Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

5026516164

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2016**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **5** printed pages and **3** blank pages.

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1 **(i)** Find the coefficients of x^4 and x^5 in the expansion of $(1 - 2x)^5$. [2]

(ii) It is given that, when $(1 + px)(1 - 2x)^5$ is expanded, there is no term in x^5. Find the value of the constant p. [2]

2 A curve for which $\dfrac{dy}{dx} = 3x^2 - \dfrac{2}{x^3}$ passes through $(-1, 3)$. Find the equation of the curve. [4]

3 The 12th term of an arithmetic progression is 17 and the sum of the first 31 terms is 1023. Find the 31st term. [5]

4 **(a)** Solve the equation $\sin^{-1}(3x) = -\frac{1}{3}\pi$, giving the solution in an exact form. [2]

(b) Solve, by factorising, the equation $2\cos\theta\sin\theta - 2\cos\theta - \sin\theta + 1 = 0$ for $0 \leqslant \theta \leqslant \pi$. [4]

5 Two points have coordinates $A\,(5, 7)$ and $B\,(9, -1)$.

(i) Find the equation of the perpendicular bisector of AB. [3]

The line through $C\,(1, 2)$ parallel to AB meets the perpendicular bisector of AB at the point X.

(ii) Find, by calculation, the distance BX. [5]

6 A vacuum flask (for keeping drinks hot) is modelled as a closed cylinder in which the internal radius is r cm and the internal height is h cm. The volume of the flask is $1000\,\text{cm}^3$. A flask is most efficient when the total internal surface area, $A\,\text{cm}^2$, is a minimum.

(i) Show that $A = 2\pi r^2 + \dfrac{2000}{r}$. [3]

(ii) Given that r can vary, find the value of r, correct to 1 decimal place, for which A has a stationary value and verify that the flask is most efficient when r takes this value. [5]

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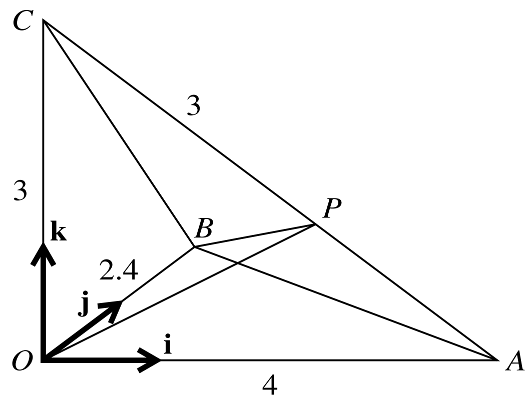

The diagram shows a pyramid $OABC$ with a horizontal triangular base OAB and vertical height OC. Angles AOB, BOC and AOC are each right angles. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OB and OC respectively, with $OA = 4$ units, $OB = 2.4$ units and $OC = 3$ units. The point P on CA is such that $CP = 3$ units.

(i) Show that $\overrightarrow{CP} = 2.4\mathbf{i} - 1.8\mathbf{k}$. [2]

(ii) Express $\overrightarrow{OP}$ and $\overrightarrow{BP}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(iii) Use a scalar product to find angle BPC. [4]

8 The function f is such that $\mathrm{f}(x) = a^2x^2 - ax + 3b$ for $x \leqslant \dfrac{1}{2a}$, where a and b are constants.

(i) For the case where $\mathrm{f}(-2) = 4a^2 - b + 8$ and $\mathrm{f}(-3) = 7a^2 - b + 14$, find the possible values of a and b. [5]

(ii) For the case where $a = 1$ and $b = -1$, find an expression for $\mathrm{f}^{-1}(x)$ and give the domain of f^{-1}. [5]

9 **(a)**

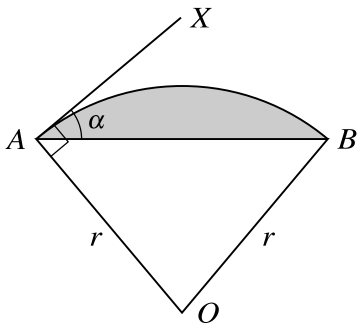

Fig. 1

In Fig. 1, OAB is a sector of a circle with centre O and radius r. AX is the tangent at A to the arc AB and angle $BAX = \alpha$.

(i) Show that angle $AOB = 2\alpha$. [2]

(ii) Find the area of the shaded segment in terms of r and α. [2]

(b)

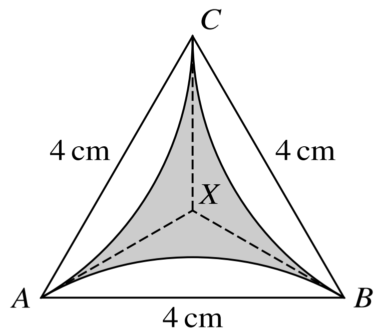

Fig. 2

In Fig. 2, ABC is an equilateral triangle of side 4 cm. The lines AX, BX and CX are tangents to the equal circular arcs AB, BC and CA. Use the results in part **(a)** to find the area of the shaded region, giving your answer in terms of π and $\sqrt{3}$. [6]

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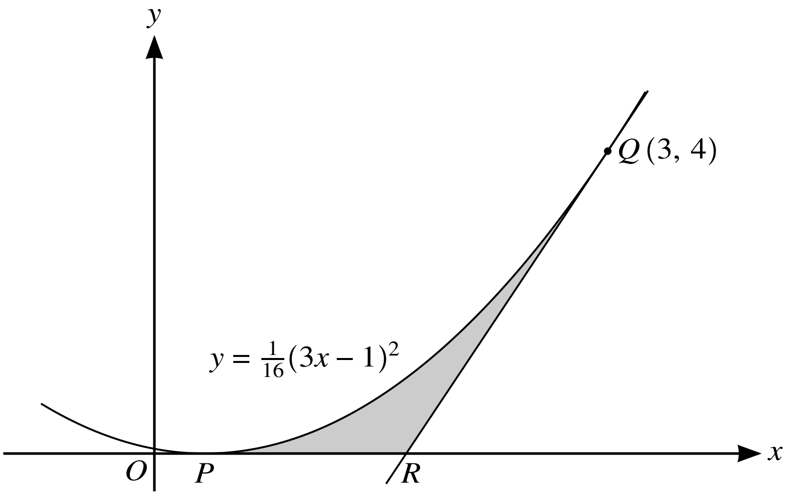

The diagram shows part of the curve $y = \frac{1}{16}(3x - 1)^2$, which touches the x-axis at the point P. The point $Q\,(3, 4)$ lies on the curve and the tangent to the curve at Q crosses the x-axis at R.

(i) State the x-coordinate of P. [1]

Showing all necessary working, find by calculation

(ii) the x-coordinate of R, [5]

(iii) the area of the shaded region PQR. [6]

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CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

8253559189

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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This document consists of **20** printed pages.

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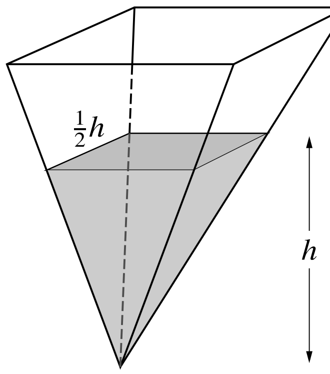
[Turn over

1 Find the set of values of k for which the equation $2x^2 + 3kx + k = 0$ has distinct real roots. [4]

2 In the expansion of $\left(\frac{1}{ax} + 2ax^2\right)^5$, the coefficient of x is 5. Find the value of the constant a. [4]

3



The diagram shows a water container in the form of an inverted pyramid, which is such that when the height of the water level is h cm the surface of the water is a square of side $\frac{1}{2}h$ cm.

(i) Express the volume of water in the container in terms of h. [1]

[The volume of a pyramid having a base area A and vertical height h is $\frac{1}{3}Ah$.]

Water is steadily dripping into the container at a constant rate of $20\,cm^3$ per minute.

(ii) Find the rate, in cm per minute, at which the water level is rising when the height of the water level is 10 cm. [4]

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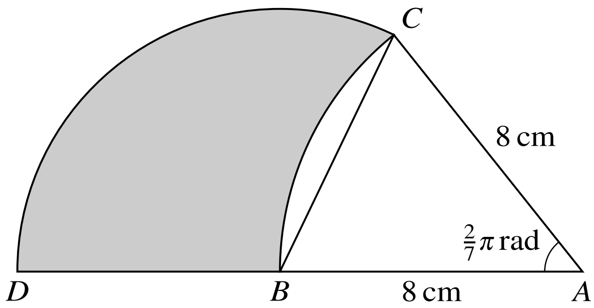

In the diagram, $AB = AC = 8$ cm and angle $CAB = \frac{2}{7}\pi$ radians. The circular arc BC has centre A, the circular arc CD has centre B and ABD is a straight line.

(i) Show that angle $CBD = \frac{9}{14}\pi$ radians. [1]

(ii) Find the perimeter of the shaded region. [5]

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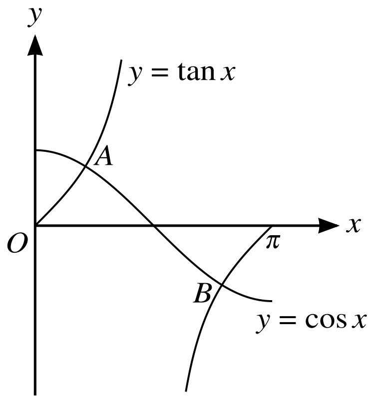

The diagram shows the graphs of $y = \tan x$ and $y = \cos x$ for $0 \leqslant x \leqslant \pi$. The graphs intersect at points A and B.

(i) Find by calculation the x-coordinate of A. [4]

(ii) Find by calculation the coordinates of B. [3]

6 Relative to an origin O, the position vectors of the points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 7\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}.$$

(i) Use a scalar product to find angle OAB. [5]

(ii) Find the area of triangle *OAB*. [2]

7 The function f is defined for $x \geqslant 0$ by $f(x) = (4x + 1)^{\frac{3}{2}}$.

(i) Find $f'(x)$ and $f''(x)$. [3]

The first, second and third terms of a geometric progression are respectively $f(2)$, $f'(2)$ and $kf''(2)$.

(ii) Find the value of the constant k. [5]

8 The functions f and g are defined for $x \geqslant 0$ by

$$\mathrm{f} : x \mapsto 2x^2 + 3,$$
$$\mathrm{g} : x \mapsto 3x + 2.$$

(i) Show that $\mathrm{gf}(x) = 6x^2 + 11$ and obtain an unsimplified expression for $\mathrm{fg}(x)$. [2]

(ii) Find an expression for $(\mathrm{fg})^{-1}(x)$ and determine the domain of $(\mathrm{fg})^{-1}$. [5]

(iii) Solve the equation $gf(2x) = fg(x)$. [3]

9 The point $A\,(2,\ 2)$ lies on the curve $y = x^2 - 2x + 2$.

(i) Find the equation of the tangent to the curve at A. [3]

The normal to the curve at A intersects the curve again at B.

(ii) Find the coordinates of B. [4]

The tangents at A and B intersect each other at C.

(iii) Find the coordinates of C. [4]

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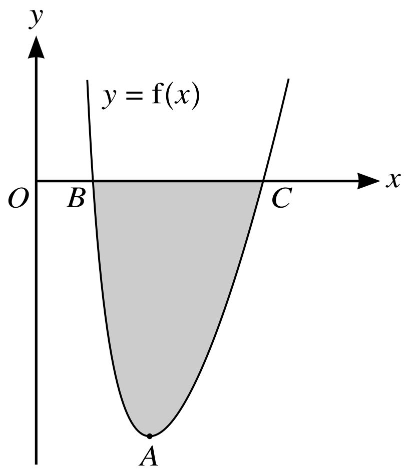

The diagram shows the curve $y = \mathrm{f}(x)$ defined for $x > 0$. The curve has a minimum point at A and crosses the x-axis at B and C. It is given that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2x - \dfrac{2}{x^3}$ and that the curve passes through the point $\left(4, \frac{189}{16}\right)$.

(i) Find the x-coordinate of A. [2]

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(ii) Find $\mathrm{f}(x)$. [3]

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(iii) Find the x-coordinates of B and C. [4]

[Question 10 (iv) is printed on the next page.]

(iv) Find, showing all necessary working, the area of the shaded region. [4]

Cambridge International AS & A Level

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8348568421

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **February/March 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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[Turn over

1 A curve passes through the point (4, −6) and has an equation for which $\frac{dy}{dx} = x^{-\frac{1}{2}} - 3$. Find the equation of the curve. [4]

2 **(i)** Find the coefficients of x^2 and x^3 in the expansion of $(1 - 2x)^7$. [3]

(ii) Hence find the coefficient of x^3 in the expansion of $(2 + 5x)(1 - 2x)^7$. [2]

3 On a certain day, the height of a young bamboo plant was found to be 40 cm. After exactly one day its height was found to be 41.2 cm. Two different models are used to predict its height exactly 60 days after it was first measured.

- Model A assumes that the daily amount of growth continues to be constant at the amount found for the first day.
- Model B assumes that the daily percentage rate of growth continues to be constant at the percentage rate of growth found for the first day.

(i) Using model A, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [2]

(ii) Using model B, find the predicted height in cm of the bamboo plant exactly 60 days after it was first measured. [3]

4 A straight line cuts the positive x-axis at A and the positive y-axis at $B\,(0,\ 2)$. Angle $BAO = \frac{1}{6}\pi$ radians, where O is the origin.

(i) Find the exact value of the x-coordinate of A. [2]

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(ii) Find the equation of the perpendicular bisector of AB, giving your answer in the form $y = mx + c$, where m is given exactly and c is an integer. [4]

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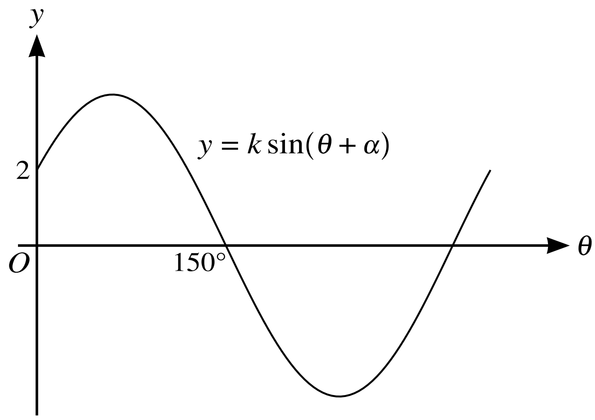
5 **(a)** Express the equation $\frac{5 + 2\tan x}{3 + 2\tan x} = 1 + \tan x$ as a quadratic equation in $\tan x$ and hence solve the equation for $0 \leqslant x \leqslant \pi$. [4]

(b)



The diagram shows part of the graph of $y = k\sin(\theta + \alpha)$, where k and α are constants and $0° < \alpha < 180°$. Find the value of α and the value of k. [2]

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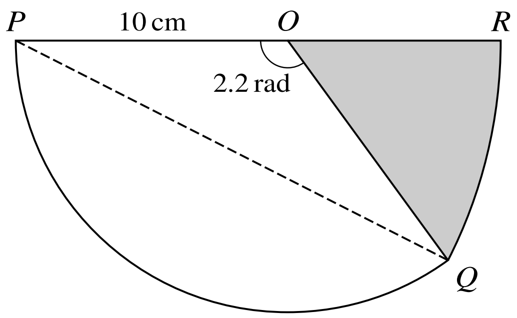

The diagram shows a sector POQ of a circle of radius 10 cm and centre O. Angle POQ is 2.2 radians. QR is an arc of a circle with centre P and POR is a straight line.

