15 and the probability that she eats pasta is 0.1.

(i) Find the probability that, in any week of 7 days, Annabel eats pasta on exactly 2 days. [2]

(ii) Find the probability that, in a period of 5 days, Annabel eats rice on 2 days, potato on 1 day and pasta on 2 days. [3]

(iii) Find the probability that Annabel eats potato on more than 44 days in a year of 365 days. [5]

351. 9709_s15_qp_61 Q: 1

The lengths, in metres, of cars in a city are normally distributed with mean μ and standard deviation 0.714. The probability that a randomly chosen car has a length more than 3.2 metres and less than μ metres is 0.475. Find μ. [4]

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352. 9709_s15_qp_61 Q: 6

(i) In a certain country, 68% of households have a printer. Find the probability that, in a random sample of 8 households, 5, 6 or 7 households have a printer. [4]

(ii) Use an approximation to find the probability that, in a random sample of 500 households, more than 337 households have a printer. [5]

(iii) Justify your use of the approximation in part **(ii)**. [1]

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353. 9709_s15_qp_62 Q: 7

(a) Once a week Zak goes for a run. The time he takes, in minutes, has a normal distribution with mean 35.2 and standard deviation 4.7.

(i) Find the expected number of days during a year (52 weeks) for which Zak takes less than 30 minutes for his run. [4]

(ii) The probability that Zak's time is between 35.2 minutes and t minutes, where $t > 35.2$, is 0.148. Find the value of t. [3]

(b) The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 7) = 0.2119$ and $P(X < 10) = 0.6700$. Find the values of μ and σ. [5]

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354. 9709_s15_qp_63 Q: 1

The weights, in grams, of onions in a supermarket have a normal distribution with mean μ and standard deviation 22. The probability that a randomly chosen onion weighs more than 195 grams is 0.128. Find the value of μ. [3]

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355. 9709_s15_qp_63 Q: 3

On a production line making cameras, the probability of a randomly chosen camera being substandard is 0.072. A random sample of 300 cameras is checked. Find the probability that there are fewer than 18 cameras which are substandard. [5]

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356. 9709_s15_qp_63 Q: 5

The heights of books in a library, in cm, have a normal distribution with mean 21.7 and standard deviation 6.5. A book with a height of more than 29 cm is classified as 'large'.

(i) Find the probability that, of 8 books chosen at random, fewer than 2 books are classified as large. [6]

(ii) n books are chosen at random. The probability of there being at least 1 large book is more than 0.98. Find the least possible value of n. [3]

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357. 9709_w15_qp_61 Q: 2

The random variable X has the distribution $N(\mu, \sigma^2)$. It is given that $P(X < 54.1) = 0.5$ and $P(X > 50.9) = 0.8665$. Find the values of μ and σ. [4]

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358. 9709_w15_qp_61 Q: 4

(a) Amy measured her pulse rate while resting, x beats per minute, at the same time each day on 30 days. The results are summarised below.

$$\Sigma(x - 80) = -147 \qquad \Sigma(x - 80)^2 = 952$$

Find the mean and standard deviation of Amy's pulse rate. [4]

(b) Amy's friend Marok measured her pulse rate every day after running for half an hour. Marok's pulse rate, in beats per minute, was found to have a mean of 148.6 and a standard deviation of 18.5. Assuming that pulse rates have a normal distribution, find what proportion of Marok's pulse rates, after running for half an hour, were above 160 beats per minute. [3]

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359. 9709_w15_qp_61 Q: 7

The faces of a biased die are numbered 1, 2, 3, 4, 5 and 6. The probabilities of throwing odd numbers are all the same. The probabilities of throwing even numbers are all the same. The probability of throwing an odd number is twice the probability of throwing an even number.

(i) Find the probability of throwing a 3. [3]

(ii) The die is thrown three times. Find the probability of throwing two 5s and one 4. [3]

(iii) The die is thrown 100 times. Use an approximation to find the probability that an even number is thrown at most 37 times. [5]

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360. 9709_w15_qp_62 Q: 7

(a) A petrol station finds that its daily sales, in litres, are normally distributed with mean 4520 and standard deviation 560.

(i) Find on how many days of the year (365 days) the daily sales can be expected to exceed 3900 litres. [4]

The daily sales at another petrol station are X litres, where X is normally distributed with mean m and standard deviation 560. It is given that $P(X > 8000) = 0.122$.

(ii) Find the value of m. [3]

(iii) Find the probability that daily sales at this petrol station exceed 8000 litres on fewer than 2 of 6 randomly chosen days. [3]

(b) The random variable Y is normally distributed with mean μ and standard deviation σ. Given that $\sigma = \frac{2}{3}\mu$, find the probability that a random value of Y is less than 2μ. [3]

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361. 9709_w15_qp_63 Q: 4

The time taken for cucumber seeds to germinate under certain conditions has a normal distribution with mean 125 hours and standard deviation σ hours.

(i) It is found that 13% of seeds take longer than 136 hours to germinate. Find the value of σ. [3]

(ii) 170 seeds are sown. Find the expected number of seeds which take between 131 and 141 hours to germinate. [4]

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362. 9709_w15_qp_63 Q: 7

A factory makes water pistols, 8% of which do not work properly.

(i) A random sample of 19 water pistols is taken. Find the probability that at most 2 do not work properly. [3]

(ii) In a random sample of n water pistols, the probability that at least one does not work properly is greater than 0.9. Find the smallest possible value of n. [3]

(iii) A random sample of 1800 water pistols is taken. Use an approximation to find the probability that there are at least 152 that do not work properly. [5]

(iv) Justify the use of your approximation in part **(iii)**. [1]