(i) Show that the length of PQ is 17.8 cm, correct to 3 significant figures. [2]

(ii) Find the perimeter of the shaded region. [4]

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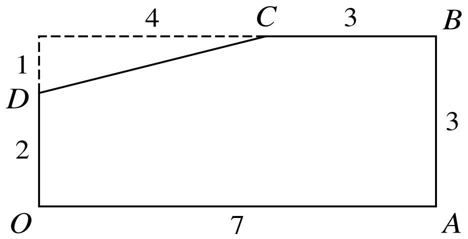

Fig. 1

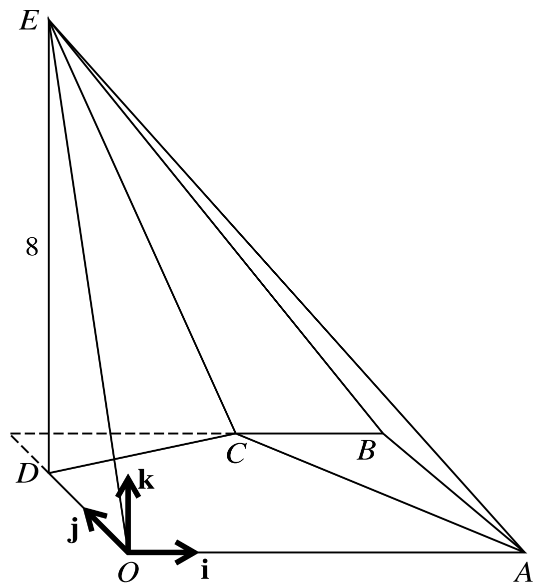

Fig. 2

Fig. 1 shows a rectangle with sides of 7 units and 3 units from which a triangular corner has been removed, leaving a 5-sided polygon $OABCD$. The sides OA, AB, BC and DO have lengths of 7 units, 3 units, 3 units and 2 units respectively. Fig. 2 shows the polygon $OABCD$ forming the horizontal base of a pyramid in which the point E is 8 units vertically above D. Unit vectors **i**, **j** and **k** are parallel to OA, OD and DE respectively.

(i) Find $\overrightarrow{CE}$ and the length of CE. [3]

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(ii) Use a scalar product to find angle ECA, giving your answer in the form $\cos^{-1}\left(\frac{m}{\sqrt{n}}\right)$, where m and n are integers. [5]

8 A curve has equation $y = \frac{1}{2}x^2 - 4x^{\frac{3}{2}} + 8x$.

(i) Find the x-coordinates of the stationary points. [5]

(ii) Find $\frac{d^2y}{dx^2}$. [1]

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(iii) Find, showing all necessary working, the nature of each stationary point. [2]

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9 A curve has equation $y = \frac{1}{x} + c$ and a line has equation $y = cx - 3$, where c is a constant.

(i) Find the set of values of c for which the curve and the line meet. [4]

(ii) The line is a tangent to the curve for two particular values of c. For each of these values find the x-coordinate of the point at which the tangent touches the curve. [4]

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10 Functions f and g are defined by

$$f(x) = \frac{8}{x-2} + 2 \quad \text{for } x > 2,$$
$$g(x) = \frac{8}{x-2} + 2 \quad \text{for } 2 < x < 4.$$

(i) **(a)** State the range of the function f. [1]

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(b) State the range of the function g. [1]

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(c) State the range of the function fg. [1]

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(ii) Explain why the function gf cannot be formed. [1]

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(iii) Find the set of values of x satisfying the inequality $6\mathrm{f}'(x) + 2\mathrm{f}^{-1}(x) - 5 < 0$. [6]

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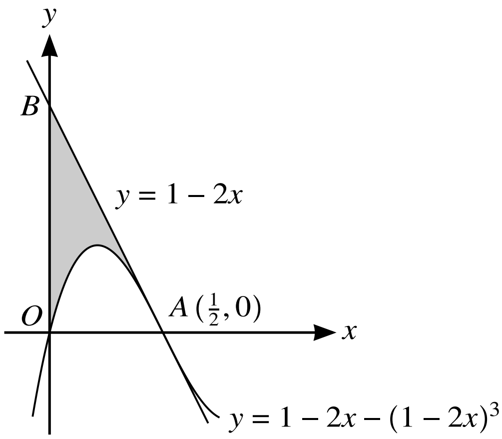

The diagram shows part of the curve $y = 1 - 2x - (1 - 2x)^3$ intersecting the x-axis at the origin O and at $A\left(\frac{1}{2}, 0\right)$. The line AB intersects the y-axis at B and has equation $y = 1 - 2x$.

(i) Show that AB is the tangent to the curve at A. [4]

(ii) Show that the area of the shaded region can be expressed as $\int_0^{\frac{1}{2}} (1-2x)^3 \, dx$. [2]

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(iii) Hence, showing all necessary working, find the area of the shaded region. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

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* 8 8 0 9 6 1 8 5 8 4 *

MATHEMATICS **9709/12**

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1 hour 45 minutes

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JC19 03_9709_12/RP

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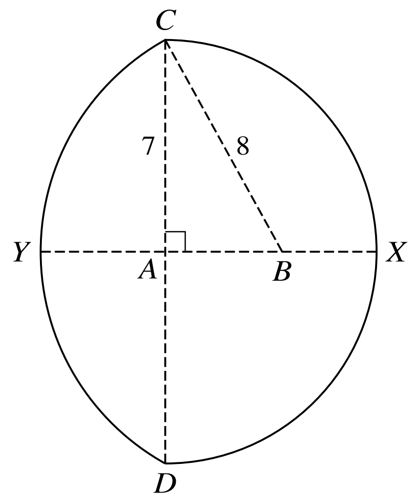
1 The coefficient of x^3 in the expansion of $(1 - px)^5$ is -2160. Find the value of the constant p. [3]

2 A curve with equation $y = f(x)$ passes through the points (0, 2) and (3, −1). It is given that $f'(x) = kx^2 - 2x$, where k is a constant. Find the value of k. [5]

3



In the diagram, CXD is a semicircle of radius 7 cm with centre A and diameter CD. The straight line $YABX$ is perpendicular to CD, and the arc CYD is part of a circle with centre B and radius 8 cm. Find the total area of the region enclosed by the two arcs. [6]

4 A curve has equation $y = (2x - 1)^{-1} + 2x$.

(i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(ii) Find the x-coordinates of the stationary points and, showing all necessary working, determine the nature of each stationary point. [4]

5 Two vectors, **u** and **v**, are such that

$$\mathbf{u} = \begin{pmatrix} q \\ 2 \\ 6 \end{pmatrix} \quad \text{and} \quad \mathbf{v} = \begin{pmatrix} 8 \\ q-1 \\ q^2-7 \end{pmatrix},$$

where q is a constant.

(i) Find the values of q for which **u** is perpendicular to **v**. [3]

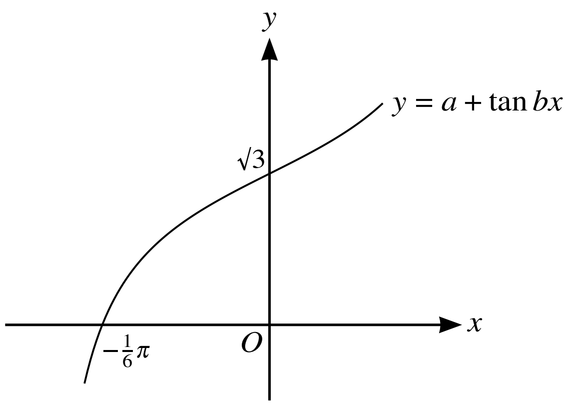
(ii) Find the angle between **u** and **v** when $q = 0$. [4]

6 **(i)** The first and second terms of a geometric progression are p and $2p$ respectively, where p is a positive constant. The sum of the first n terms is greater than $1000p$. Show that $2^n > 1001$. [2]

(ii) In another case, p and $2p$ are the first and second terms respectively of an arithmetic progression. The nth term is 336 and the sum of the first n terms is 7224. Write down two equations in n and p and hence find the values of n and p. [5]

7 **(a)** Solve the equation $3\sin^2 2\theta + 8\cos 2\theta = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

(b)



The diagram shows part of the graph of $y = a + \tan bx$, where x is measured in radians and a and b are constants. The curve intersects the x-axis at $\left(-\frac{1}{6}\pi, 0\right)$ and the y-axis at $(0, \sqrt{3})$. Find the values of a and b. [3]

8 **(i)** Express $x^2 - 4x + 7$ in the form $(x + a)^2 + b$. [2]

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The function f is defined by $f(x) = x^2 - 4x + 7$ for $x < k$, where k is a constant.

(ii) State the largest value of k for which f is a decreasing function. [1]

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The value of k is now given to be 1.

(iii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(iv) The function g is defined by $g(x) = \dfrac{2}{x-1}$ for $x > 1$. Find an expression for $gf(x)$ and state the range of gf. [4]

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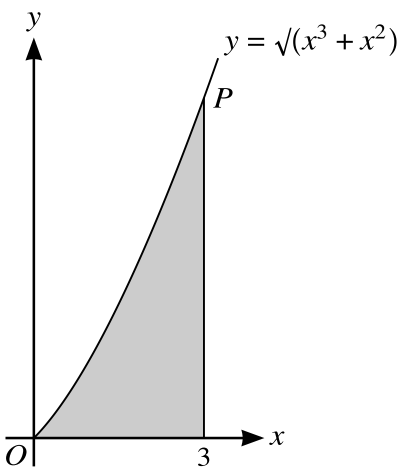

The diagram shows part of the curve with equation $y = \sqrt{(x^3 + x^2)}$. The shaded region is bounded by the curve, the x-axis and the line $x = 3$.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

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(ii) P is the point on the curve with x-coordinate 3. Find the y-coordinate of the point where the normal to the curve at P crosses the y-axis. [6]

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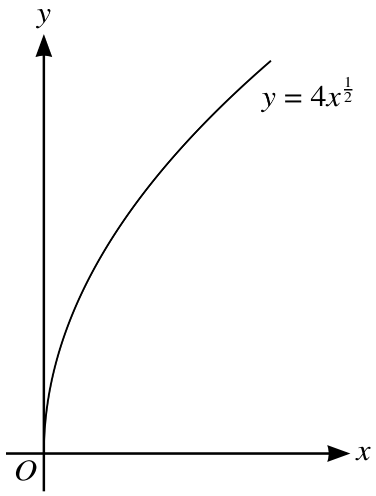

The diagram shows the curve with equation $y = 4x^{\frac{1}{2}}$.

(i) The straight line with equation $y = x + 3$ intersects the curve at points A and B. Find the length of AB. [6]

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(ii) The tangent to the curve at a point T is parallel to AB. Find the coordinates of T. [3]

(iii) Find the coordinates of the point of intersection of the normal to the curve at T with the line AB. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME			
CENTRE NUMBER		CANDIDATE NUMBER	

1939478240

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 03_9709_12/FP

1 The function f is defined by $f(x) = \dfrac{1}{3x+2} + x^2$ for $x < -1$.

Determine whether f is an increasing function, a decreasing function or neither. [3]

2 The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 1 + \mathrm{f}\left(\frac{1}{2}x\right)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [4]

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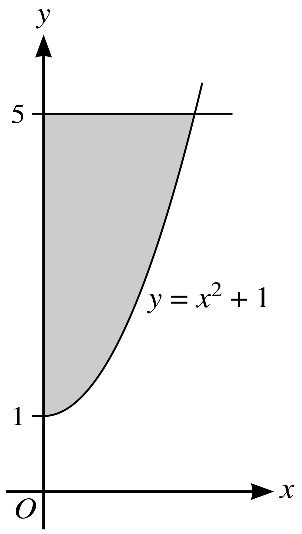

The diagram shows part of the curve with equation $y = x^2 + 1$. The shaded region enclosed by the curve, the y-axis and the line $y = 5$ is rotated through 360° about the ***y*-axis**.

Find the volume obtained. [4]

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4 A curve has equation $y = x^2 - 2x - 3$. A point is moving along the curve in such a way that at P the y-coordinate is increasing at 4 units per second and the x-coordinate is increasing at 6 units per second.

Find the x-coordinate of P. [4]

5 Solve the equation

$$\frac{\tan\theta + 3\sin\theta + 2}{\tan\theta - 3\sin\theta + 1} = 2$$

for $0° \leqslant \theta \leqslant 90°$. [5]

6 The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{a}{x^2}\right)^5$ is 720.

(a) Find the possible values of the constant a. [3]

(b) Hence find the coefficient of $\frac{1}{x^7}$ in the expansion. [2]

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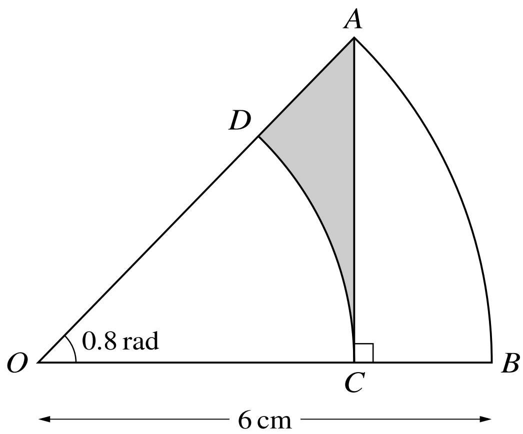

The diagram shows a sector AOB which is part of a circle with centre O and radius 6 cm and with angle $AOB = 0.8$ radians. The point C on OB is such that AC is perpendicular to OB. The arc CD is part of a circle with centre O, where D lies on OA.

Find the area of the shaded region. [6]

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8 A woman's basic salary for her first year with a particular company is \$30 000 and at the end of the year she also gets a bonus of \$600.

(a) For her first year, express her bonus as a percentage of her basic salary. [1]

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At the end of each complete year, the woman's basic salary will increase by 3% and her bonus will increase by \$100.

(b) Express the bonus she will be paid at the end of her 24th year as a percentage of the basic salary paid during that year. [5]

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9 **(a)** Express $2x^2 + 12x + 11$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

The function f is defined by $f(x) = 2x^2 + 12x + 11$ for $x \leqslant -4$.

(b) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

The function g is defined by $g(x) = 2x - 3$ for $x \leqslant k$.

(c) For the case where $k = -1$, solve the equation $fg(x) = 193$. [2]

(d) State the largest value of k possible for the composition fg to be defined. [1]

10 The gradient of a curve at the point (x, y) is given by $\frac{dy}{dx} = 2(x+3)^{\frac{1}{2}} - x$. The curve has a stationary point at $(a, 14)$, where a is a positive constant.

(a) Find the value of a. [3]

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(b) Determine the nature of the stationary point. [3]

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(c) Find the equation of the curve. [4]

11 **(a)** Solve the equation $3\tan^2 x - 5\tan x - 2 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

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(b) Find the set of values of k for which the equation $3\tan^2 x - 5\tan x + k = 0$ has no solutions. [2]

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(c) For the equation $3\tan^2 x - 5\tan x + k = 0$, state the value of k for which there are three solutions in the interval $0° \leqslant x \leqslant 180°$, and find these solutions. [3]

12 A diameter of a circle C_1 has end-points at $(-3, -5)$ and $(7, 3)$.

(a) Find an equation of the circle C_1. [3]

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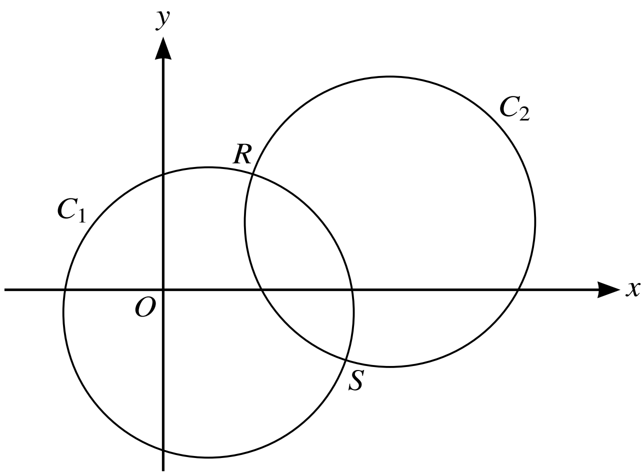

The circle C_1 is translated by $\begin{pmatrix} 8 \\ 4 \end{pmatrix}$ to give circle C_2, as shown in the diagram.

(b) Find an equation of the circle C_2. [2]

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The two circles intersect at points R and S.

(c) Show that the equation of the line RS is $y = -2x + 13$. [4]

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(d) Hence show that the x-coordinates of R and S satisfy the equation $5x^2 - 60x + 159 = 0$. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5712505207

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **February/March 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 **(a)** Find the first three terms in the expansion, in ascending powers of x, of $(1 + x)^5$. [1]

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(b) Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^6$. [2]

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(c) Hence find the coefficient of x^2 in the expansion of $(1 + x)^5(1 - 2x)^6$. [2]

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2 By using a suitable substitution, solve the equation

$$(2x-3)^2 - \frac{4}{(2x-3)^2} - 3 = 0.$$ [4]

3 Solve the equation $\dfrac{\tan\theta + 2\sin\theta}{\tan\theta - 2\sin\theta} = 3$ for $0° < \theta < 180°$. [4]

4 A line has equation $y = 3x + k$ and a curve has equation $y = x^2 + kx + 6$, where k is a constant.

Find the set of values of k for which the line and curve have two distinct points of intersection. [5]

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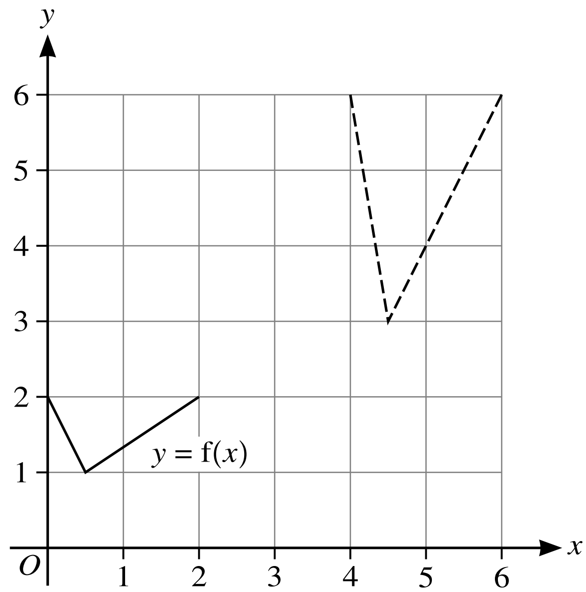

In the diagram, the graph of $y = f(x)$ is shown with solid lines. The graph shown with broken lines is a transformation of $y = f(x)$.

(a) Describe fully the two single transformations of $y = f(x)$ that have been combined to give the resulting transformation. [4]

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(b) State in terms of y, f and x, the equation of the graph shown with broken lines. [2]

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6 A curve is such that $\frac{dy}{dx} = \frac{6}{(3x-2)^3}$ and $A\,(1, -3)$ lies on the curve. A point is moving along the curve and at A the y-coordinate of the point is increasing at 3 units per second.

(a) Find the rate of increase at A of the x-coordinate of the point. [3]

(b) Find the equation of the curve. [4]

7 Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 + 2x + 3 \text{ for } x \leqslant -1,$$
$$\mathrm{g} : x \mapsto 2x + 1 \text{ for } x \geqslant -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x + a)^2 + b$ and state the range of f. [3]

(b) Find an expression for $f^{-1}(x)$. [2]

(c) Solve the equation $gf(x) = 13$. [3]

8 The points $A\,(7,\ 1)$, $B\,(7,\ 9)$ and $C\,(1,\ 9)$ are on the circumference of a circle.

(a) Find an equation of the circle. [5]

(b) Find an equation of the tangent to the circle at B. [2]

9 The first term of a progression is $\cos\theta$, where $0 < \theta < \frac{1}{2}\pi$.

(a) For the case where the progression is geometric, the sum to infinity is $\dfrac{1}{\cos\theta}$.

(i) Show that the second term is $\cos\theta\sin^2\theta$. [3]

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(ii) Find the sum of the first 12 terms when $\theta = \frac{1}{3}\pi$, giving your answer correct to 4 significant figures. [2]

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(b) For the case where the progression is arithmetic, the first two terms are again $\cos\theta$ and $\cos\theta\sin^2\theta$ respectively.

Find the 85th term when $\theta = \frac{1}{3}\pi$. [4]

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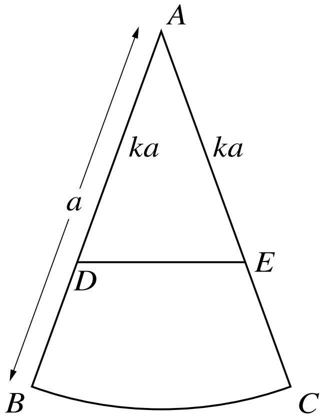

The diagram shows a sector ABC which is part of a circle of radius a. The points D and E lie on AB and AC respectively and are such that $AD = AE = ka$, where $k < 1$. The line DE divides the sector into two regions which are equal in area.

(a) For the case where angle $BAC = \frac{1}{6}\pi$ radians, find k correct to 4 significant figures. [5]

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(b) For the general case in which angle $BAC = \theta$ radians, where $0 < \theta < \frac{1}{2}\pi$, it is given that $\frac{\theta}{\sin\theta} > 1$.

Find the set of possible values of k. [3]

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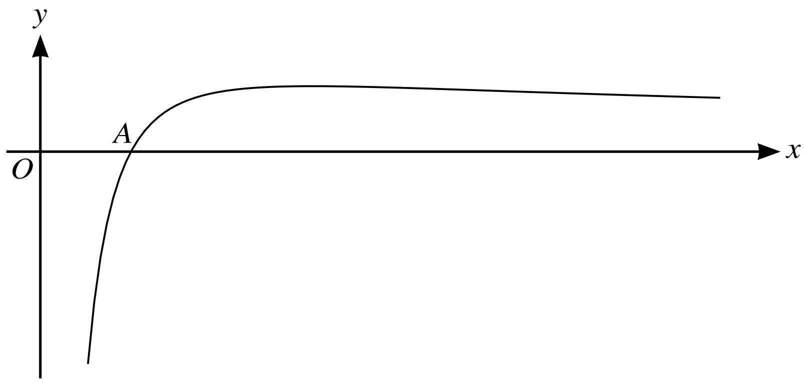

The diagram shows the curve with equation $y = 9\left(x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}\right)$. The curve crosses the x-axis at the point A.

(a) Find the x-coordinate of A. [2]

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(b) Find the equation of the tangent to the curve at A. [4]

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(c) Find the x-coordinate of the maximum point of the curve. [2]

(d) Find the area of the region bounded by the curve, the x-axis and the line $x = 9$. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

1120942846

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **4** printed pages and **1** insert.

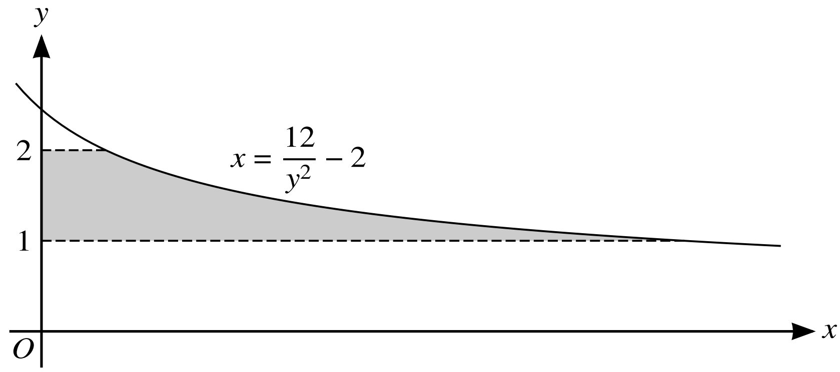
JC16 06_9709_11/2R

CAMBRIDGE International Examinations

1 Find the term independent of x in the expansion of $\left(x - \frac{3}{2x}\right)^6$. [3]

2 Solve the equation $3\sin^2\theta = 4\cos\theta - 1$ for $0° \leqslant \theta \leqslant 360°$. [4]

3



The diagram shows part of the curve $x = \frac{12}{y^2} - 2$. The shaded region is bounded by the curve, the y-axis and the lines $y = 1$ and $y = 2$. Showing all necessary working, find the volume, in terms of π, when this shaded region is rotated through 360° about the y-axis. [5]

4 A curve is such that $\frac{dy}{dx} = 2 - 8(3x + 4)^{-\frac{1}{2}}$.

(i) A point P moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of change of the y-coordinate as P crosses the y-axis. [2]

The curve intersects the y-axis where $y = \frac{4}{3}$.

(ii) Find the equation of the curve. [4]

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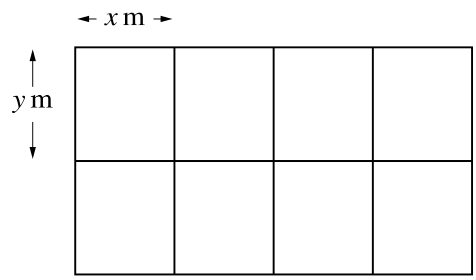

A farmer divides a rectangular piece of land into 8 equal-sized rectangular sheep pens as shown in the diagram. Each sheep pen measures x m by y m and is fully enclosed by metal fencing. The farmer uses 480 m of fencing.

(i) Show that the total area of land used for the sheep pens, A m^2, is given by

$$A = 384x - 9.6x^2.$$ [3]

(ii) Given that x and y can vary, find the dimensions of each sheep pen for which the value of A is a maximum. (There is no need to verify that the value of A is a maximum.) [3]

6 **(a)** Find the values of the constant m for which the line $y = mx$ is a tangent to the curve $y = 2x^2 - 4x + 8$. [3]

(b) The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 + ax + b$, where a and b are constants. The solutions of the equation $f(x) = 0$ are $x = 1$ and $x = 9$. Find

(i) the values of a and b, [2]

(ii) the coordinates of the vertex of the curve $y = f(x)$. [2]

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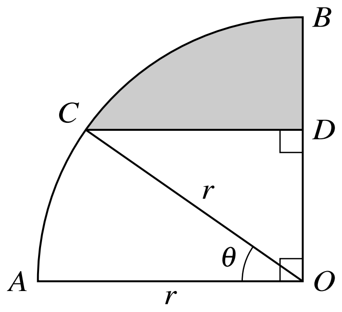

In the diagram, AOB is a quarter circle with centre O and radius r. The point C lies on the arc AB and the point D lies on OB. The line CD is parallel to AO and angle $AOC = \theta$ radians.

(i) Express the perimeter of the shaded region in terms of r, θ and π. [4]

(ii) For the case where $r = 5$ cm and $\theta = 0.6$, find the area of the shaded region. [3]

8 A curve has equation $y = 3x - \dfrac{4}{x}$ and passes through the points $A(1, -1)$ and $B(4, 11)$. At each of the points C and D on the curve, the tangent is parallel to AB. Find the equation of the perpendicular bisector of CD. [7]

9 **(a)** The first term of a geometric progression in which all the terms are positive is 50. The third term is 32. Find the sum to infinity of the progression. [3]

(b) The first three terms of an arithmetic progression are $2\sin x$, $3\cos x$ and $(\sin x + 2\cos x)$ respectively, where x is an acute angle.

(i) Show that $\tan x = \frac{4}{3}$. [3]

(ii) Find the sum of the first twenty terms of the progression. [3]

[Questions 10 and 11 are printed on the next page.]

10 Relative to an origin O, the position vectors of points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$$

respectively, where k is a constant.

(i) Find the value of k in the case where angle $AOB = 90°$. [2]

(ii) Find the possible values of k for which the lengths of AB and OC are equal. [4]

The point D is such that $\overrightarrow{OD}$ is in the same direction as $\overrightarrow{OA}$ and has magnitude 9 units. The point E is such that $\overrightarrow{OE}$ is in the same direction as $\overrightarrow{OC}$ and has magnitude 14 units.

(iii) Find the magnitude of $\overrightarrow{DE}$ in the form $\sqrt{n}$ where n is an integer. [4]

11 The function f is defined by $\mathrm{f} : x \mapsto 4\sin x - 1$ for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) State the range of f. [2]

(ii) Find the coordinates of the points at which the curve $y = \mathrm{f}(x)$ intersects the coordinate axes. [3]

(iii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iv) Obtain an expression for $\mathrm{f}^{-1}(x)$, stating both the domain and range of f^{-1}. [4]

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **4** printed pages and **1** insert.

JC16 06_9709_12/2R

1 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R},\ x \neq \tfrac{3}{2}.$$

Solve the equation $\mathrm{ff}(x) = \mathrm{gf}(2)$. [3]

2 A curve is such that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{8}{(5 - 2x)^2}$. Given that the curve passes through $(2, 7)$, find the equation of the curve. [4]

3 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$$

The point C is such that $\overrightarrow{AB} = \overrightarrow{BC}$. Find the unit vector in the direction of $\overrightarrow{OC}$. [4]

4 Find the term that is independent of x in the expansion of

(i) $\left(x - \frac{2}{x}\right)^6$, [2]

(ii) $\left(2 + \frac{3}{x^2}\right)\left(x - \frac{2}{x}\right)^6$. [4]

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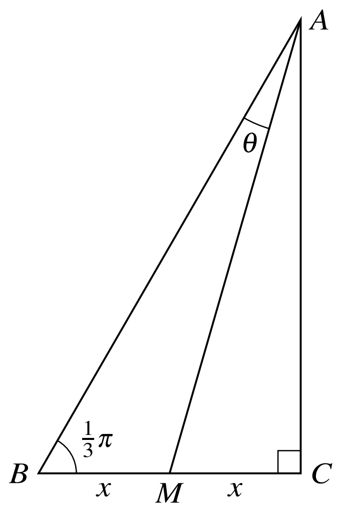

In the diagram, triangle ABC is right-angled at C and M is the mid-point of BC. It is given that angle $ABC = \frac{1}{3}\pi$ radians and angle $BAM = \theta$ radians. Denoting the lengths of BM and MC by x,

(i) find AM in terms of x, [3]

(ii) show that $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$. [2]

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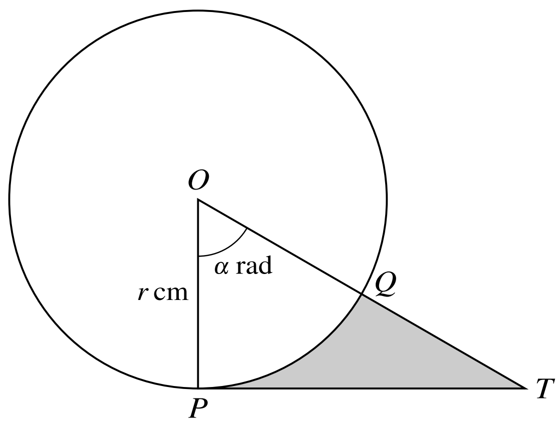


The diagram shows a circle with radius r cm and centre O. The line PT is the tangent to the circle at P and angle $POT = \alpha$ radians. The line OT meets the circle at Q.

(i) Express the perimeter of the shaded region PQT in terms of r and α. [3]

(ii) In the case where $\alpha = \frac{1}{3}\pi$ and $r = 10$, find the area of the shaded region correct to 2 significant figures. [3]

7 **(i)** Prove the identity $\dfrac{1 + \cos\theta}{1 - \cos\theta} - \dfrac{1 - \cos\theta}{1 + \cos\theta} \equiv \dfrac{4}{\sin\theta\tan\theta}$. [4]

(ii) Hence solve, for $0° < \theta < 360°$, the equation

$$\sin\theta\left(\frac{1 + \cos\theta}{1 - \cos\theta} - \frac{1 - \cos\theta}{1 + \cos\theta}\right) = 3.$$ [3]

8 Three points have coordinates $A\,(0,\ 7)$, $B\,(8,\ 3)$ and $C\,(3k,\ k)$. Find the value of the constant k for which

(i) C lies on the line that passes through A and B, [4]

(ii) C lies on the perpendicular bisector of AB. [4]

9 A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

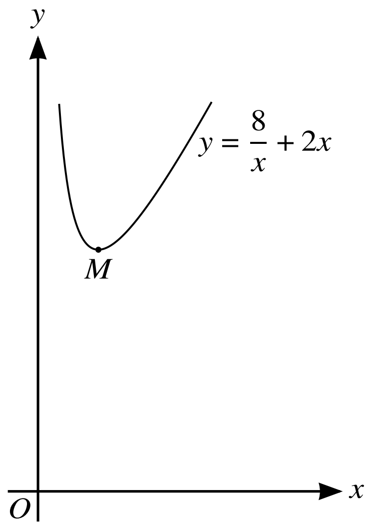
(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the nth day after filling. Find the value of n. [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

[Questions 10 and 11 are printed on the next page.]

10



The diagram shows the part of the curve $y = \frac{8}{x} + 2x$ for $x > 0$, and the minimum point M.

(i) Find expressions for $\frac{dy}{dx}$, $\frac{d^2y}{dx^2}$ and $\int y^2\,dx$. [5]

(ii) Find the coordinates of M and determine the coordinates and nature of the stationary point on the part of the curve for which $x < 0$. [5]

(iii) Find the volume obtained when the region bounded by the curve, the x-axis and the lines $x = 1$ and $x = 2$ is rotated through 360° about the x-axis. [2]

11 The function f is defined by $f : x \mapsto 6x - x^2 - 5$ for $x \in \mathbb{R}$.

(i) Find the set of values of x for which $f(x) \leqslant 3$. [3]

(ii) Given that the line $y = mx + c$ is a tangent to the curve $y = f(x)$, show that $4c = m^2 - 12m + 16$. [3]

The function g is defined by $g : x \mapsto 6x - x^2 - 5$ for $x \geqslant k$, where k is a constant.

(iii) Express $6x - x^2 - 5$ in the form $a - (x - b)^2$, where a and b are constants. [2]

(iv) State the smallest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $g^{-1}(x)$. [2]

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

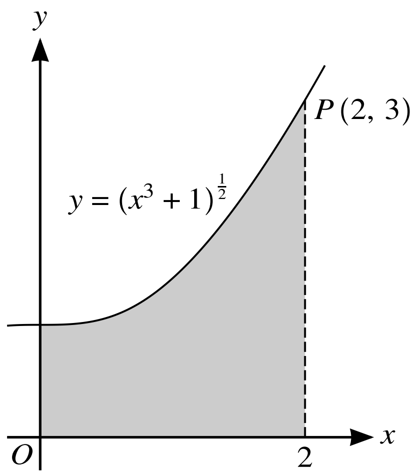
An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

1 Find the coefficient of x in the expansion of $\left(\frac{1}{x} + 3x^2\right)^5$. [3]

2



The diagram shows part of the curve $y = (x^3 + 1)^{\frac{1}{2}}$ and the point $P\,(2, 3)$ lying on the curve. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

3 A curve is such that $\frac{dy}{dx} = 6x^2 + \frac{k}{x^3}$ and passes through the point $P\,(1, 9)$. The gradient of the curve at P is 2.

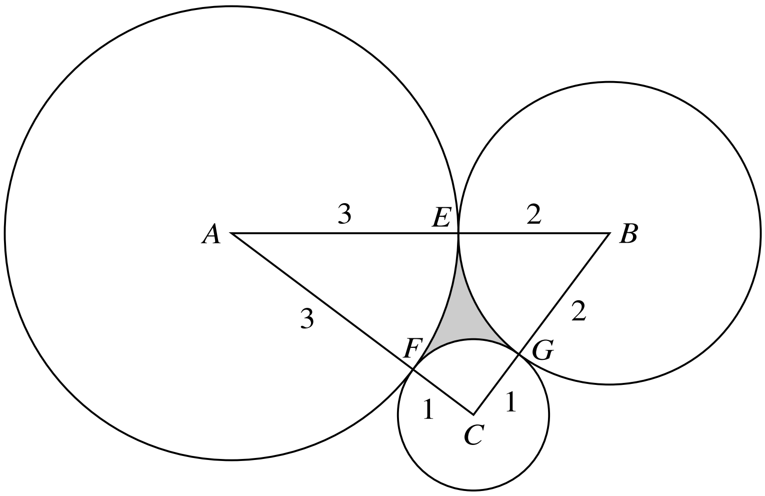
(i) Find the value of the constant k. [1]

(ii) Find the equation of the curve. [4]

4 The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

5 A curve has equation $y = 8x + (2x - 1)^{-1}$. Find the values of x at which the curve has a stationary point and determine the nature of each stationary point, justifying your answers. [7]

6



The diagram shows triangle ABC where $AB = 5\text{ cm}$, $AC = 4\text{ cm}$ and $BC = 3\text{ cm}$. Three circles with centres at A, B and C have radii 3 cm, 2 cm and 1 cm respectively. The circles touch each other at points E, F and G, lying on AB, AC and BC respectively. Find the area of the shaded region EFG. [7]

7 The point $P(x, y)$ is moving along the curve $y = x^2 - \frac{10}{3}x^{\frac{3}{2}} + 5x$ in such a way that the rate of change of y is constant. Find the values of x at the points at which the rate of change of x is equal to half the rate of change of y. [7]

8 **(i)** Show that $3\sin x \tan x - \cos x + 1 = 0$ can be written as a quadratic equation in $\cos x$ and hence solve the equation $3\sin x \tan x - \cos x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [5]

(ii) Find the solutions to the equation $3\sin 2x \tan 2x - \cos 2x + 1 = 0$ for $0 \leqslant x \leqslant \pi$. [3]

9 The position vectors of A, B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix},$$

where p is a constant.

(i) Find the value of p for which the lengths of AB and CB are equal. [4]

(ii) For the case where $p = 1$, use a scalar product to find angle ABC. [4]

[Questions 10 and 11 are printed on the next page.]

10 The function f is such that $f(x) = 2x + 3$ for $x \geqslant 0$. The function g is such that $g(x) = ax^2 + b$ for $x \leqslant q$, where a, b and q are constants. The function fg is such that $fg(x) = 6x^2 - 21$ for $x \leqslant q$.

(i) Find the values of a and b. [3]

(ii) Find the greatest possible value of q. [2]

It is now given that $q = -3$.

(iii) Find the range of fg. [1]

(iv) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [3]

11 Triangle ABC has vertices at $A\,(-2, -1)$, $B\,(4, 6)$ and $C\,(6, -3)$.

(i) Show that triangle ABC is isosceles and find the exact area of this triangle. [6]

(ii) The point D is the point on AB such that CD is perpendicular to AB. Calculate the x-coordinate of D. [6]

Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6898538094

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC17 06_9709_11/RP

1 The coefficients of x^2 and x^3 in the expansion of $(3 - 2x)^6$ are a and b respectively. Find the value of $\frac{a}{b}$. [4]

2 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix},$$

and angle $AOB = 90°$.

(i) Find the value of p. [2]

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The point C is such that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$.

(ii) Find the unit vector in the direction of $\overrightarrow{BC}$. [4]

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3 **(i)** Prove the identity $\dfrac{1+\cos\theta}{\sin\theta}+\dfrac{\sin\theta}{1+\cos\theta}\equiv\dfrac{2}{\sin\theta}$. [3]

(ii) Hence solve the equation $\dfrac{1 + \cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1 + \cos\theta} = \dfrac{3}{\cos\theta}$ for $0° \leqslant \theta \leqslant 360°$. [3]

4 **(a)** An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of −28. Find the sum of all the terms in the progression. [4]

(b) Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated \$2000. Find the total amount allocated in the years 2005 to 2014 inclusive. [3]

5 The equation of a curve is $y = 2\cos x$.

(i) Sketch the graph of $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$, stating the coordinates of the point of intersection with the y-axis. [2]

Points P and Q lie on the curve and have x-coordinates of $\frac{1}{3}\pi$ and π respectively.

(ii) Find the length of PQ correct to 1 decimal place. [2]

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The line through P and Q meets the x-axis at $H\,(h,\ 0)$ and the y-axis at $K\,(0,\ k)$.

(iii) Show that $h = \frac{5}{9}\pi$ and find the value of k. [3]

6 The horizontal base of a solid prism is an equilateral triangle of side x cm. The sides of the prism are vertical. The height of the prism is h cm and the volume of the prism is $2000\,\text{cm}^3$.

(i) Express h in terms of x and show that the total surface area of the prism, $A\,\text{cm}^2$, is given by

$$A = \frac{\sqrt{3}}{2}x^2 + \frac{24\,000}{\sqrt{3}}x^{-1}.$$ [3]

(ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

(iii) Determine, showing all necessary working, the nature of this stationary value. [2]

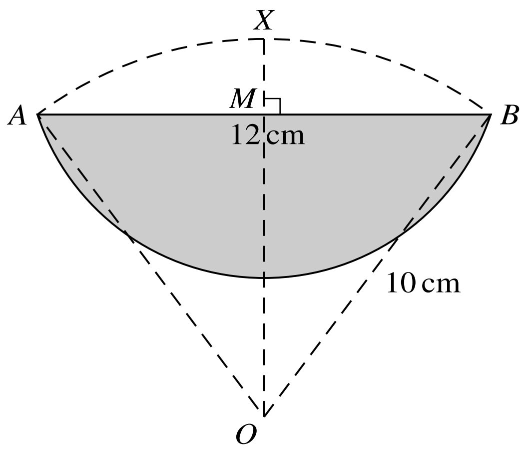
7 A curve for which $\frac{dy}{dx} = 7 - x^2 - 6x$ passes through the point $(3, -10)$.

(i) Find the equation of the curve. [3]

(ii) Express $7 - x^2 - 6x$ in the form $a - (x + b)^2$, where a and b are constants. [2]

(iii) Find the set of values of x for which the gradient of the curve is positive. [3]

8



In the diagram, $OAXB$ is a sector of a circle with centre O and radius 10 cm. The length of the chord AB is 12 cm. The line OX passes through M, the mid-point of AB, and OX is perpendicular to AB. The shaded region is bounded by the chord AB and by the arc of a circle with centre X and radius XA.

(i) Show that angle AXB is 2.498 radians, correct to 3 decimal places. [3]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [3]

9 The function f is defined by $f : x \mapsto \dfrac{2}{3 - 2x}$ for $x \in \mathbb{R},\ x \neq \frac{3}{2}$.

(i) Find an expression for $f^{-1}(x)$. [3]

The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

(ii) Find the value of a for which $gf(-1) = 3$. [3]

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(iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

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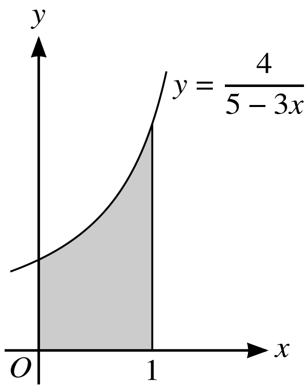

The diagram shows part of the curve $y = \dfrac{4}{5 - 3x}$.

(i) Find the equation of the normal to the curve at the point where $x = 1$ in the form $y = mx + c$, where m and c are constants. [5]

The shaded region is bounded by the curve, the coordinate axes and the line $x = 1$.

(ii) Find, showing all necessary working, the volume obtained when this shaded region is rotated through 360° about the x-axis. [5]

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Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

9022343494

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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JC17 06_9709_12/RP

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[Turn over

1 (i) Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [2]

(ii) Hence find the coefficient of x in the expansion of $(1 + 3x^2)\left(2x - \frac{1}{x}\right)^5$. [4]

2 The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

(i) Find the equation of AB. [3]

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(ii) Find the coordinates of B. [3]

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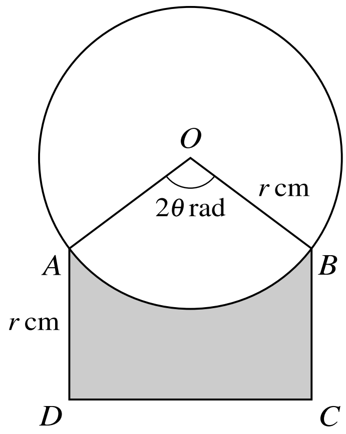
3 **(i)** Prove the identity $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 \equiv \frac{1 - \sin\theta}{1 + \sin\theta}$. [3]

(ii) Hence solve the equation $\left(\frac{1}{\cos\theta} - \tan\theta\right)^2 = \frac{1}{2}$, for $0° \leqslant \theta \leqslant 360°$. [3]

4



The diagram shows a circle with radius r cm and centre O. Points A and B lie on the circle and $ABCD$ is a rectangle. Angle $AOB = 2\theta$ radians and $AD = r$ cm.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

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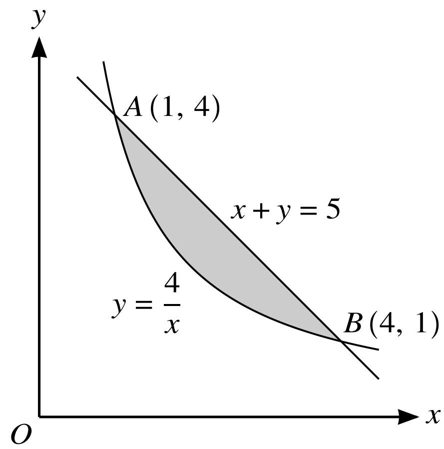
(ii) In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. [4]

5 A curve has equation $y = 3 + \dfrac{12}{2 - x}$.

(i) Find the equation of the tangent to the curve at the point where the curve crosses the x-axis. [5]

(ii) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.04 units per second. Find the rate of change of the y-coordinate when $x = 4$. [2]

6



The diagram shows the straight line $x + y = 5$ intersecting the curve $y = \dfrac{4}{x}$ at the points $A\,(1,\ 4)$ and $B\,(4,\ 1)$. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [7]

7 **(a)** The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]

(b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

8 Relative to an origin O, the position vectors of three points A, B and C are given by

$$\overrightarrow{OA} = 3\mathbf{i} + p\mathbf{j} - 2p\mathbf{k}, \quad \overrightarrow{OB} = 6\mathbf{i} + (p+4)\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = (p-1)\mathbf{i} + 2\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) In the case where $p = 2$, use a scalar product to find angle AOB. [4]

(ii) In the case where $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$, find the values of p and q. [4]

9 The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve. [3]

(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

(iii) Find the values of x at which the line $y = 6$ meets the curve. [3]

(iv) State the set of values of k for which the line $y = k$ does not meet the curve. [1]

10 The function f is defined by $f(x) = 3\tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

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(ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1}. [5]

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(iii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [3]

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Cambridge International Examinations
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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3951355360

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC17 06_9709_13/RP

CAMBRIDGE International Examinations

[Turn over

1 The coefficients of x and x^2 in the expansion of $(2 + ax)^7$ are equal. Find the value of the non-zero constant a. [3]

2 The common ratio of a geometric progression is r. The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S.

(i) Show that $S = 2 - r$. [2]

(ii) Find the set of possible values that S can take. [2]

3 Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

4 Relative to an origin O, the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}.$$

The point P lies on AB and is such that $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$.

(i) Find the position vector of P. [3]

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(ii) Find the distance OP. [1]

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(iii) Determine whether OP is perpendicular to AB. Justify your answer. [2]

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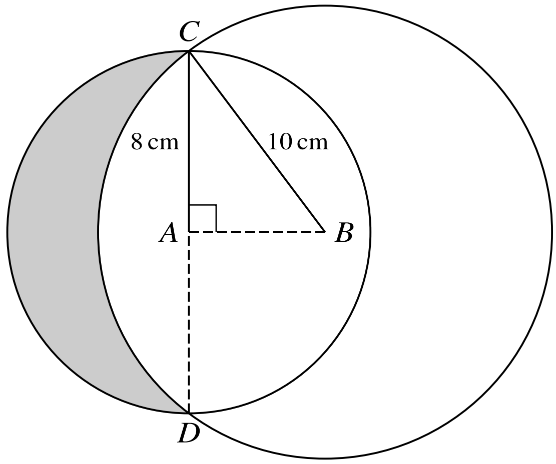
5 **(i)** Show that the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ may be expressed as $\cos^2\theta = 2\sin^2\theta$. [3]

(ii) Hence solve the equation $\dfrac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = 2\tan\theta$ for $0° < \theta < 180°$. [3]

6 The line $3y + x = 25$ is a normal to the curve $y = x^2 - 5x + k$. Find the value of the constant k. [6]

7



The diagram shows two circles with centres A and B having radii 8 cm and 10 cm respectively. The two circles intersect at C and D where CAD is a straight line and AB is perpendicular to CD.

(i) Find angle ABC in radians. [1]

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(ii) Find the area of the shaded region. [6]

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8 $A(-1, 1)$ and $P(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

(i) Find an expression for b in terms of a. [2]

(ii) $B(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P. [6]

9 **(i)** Express $9x^2 - 6x + 6$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

The function f is defined by $f(x) = 9x^2 - 6x + 6$ for $x \geqslant p$, where p is a constant.

(ii) State the smallest value of p for which f is a one-one function. [1]

(iii) For this value of p, obtain an expression for $f^{-1}(x)$, and state the domain of f^{-1}. [4]

(iv) State the set of values of q for which the equation $f(x) = q$ has no solution. [1]

10 (a)

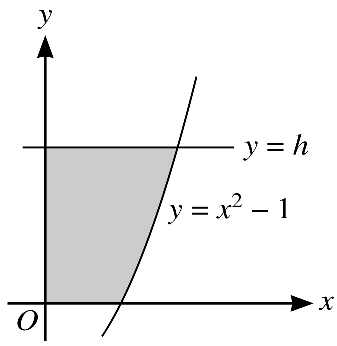

Fig. 1

Fig. 1 shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.

(i) The shaded region is rotated through 360° about the ***y*-axis**. Show that the volume of revolution, V, is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. [3]

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(ii) Find, showing all necessary working, the area of the shaded region when $h = 3$. [4]

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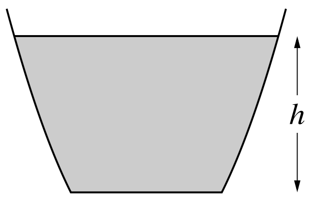

Fig. 2

Fig. 2 shows a cross-section of a bowl containing water. When the height of the water level is h cm, the volume, $V\,\text{cm}^3$, of water is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. Water is poured into the bowl at a constant rate of $2\,\text{cm}^3\,\text{s}^{-1}$. Find the rate, in $\text{cm}\,\text{s}^{-1}$, at which the height of the water level is increasing when the height of the water level is 3 cm. [4]

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11 The function f is defined for $x \geqslant 0$. It is given that f has a minimum value when $x = 2$ and that $\mathrm{f}''(x) = (4x + 1)^{-\frac{1}{2}}$.

(i) Find $\mathrm{f}'(x)$. [3]

It is now given that $\mathrm{f}''(0)$, $\mathrm{f}'(0)$ and $\mathrm{f}(0)$ are the first three terms respectively of an arithmetic progression.

(ii) Find the value of $\mathrm{f}(0)$. [3]

(iii) Find $f(x)$, and hence find the minimum value of f. [5]

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Cambridge International AS & A Level

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CANDIDATE NAME

CENTRE NUMBER

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC18 06_9709_11/2R

CAMBRIDGE International Examinations

[Turn over

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1 **(i)** Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

(ii) Given that the coefficient of x^2 in the expansion of $(1 + ax + 2x^2)(1 - 2x)^5$ is 12, find the value of the constant a. [3]

2 A point is moving along the curve $y = 2x + \frac{5}{x}$ in such a way that the x-coordinate is increasing at a constant rate of 0.02 units per second. Find the rate of change of the y-coordinate when $x = 1$. [4]

3 A curve is such that $\frac{dy}{dx} = \frac{12}{(2x+1)^2}$. The point $(1, 1)$ lies on the curve. Find the coordinates of the point at which the curve intersects the x-axis. [6]

4 **(i)** Prove the identity $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$. [3]

(ii) Hence solve the equation $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta$ for $0° \leqslant \theta \leqslant 360°$. [3]

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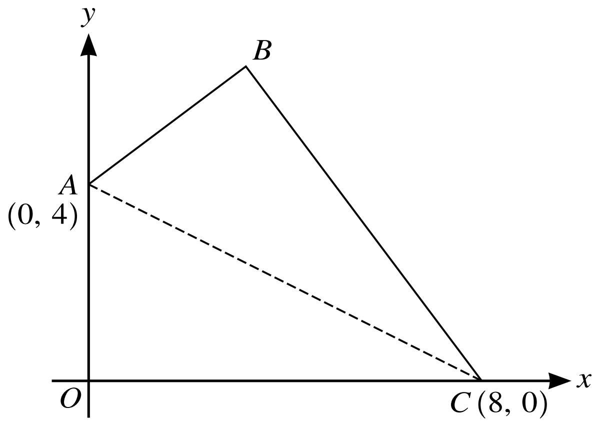

The diagram shows a kite $OABC$ in which AC is the line of symmetry. The coordinates of A and C are (0, 4) and (8, 0) respectively and O is the origin.

(i) Find the equations of AC and OB. [4]

(ii) Find, by calculation, the coordinates of B. [3]

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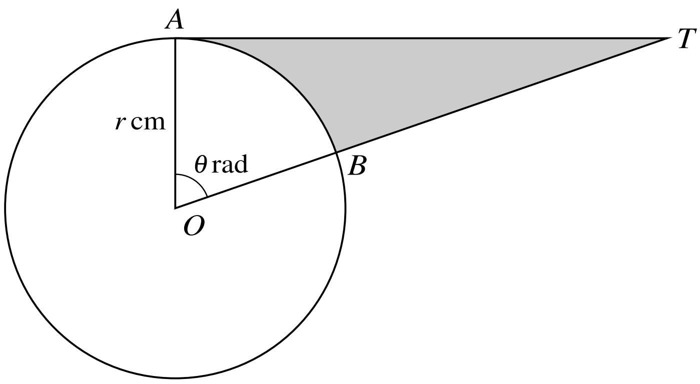

The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle. Angle $AOB = \theta$ radians and OBT is a straight line.

(i) Express the area of the shaded region in terms of r and θ. [3]

(ii) In the case where $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region. [4]

7 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}.$$

(i) Find $\overrightarrow{AC}$. [1]

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(ii) The point M is the mid-point of AC. Find the unit vector in the direction of $\overrightarrow{OM}$. [3]

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(iii) Evaluate $\overrightarrow{AB} \cdot \overrightarrow{AC}$ and hence find angle BAC. [4]

8 **(a)** A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]

(b) The nth term of a progression is $p + qn$, where p and q are constants, and S_n is the sum of the first n terms.

(i) Find an expression, in terms of p, q and n, for S_n. [3]

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(ii) Given that $S_4 = 40$ and $S_6 = 72$, find the values of p and q. [2]

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9 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \tfrac{1}{2}x - 2,$$
$$g : x \mapsto 4 + x - \tfrac{1}{2}x^2.$$

(i) Find the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$. [3]

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(ii) Find the set of values of x for which $f(x) > g(x)$. [2]

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(iii) Find an expression for $\mathrm{fg}(x)$ and deduce the range of fg. [4]

The function h is defined by $\mathrm{h} : x \mapsto 4 + x - \frac{1}{2}x^2$ for $x \geqslant k$.

(iv) Find the smallest value of k for which h has an inverse. [2]

10 The curve with equation $y = x^3 - 2x^2 + 5x$ passes through the origin.

(i) Show that the curve has no stationary points. [3]

(ii) Denoting the gradient of the curve by m, find the stationary value of m and determine its nature. [5]

(iii) Showing all necessary working, find the area of the region enclosed by the curve, the x-axis and the line $x = 6$. [4]

Additional Page

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6653000674

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

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JC18 06_9709_12/2RP **R**

CAMBRIDGE International Examinations

1 The coefficient of x^2 in the expansion of $\left(2 + \frac{x}{2}\right)^6 + (a + x)^5$ is 330. Find the value of the constant a. [5]

2 The equation of a curve is $y = x^2 - 6x + k$, where k is a constant.

(i) Find the set of values of k for which the whole of the curve lies above the x-axis. [2]

(ii) Find the value of k for which the line $y + 2x = 7$ is a tangent to the curve. [3]

3 A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

(ii) Find the total amount of salt obtained in the first 12 weeks after the change. [2]

4 The function f is such that $\mathrm{f}(x) = a + b\cos x$ for $0 \leqslant x \leqslant 2\pi$. It is given that $\mathrm{f}\left(\frac{1}{3}\pi\right) = 5$ and $\mathrm{f}(\pi) = 11$.

(i) Find the values of the constants a and b. [3]

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(ii) Find the set of values of k for which the equation $\mathrm{f}(x) = k$ has no solution. [3]

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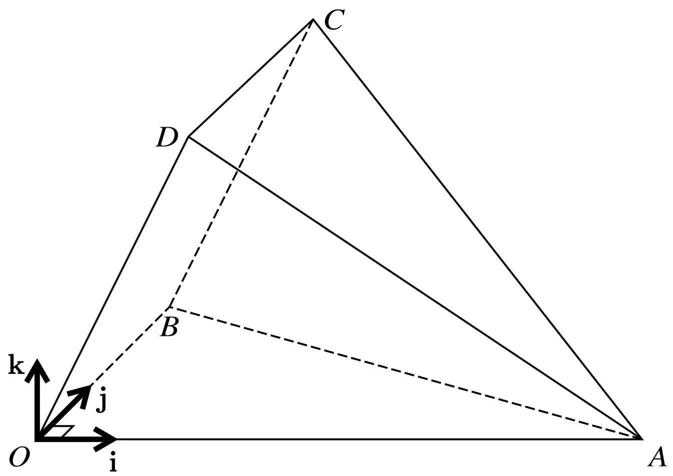

The diagram shows a three-dimensional shape. The base OAB is a horizontal triangle in which angle AOB is 90°. The side $OBCD$ is a rectangle and the side OAD lies in a vertical plane. Unit vectors **i** and **j** are parallel to OA and OB respectively and the unit vector **k** is vertical. The position vectors of A, B and D are given by $\overrightarrow{OA} = 8\mathbf{i}$, $\overrightarrow{OB} = 5\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 4\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{DA}$ and $\overrightarrow{CA}$ in terms of **i**, **j** and **k**. [2]

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(ii) Use a scalar product to find angle *CAD*. [4]

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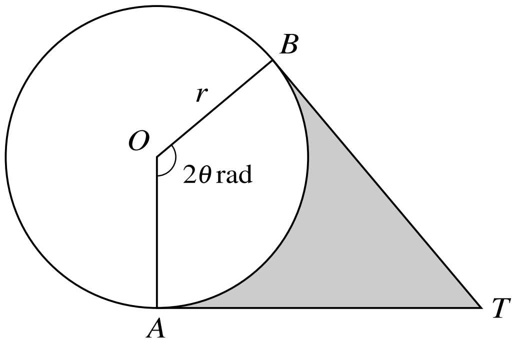

The diagram shows points A and B on a circle with centre O and radius r. The tangents to the circle at A and B meet at T. The shaded region is bounded by the minor arc AB and the lines AT and BT. Angle AOB is 2θ radians.

(i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan\theta = 2\theta$. [3]

(ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region. [3]

7 The function f is defined by $f : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants. [2]

(ii) State the coordinates of the stationary point on the curve $y = f(x)$. [1]

The function g is defined by g : $x \mapsto 7 - 2x^2 - 12x$ for $x \geqslant k$.

(iii) State the smallest value of k for which g has an inverse. [1]

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(iv) For this value of k, find $g^{-1}(x)$. [3]

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8 Points A and B have coordinates $(h,\ h)$ and $(4h+6,\ 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x+2y=k$. Find the values of the constants h and k. [7]

9 A curve is such that $\frac{dy}{dx} = \sqrt{(4x + 1)}$ and (2, 5) is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) A point P moves along the curve in such a way that the y-coordinate is increasing at a constant rate of 0.06 units per second. Find the rate of change of the x-coordinate when P passes through (2, 5). [2]

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(iii) Show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx}$ is constant. [2]

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10 **(i)** Solve the equation $2\cos x + 3\sin x = 0$, for $0° \leqslant x \leqslant 360°$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = 2\cos x$ and $y = -3\sin x$ for $0° \leqslant x \leqslant 360°$. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to find the set of values of x for $0° \leqslant x \leqslant 360°$ for which $2\cos x + 3\sin x > 0$. [2]

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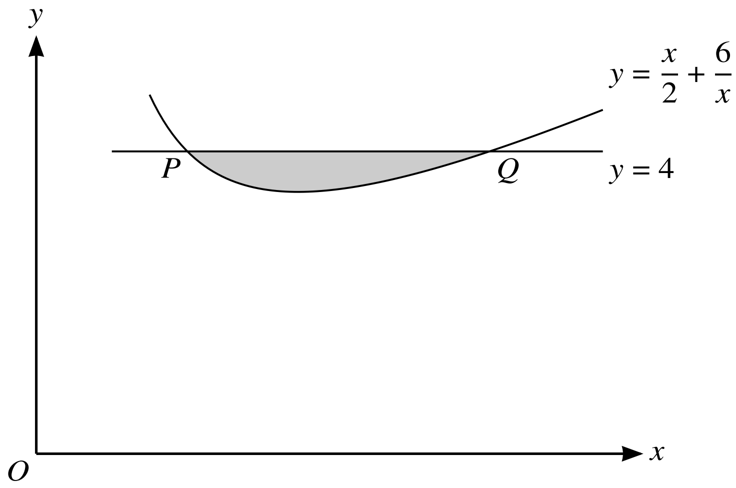

The diagram shows part of the curve $y = \frac{x}{2} + \frac{6}{x}$. The line $y = 4$ intersects the curve at the points P and Q.

(i) Show that the tangents to the curve at P and Q meet at a point on the line $y = x$. [6]

(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. Give your answer in terms of π. [6]

Additional Page

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **20** printed pages.

JC18 06_9709_13/RP

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[Turn over

1 Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

2 Find the coefficient of $\frac{1}{x}$ in the expansion of $\left(x - \frac{2}{x}\right)^5$. [3]

3 The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

4 A curve with equation $y = \mathrm{f}(x)$ passes through the point $A\ (3,\ 1)$ and crosses the y-axis at B. It is given that $\mathrm{f}'(x) = (3x - 1)^{-\frac{1}{3}}$. Find the y-coordinate of B. [6]

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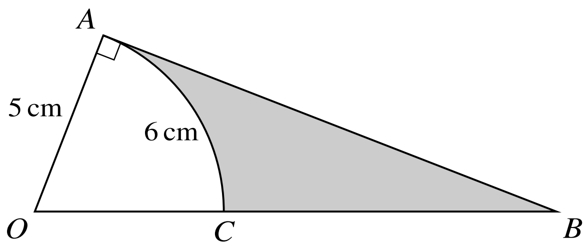


The diagram shows a triangle OAB in which angle $OAB = 90°$ and $OA = 5$ cm. The arc AC is part of a circle with centre O. The arc has length 6 cm and it meets OB at C. Find the area of the shaded region. [5]

6 The coordinates of points A and B are $(-3k - 1,\ k + 3)$ and $(k + 3,\ 3k + 5)$ respectively, where k is a constant ($k \neq -1$).

(i) Find and simplify the gradient of AB, showing that it is independent of k. [2]

(ii) Find and simplify the equation of the perpendicular bisector of AB. [5]

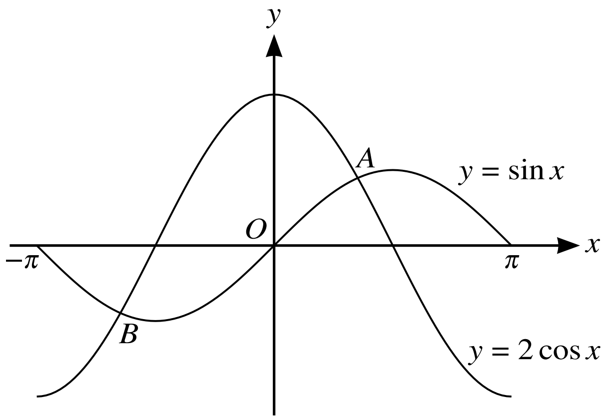
7 **(a)** **(i)** Express $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1}$ in the form $a\sin^2\theta + b$, where a and b are constants to be found. [3]

(ii) Hence, or otherwise, and showing all necessary working, solve the equation

$$\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{1}{4}$$

for $-90° \leqslant \theta \leqslant 0°$. [2]

(b)



The diagram shows the graphs of $y = \sin x$ and $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$. The graphs intersect at the points A and B.

(i) Find the x-coordinate of A. [2]

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(ii) Find the y-coordinate of B. [2]

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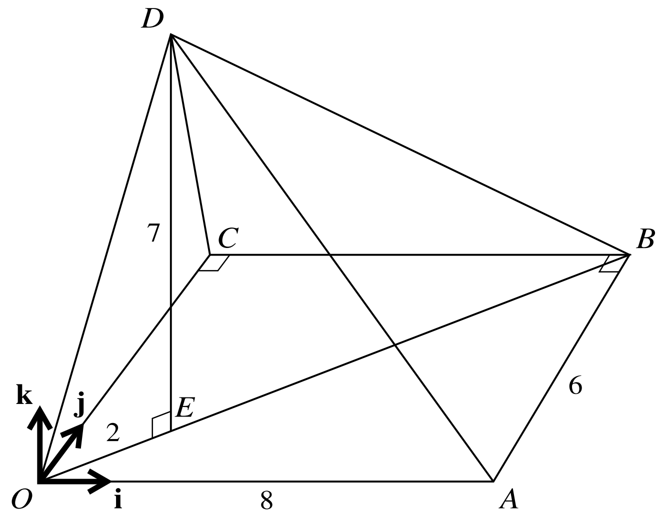
8 **(i)** The tangent to the curve $y = x^3 - 9x^2 + 24x - 12$ at a point A is parallel to the line $y = 2 - 3x$. Find the equation of the tangent at A. [6]

(ii) The function f is defined by $f(x) = x^3 - 9x^2 + 24x - 12$ for $x > k$, where k is a constant. Find the smallest value of k for f to be an increasing function. [2]

9



The diagram shows a pyramid $OABCD$ with a horizontal rectangular base $OABC$. The sides OA and AB have lengths of 8 units and 6 units respectively. The point E on OB is such that $OE = 2$ units. The point D of the pyramid is 7 units vertically above E. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and ED respectively.

(i) Show that $\overrightarrow{OE} = 1.6\mathbf{i} + 1.2\mathbf{j}$. [2]

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(ii) Use a scalar product to find angle BDO. [7]

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10 The one-one function f is defined by $f(x) = (x-2)^2 + 2$ for $x \geqslant c$, where c is a constant.

(i) State the smallest possible value of c. [1]

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In parts **(ii)** and **(iii)** the value of c is 4.

(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(iii) Solve the equation $ff(x) = 51$, giving your answer in the form $a + \sqrt{b}$. [5]

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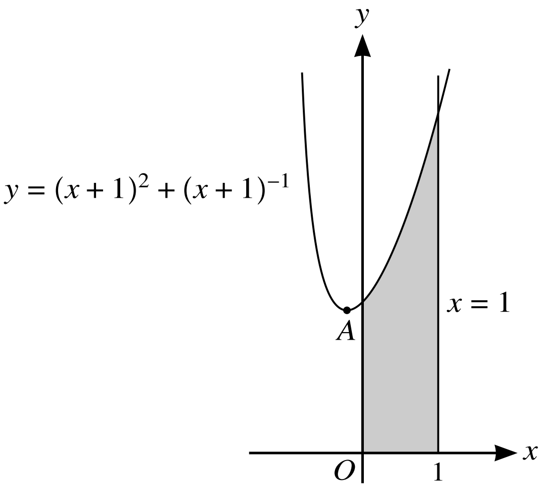

The diagram shows part of the curve $y = (x+1)^2 + (x+1)^{-1}$ and the line $x = 1$. The point A is the minimum point on the curve.

(i) Show that the x-coordinate of A satisfies the equation $2(x+1)^3 = 1$ and find the exact value of $\frac{d^2y}{dx^2}$ at A. [5]

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(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

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Cambridge Assessment International Education

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1 The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k. [3]

(ii) For this value of k, find the coefficient of x^2 in the expansion. [2]

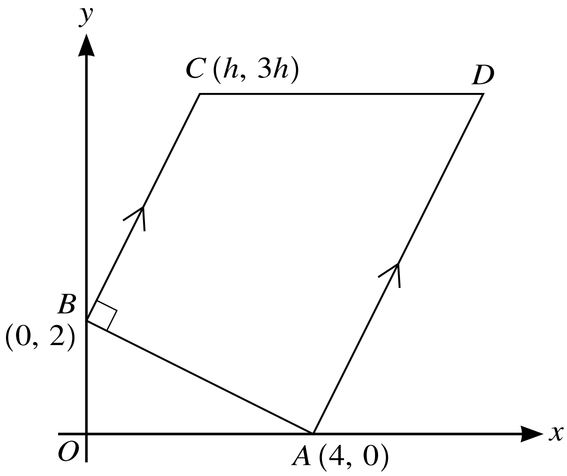
2 The line $4y = x + c$, where c is a constant, is a tangent to the curve $y^2 = x + 3$ at the point P on the curve.

(i) Find the value of c. [3]

(ii) Find the coordinates of P. [2]

3 A sector of a circle of radius r cm has an area of A cm^2. Express the perimeter of the sector in terms of r and A. [4]

4



The diagram shows a trapezium $ABCD$ in which the coordinates of A, B and C are (4, 0), (0, 2) and $(h, 3h)$ respectively. The lines BC and AD are parallel, angle $ABC = 90°$ and CD is parallel to the x-axis.

(i) Find, by calculation, the value of h. [3]

(ii) Hence find the coordinates of D. [3]

5 The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

(i) Express $-2x^2 + 12x - 3$ in the form $-2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the greatest value of $f(x)$. [1]

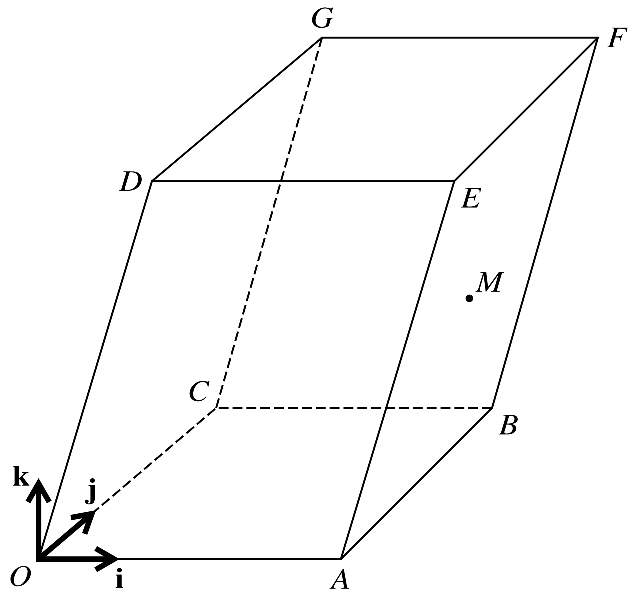
The function g is defined by $g(x) = 2x + 5$ for $x \in \mathbb{R}$.

(iii) Find the values of x for which $gf(x) + 1 = 0$. [3]

6 **(i)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}$. [4]

(ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leqslant x \leqslant \pi$. [3]

7



The diagram shows a three-dimensional shape in which the base $OABC$ and the upper surface $DEFG$ are identical horizontal squares. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. The point M is the mid-point of AF.

Unit vectors $\mathbf{i}$ and $\mathbf{j}$ are parallel to OA and OC respectively and the unit vector $\mathbf{k}$ is vertically upwards. The position vectors of A and D are given by $\overrightarrow{OA} = 8\mathbf{i}$ and $\overrightarrow{OD} = 3\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{AM}$ and $\overrightarrow{GM}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

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(ii) Use a scalar product to find angle GMA correct to the nearest degree. [4]

8 **(a)** The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

(b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme A is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme B is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period. [5]

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9 The function f is defined by $f(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant 2\pi$.

(i) State the range of f. [2]

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(ii) Sketch the graph of $y = f(x)$. [2]

The function g is defined by $g(x) = 2 - 3\cos x$ for $0 \leqslant x \leqslant p$, where p is a constant.

(iii) State the largest value of p for which g has an inverse. [1]

(iv) For this value of p, find an expression for $g^{-1}(x)$. [2]

10 A curve for which $\frac{d^2y}{dx^2} = 2x - 5$ has a stationary point at (3, 6).

(i) Find the equation of the curve. [6]

(ii) Find the x-coordinate of the other stationary point on the curve. [1]

(iii) Determine the nature of each of the stationary points. [2]

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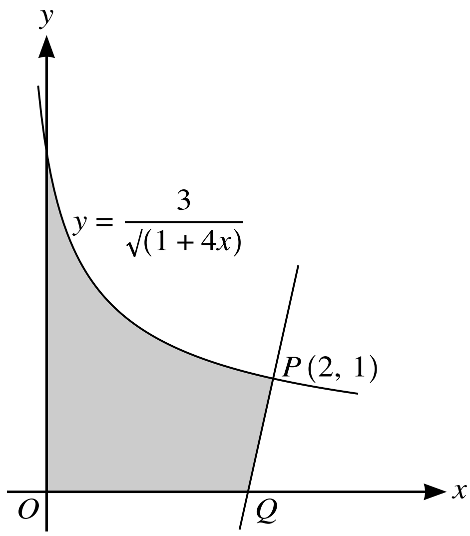

The diagram shows part of the curve $y = \dfrac{3}{\sqrt{(1+4x)}}$ and a point $P(2, 1)$ lying on the curve. The normal to the curve at P intersects the x-axis at Q.

(i) Show that the x-coordinate of Q is $\frac{16}{9}$. [5]

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(ii) Find, showing all necessary working, the area of the shaded region. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6611123410

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

JC19 06_9709_12/RP

1 Find the coefficient of x in the expansion of $\left(\frac{2}{x} - 3x\right)^5$. [3]

2 Two points A and B have coordinates $(1, 3)$ and $(9, -1)$ respectively. The perpendicular bisector of AB intersects the y-axis at the point C. Find the coordinates of C. [5]

3 A curve is such that $\frac{dy}{dx} = x^3 - \frac{4}{x^2}$. The point $P(2, 9)$ lies on the curve.

(i) A point moves on the curve in such a way that the x-coordinate is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the y-coordinate when the point is at P. [2]

(ii) Find the equation of the curve. [3]

4 Angle x is such that $\sin x = a + b$ and $\cos x = a - b$, where a and b are constants.

(i) Show that $a^2 + b^2$ has a constant value for all values of x. [3]

(ii) In the case where $\tan x = 2$, express a in terms of b. [2]

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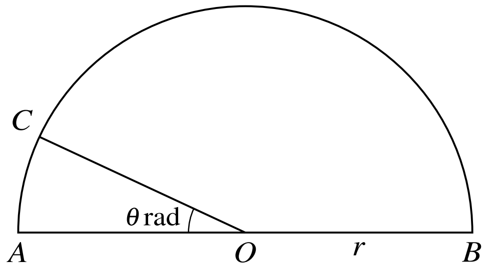

The diagram shows a semicircle with diameter AB, centre O and radius r. The point C lies on the circumference and angle $AOC = \theta$ radians. The perimeter of sector BOC is twice the perimeter of sector AOC. Find the value of θ correct to 2 significant figures. [5]

6 The equation of a curve is $y = 3\cos 2x$ and the equation of a line is $2y + \dfrac{3x}{\pi} = 5$.

(i) State the smallest and largest values of y for both the curve and the line for $0 \leqslant x \leqslant 2\pi$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = 3\cos 2x$ and $2y + \dfrac{3x}{\pi} = 5$ for $0 \leqslant x \leqslant 2\pi$. [3]

(iii) State the number of solutions of the equation $6\cos 2x = 5 - \dfrac{3x}{\pi}$ for $0 \leqslant x \leqslant 2\pi$. [1]

7 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 2, \quad x \in \mathbb{R},$$

$$\mathrm{g} : x \mapsto \frac{2x+3}{x-1}, \quad x \in \mathbb{R},\ x \neq 1.$$

(i) Obtain expressions for $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$, stating the value of x for which $\mathrm{g}^{-1}(x)$ is not defined. [4]

(ii) Solve the equation $\mathrm{fg}(x) = \frac{7}{3}$. [3]

8 The position vectors of points A and B, relative to an origin O, are given by

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -2 \\ -6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ k \\ -3 \end{pmatrix},$$

where k is a constant.

(i) Find the value of k for which angle AOB is 90°. [2]

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(ii) Find the values of k for which the lengths of OA and OB are equal. [2]

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The point C is such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

(iii) In the case where $k = 4$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]

9 The curve C_1 has equation $y = x^2 - 4x + 7$. The curve C_2 has equation $y^2 = 4x + k$, where k is a constant. The tangent to C_1 at the point where $x = 3$ is also the tangent to C_2 at the point P. Find the value of k and the coordinates of P. [8]

10 **(a)** In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is a.

(i) Show that the common difference of the progression is $\frac{1}{3}a$. [4]

(ii) Given that the tenth term is 36 more than the fourth term, find the value of a. [2]

(b) The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

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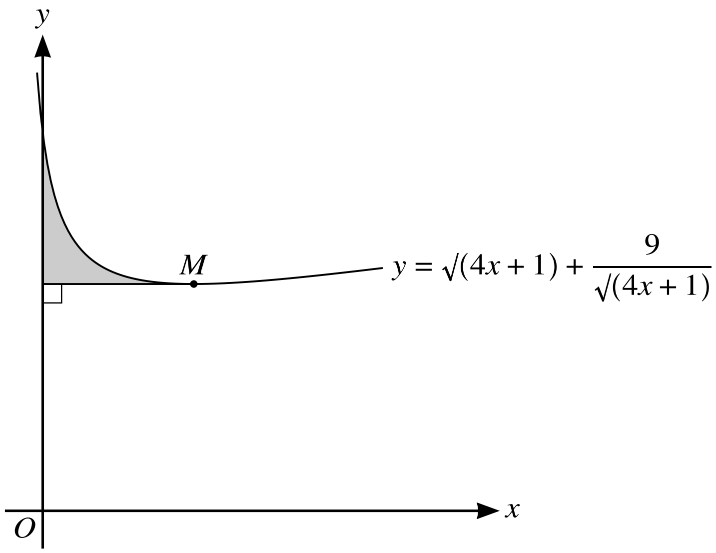

The diagram shows part of the curve $y = \sqrt{(4x+1)} + \dfrac{9}{\sqrt{(4x+1)}}$ and the minimum point M.

(i) Find expressions for $\dfrac{dy}{dx}$ and $\int y \, dx$. [6]

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(ii) Find the coordinates of M. [3]

The shaded region is bounded by the curve, the y-axis and the line through M parallel to the x-axis.

(iii) Find, showing all necessary working, the area of the shaded region. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0008858838

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC19 06_9709_13/RP

Cambridge Assessment International Education

1 The function f is defined by $f(x) = x^2 - 4x + 8$ for $x \in \mathbb{R}$.

(i) Express $x^2 - 4x + 8$ in the form $(x - a)^2 + b$. [2]

(ii) Hence find the set of values of x for which $f(x) < 9$, giving your answer in exact form. [3]

2 **(i)** In the binomial expansion of $\left(2x - \frac{1}{2x}\right)^5$, the first three terms are $32x^5 - 40x^3 + 20x$. Find the remaining three terms of the expansion. [3]

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(ii) Hence find the coefficient of x in the expansion of $(1 + 4x^2)\left(2x - \frac{1}{2x}\right)^5$. [2]

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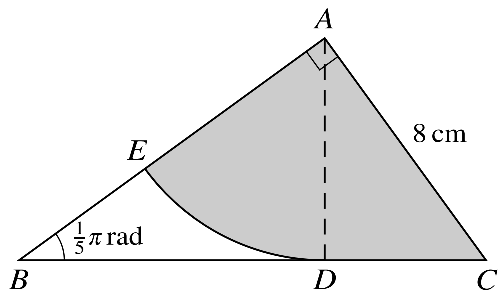

The diagram shows triangle ABC which is right-angled at A. Angle $ABC = \frac{1}{5}\pi$ radians and $AC = 8$ cm. The points D and E lie on BC and BA respectively. The sector ADE is part of a circle with centre A and is such that BDC is the tangent to the arc DE at D.

(i) Find the length of AD. [3]

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(ii) Find the area of the shaded region. [3]

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4 The function f is defined by $f(x) = \frac{48}{x-1}$ for $3 \leqslant x \leqslant 7$. The function g is defined by $g(x) = 2x - 4$ for $a \leqslant x \leqslant b$, where a and b are constants.

(i) Find the greatest value of a and the least value of b which will permit the formation of the composite function gf. [2]

It is now given that the conditions for the formation of gf are satisfied.

(ii) Find an expression for $gf(x)$. [1]

(iii) Find an expression for $(gf)^{-1}(x)$. [2]

5 Two heavyweight boxers decide that they would be more successful if they competed in a lower weight class. For each boxer this would require a total weight loss of 13 kg. At the end of week 1 they have each recorded a weight loss of 1 kg and they both find that in each of the following weeks their weight loss is slightly less than the week before.

Boxer A's weight loss in week 2 is 0.98 kg. It is given that his weekly weight loss follows an arithmetic progression.

(i) Write down an expression for his total weight loss after x weeks. [1]

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(ii) He reaches his 13 kg target during week n. Use your answer to part **(i)** to find the value of n. [2]

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Boxer B's weight loss in week 2 is 0.92 kg and it is given that his weekly weight loss follows a geometric progression.

(iii) Calculate his total weight loss after 20 weeks and show that he can never reach his target. [4]

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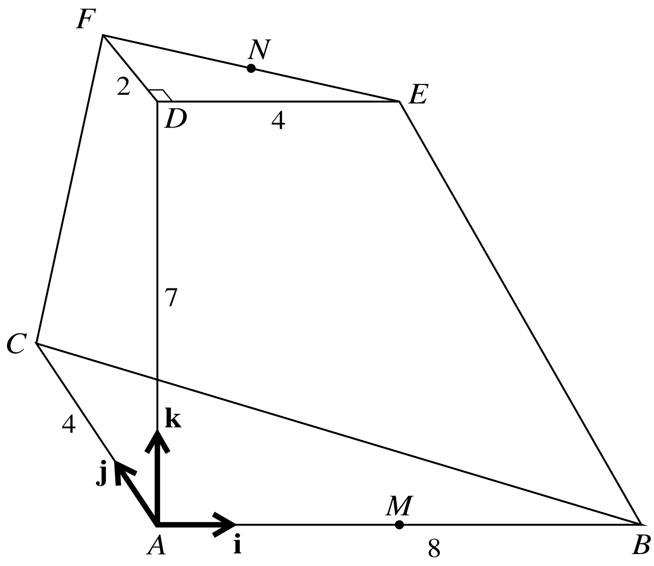

The diagram shows a solid figure $ABCDEF$ in which the horizontal base ABC is a triangle right-angled at A. The lengths of AB and AC are 8 units and 4 units respectively and M is the mid-point of AB. The point D is 7 units vertically above A. Triangle DEF lies in a horizontal plane with DE, DF and FE parallel to AB, AC and CB respectively and N is the mid-point of FE. The lengths of DE and DF are 4 units and 2 units respectively. Unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ respectively.

(i) Find $\overrightarrow{MF}$ in terms of **i**, **j** and **k**. [1]

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(ii) Find $\overrightarrow{FN}$ in terms of **i** and **j**. [1]

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(iii) Find $\overrightarrow{MN}$ in terms of **i**, **j** and **k**. [1]

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(iv) Use a scalar product to find angle FMN. [4]

7 The coordinates of two points A and B are $(1, 3)$ and $(9, -1)$ respectively and D is the mid-point of AB. A point C has coordinates (x, y), where x and y are variables.

(i) State the coordinates of D. [1]

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(ii) It is given that $CD^2 = 20$. Write down an equation relating x and y. [1]

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(iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]

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(iv) Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C. [4]

8 A curve is such that $\frac{dy}{dx} = 3x^2 + ax + b$. The curve has stationary points at $(-1, 2)$ and $(3, k)$. Find the values of the constants a, b and k. [8]

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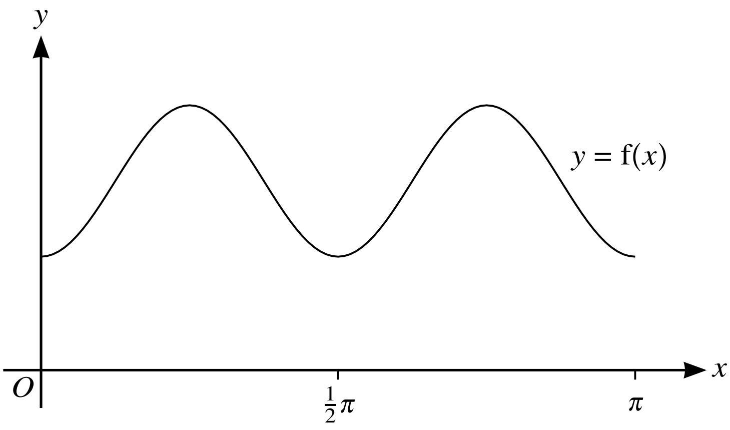

The function $f : x \mapsto p\sin^2 2x + q$ is defined for $0 \leqslant x \leqslant \pi$, where p and q are positive constants. The diagram shows the graph of $y = f(x)$.

(i) In terms of p and q, state the range of f. [2]

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(ii) State the number of solutions of the following equations.

(a) $f(x) = p + q$ [1]

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(b) $f(x) = q$ [1]

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(c) $f(x) = \frac{1}{2}p + q$ [1]

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(iii) For the case where $p = 3$ and $q = 2$, solve the equation $f(x) = 4$, showing all necessary working. [5]

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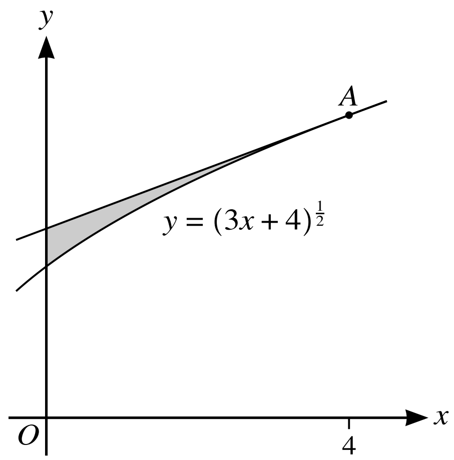

The diagram shows part of the curve with equation $y = (3x + 4)^{\frac{1}{2}}$ and the tangent to the curve at the point A. The x-coordinate of A is 4.

(i) Find the equation of the tangent to the curve at A. [5]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

[Question 10 (iii) is printed on the next page.]

(iii) A point is moving along the curve. At the point P the y-coordinate is increasing at half the rate at which the x-coordinate is increasing. Find the x-coordinate of P. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

2851798509

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 06_9709_11/2R

1 The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression. [4]

2 The coefficient of $\frac{1}{x}$ in the expansion of $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$ is 74.

Find the value of the positive constant k. [5]

3 Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

(a) Write down an expression for the selling price of the necklace n years later and hence find the selling price in 2008. [3]

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(b) The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

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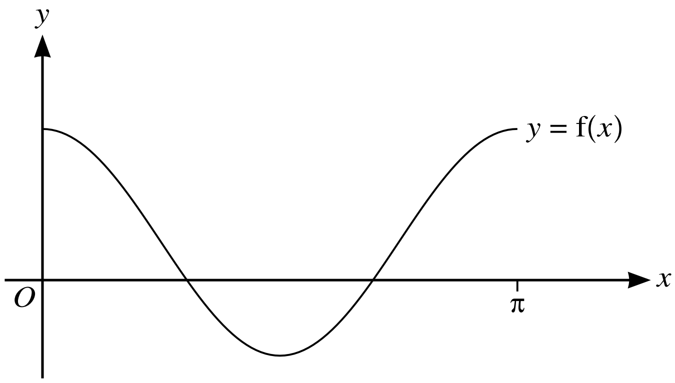

The diagram shows the graph of $y = \mathrm{f}(x)$, where $\mathrm{f}(x) = \frac{3}{2}\cos 2x + \frac{1}{2}$ for $0 \leqslant x \leqslant \pi$.

(a) State the range of f. [2]

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A function g is such that $\mathrm{g}(x) = \mathrm{f}(x) + k$, where k is a positive constant. The x-axis is a tangent to the curve $y = \mathrm{g}(x)$.

(b) State the value of k and hence describe fully the transformation that maps the curve $y = \mathrm{f}(x)$ on to $y = \mathrm{g}(x)$. [2]

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(c) State the equation of the curve which is the reflection of $y = \mathrm{f}(x)$ in the x-axis. Give your answer in the form $y = a\cos 2x + b$, where a and b are constants. [1]

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5 The equation of a line is $y = mx + c$, where m and c are constants, and the equation of a curve is $xy = 16$.

(a) Given that the line is a tangent to the curve, express m in terms of c. [3]

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(b) Given instead that $m = -4$, find the set of values of c for which the line intersects the curve at two distinct points. [3]

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6 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \tfrac{1}{2}x - a,$$
$$g : x \mapsto 3x + b,$$

where a and b are constants.

(a) Given that $gg(2) = 10$ and $f^{-1}(2) = 14$, find the values of a and b. [4]

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(b) Using these values of a and b, find an expression for $gf(x)$ in the form $cx + d$, where c and d are constants. [2]

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7 **(a)** Prove the identity $\dfrac{1+\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1+\sin\theta} \equiv \dfrac{2}{\cos\theta}$. [3]

(b) Hence solve the equation $\dfrac{1 + \sin\theta}{\cos\theta} + \dfrac{\cos\theta}{1 + \sin\theta} = \dfrac{3}{\sin\theta}$, for $0 \leqslant \theta \leqslant 2\pi$. [3]

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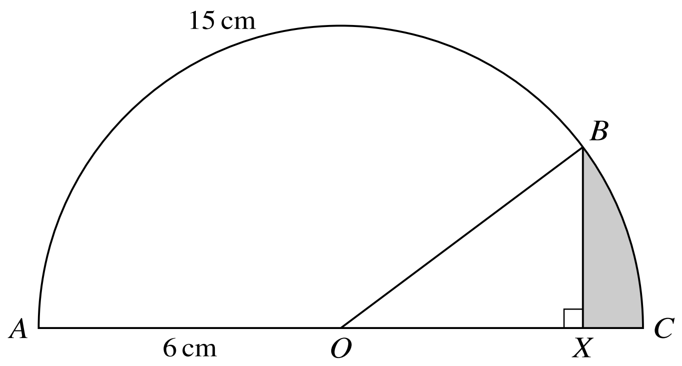

In the diagram, ABC is a semicircle with diameter AC, centre O and radius 6 cm. The length of the arc AB is 15 cm. The point X lies on AC and BX is perpendicular to AX.

Find the perimeter of the shaded region BXC. [6]

9 The equation of a curve is $y = (3 - 2x)^3 + 24x$.

(a) Find expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

(b) Find the coordinates of each of the stationary points on the curve. [3]

(c) Determine the nature of each stationary point. [2]

10 The coordinates of the points A and B are $(-1, -2)$ and $(7, 4)$ respectively.

(a) Find the equation of the circle, C, for which AB is a diameter. [4]

(b) Find the equation of the tangent, T, to circle C at the point B. [4]

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(c) Find the equation of the circle which is the reflection of circle C in the line T. [3]

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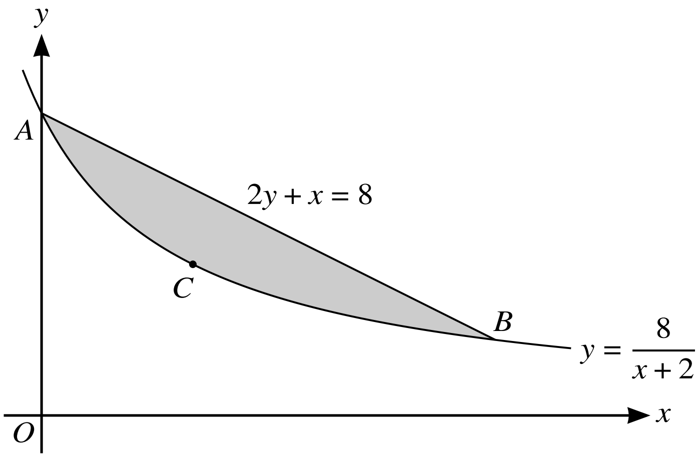

The diagram shows part of the curve $y = \dfrac{8}{x+2}$ and the line $2y + x = 8$, intersecting at points A and B. The point C lies on the curve and the tangent to the curve at C is parallel to AB.

(a) Find, by calculation, the coordinates of A, B and C. [6]

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(b) Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through 360° about the x-axis. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 5 8 7 4 7 0 1 7 4 4 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

1 **(a)** Find the coefficient of x^2 in the expansion of $\left(x - \dfrac{2}{x}\right)^6$. [2]

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(b) Find the coefficient of x^2 in the expansion of $(2 + 3x^2)\left(x - \dfrac{2}{x}\right)^6$. [3]

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2 **(a)** Express the equation $3\cos\theta = 8\tan\theta$ as a quadratic equation in $\sin\theta$. [3]

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(b) Hence find the acute angle, in degrees, for which $3\cos\theta = 8\tan\theta$. [2]

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3 A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of $600\,\text{cm}^3$ per second. The balloon was empty at the start of pumping.

(a) Find the radius of the balloon after 30 seconds. [2]

(b) Find the rate of increase of the radius after 30 seconds. [3]

4 The nth term of an arithmetic progression is $\frac{1}{2}(3n - 15)$.

Find the value of n for which the sum of the first n terms is 84. [5]

5 The function f is defined for $x \in \mathbb{R}$ by

$$f : x \mapsto a - 2x,$$

where a is a constant.

(a) Express $ff(x)$ and $f^{-1}(x)$ in terms of a and x. [4]

(b) Given that $ff(x) = f^{-1}(x)$, find x in terms of a. [2]

6 The equation of a curve is $y = 2x^2 + kx + k - 1$, where k is a constant.

(a) Given that the line $y = 2x + 3$ is a tangent to the curve, find the value of k. [3]

It is now given that $k = 2$.

(b) Express the equation of the curve in the form $y = 2(x + a)^2 + b$, where a and b are constants, and hence state the coordinates of the vertex of the curve. [3]

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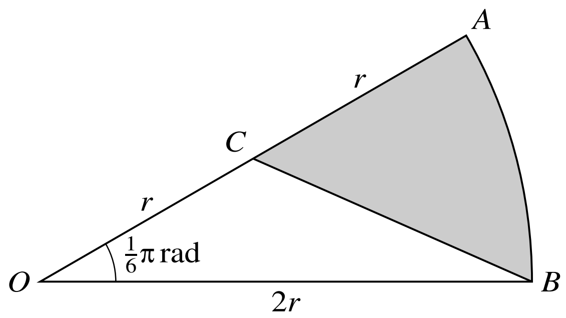

In the diagram, OAB is a sector of a circle with centre O and radius $2r$, and angle $AOB = \frac{1}{6}\pi$ radians. The point C is the midpoint of OA.

(a) Show that the exact length of BC is $r\sqrt{5 - 2\sqrt{3}}$. [2]

(b) Find the exact perimeter of the shaded region. [2]

(c) Find the exact area of the shaded region. [3]

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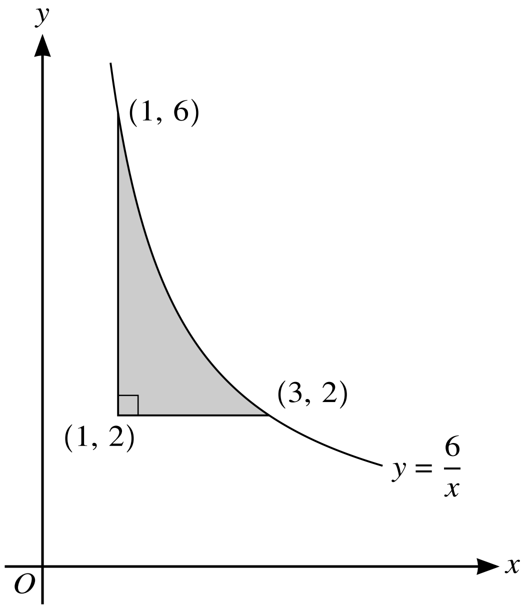

The diagram shows part of the curve $y = \frac{6}{x}$. The points $(1, 6)$ and $(3, 2)$ lie on the curve. The shaded region is bounded by the curve and the lines $y = 2$ and $x = 1$.

(a) Find the volume generated when the shaded region is rotated through 360° about the ***y*-axis**. [5]

(b) The tangent to the curve at a point X is parallel to the line $y + 2x = 0$. Show that X lies on the line $y = 2x$. [3]

9 Functions f and g are such that

$$f(x) = 2 - 3\sin 2x \quad \text{for } 0 \leqslant x \leqslant \pi,$$
$$g(x) = -2f(x) \quad \text{for } 0 \leqslant x \leqslant \pi.$$

(a) State the ranges of f and g. [3]

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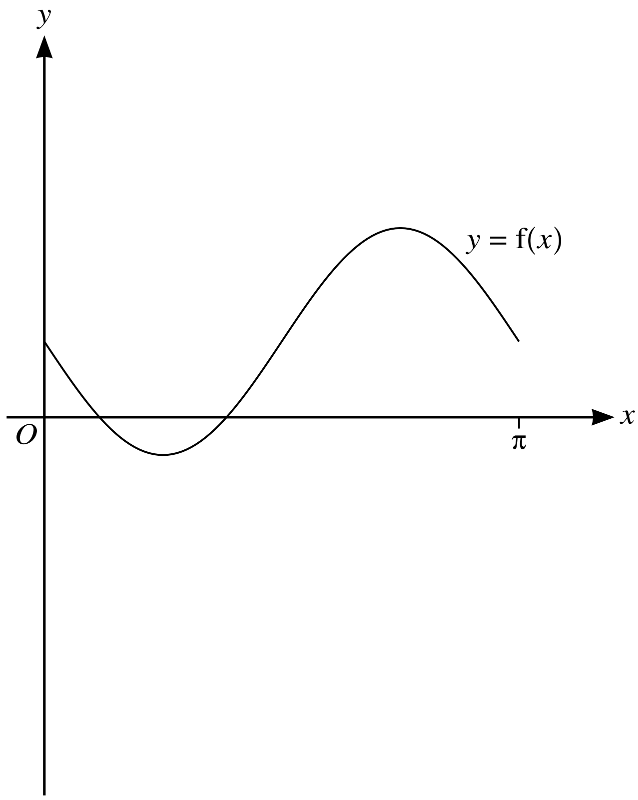
The diagram below shows the graph of $y = f(x)$.



(b) Sketch, on this diagram, the graph of $y = g(x)$. [2]

The function h is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leqslant x \leqslant 0.$$

(c) Describe fully a sequence of transformations that maps the curve $y = f(x)$ on to $y = h(x)$. [3]

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10 The equation of a curve is $y = 54x - (2x - 7)^3$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [4]

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(b) Find the coordinates of each of the stationary points on the curve. [3]

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(c) Determine the nature of each of the stationary points. [2]

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11 The equation of a circle with centre C is $x^2 + y^2 - 8x + 4y - 5 = 0$.

(a) Find the radius of the circle and the coordinates of C. [3]

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The point $P(1, 2)$ lies on the circle.

(b) Show that the equation of the tangent to the circle at P is $4y = 3x + 5$. [3]

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The point Q also lies on the circle and PQ is parallel to the x-axis.

(c) Write down the coordinates of Q. [2]

The tangents to the circle at P and Q meet at T.

(d) Find the coordinates of T. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME						
CENTRE NUMBER						
CANDIDATE NUMBER						

* 1 3 6 0 5 2 4 7 3 1 *

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 06_9709_13/FP

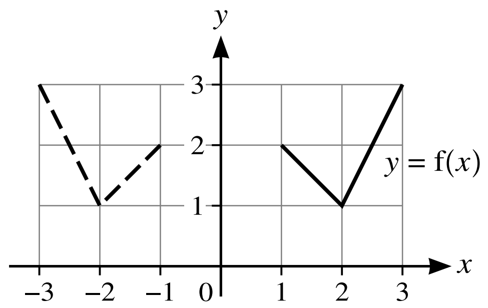
1 Find the set of values of m for which the line with equation $y = mx + 1$ and the curve with equation $y = 3x^2 + 2x + 4$ intersect at two distinct points. [4]

2 The equation of a curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$. It is given that the point $(4, 7)$ lies on the curve.

Find the equation of the curve. [4]

3 In each of parts **(a)**, **(b)** and **(c)**, the graph shown with solid lines has equation $y = \mathrm{f}(x)$. The graph shown with broken lines is a transformation of $y = \mathrm{f}(x)$.

(a)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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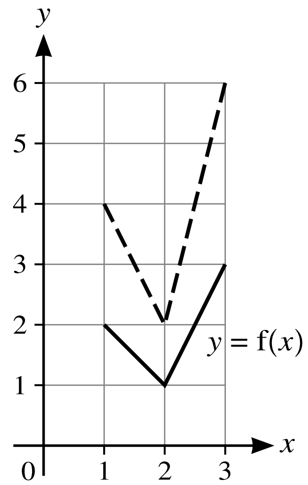
(b)



State, in terms of f, the equation of the graph shown with broken lines. [1]

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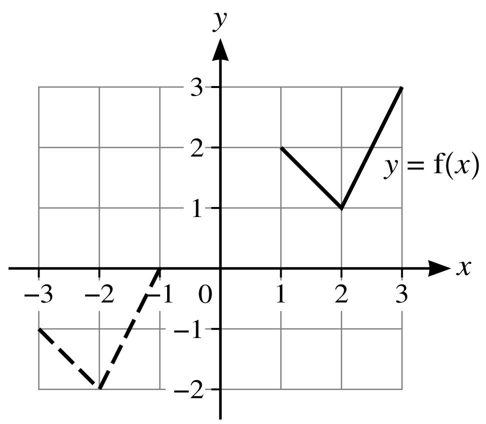
(c)



State, in terms of f, the equation of the graph shown with broken lines. [2]

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4 **(a)** Expand $(1 + a)^5$ in ascending powers of a up to and including the term in a^3. [1]

(b) Hence expand $[1 + (x + x^2)]^5$ in ascending powers of x up to and including the term in x^3, simplifying your answer. [3]

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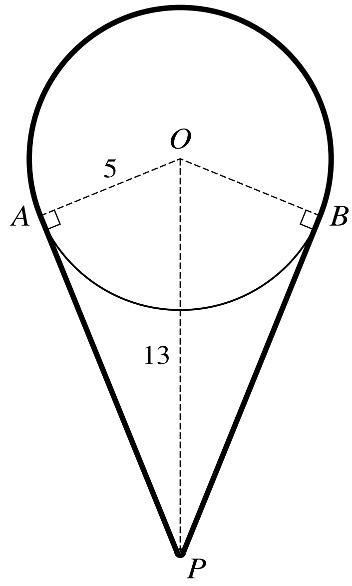

The diagram shows a cord going around a pulley and a pin. The pulley is modelled as a circle with centre O and radius 5 cm. The thickness of the cord and the size of the pin P can be neglected. The pin is situated 13 cm vertically below O. Points A and B are on the circumference of the circle such that AP and BP are tangents to the circle. The cord passes over the major arc AB of the circle and under the pin such that the cord is taut.

Calculate the length of the cord. [6]

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6 A point P is moving along a curve in such a way that the x-coordinate of P is increasing at a constant rate of 2 units per minute. The equation of the curve is $y = (5x - 1)^{\frac{1}{2}}$.

(a) Find the rate at which the y-coordinate is increasing when $x = 1$. [4]

(b) Find the value of x when the y-coordinate is increasing at $\frac{5}{8}$ units per minute. [3]

7 **(a)** Show that $\dfrac{\tan\theta}{1+\cos\theta}+\dfrac{\tan\theta}{1-\cos\theta}\equiv\dfrac{2}{\sin\theta\cos\theta}$. [4]

(b) Hence solve the equation $\dfrac{\tan\theta}{1+\cos\theta}+\dfrac{\tan\theta}{1-\cos\theta}=\dfrac{6}{\tan\theta}$ for $0° < \theta < 180°$. [4]

8 The first term of a progression is $\sin^2\theta$, where $0 < \theta < \frac{1}{2}\pi$. The second term of the progression is $\sin^2\theta\cos^2\theta$.

(a) Given that the progression is geometric, find the sum to infinity. [3]

It is now given instead that the progression is arithmetic.

(b) **(i)** Find the common difference of the progression in terms of $\sin\theta$. [3]

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(ii) Find the sum of the first 16 terms when $\theta = \frac{1}{3}\pi$. [3]

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9 The functions f and g are defined by

$$\mathrm{f}(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$\mathrm{g}(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

(a) Express $\mathrm{f}(x)$ in the form $(x - a)^2 + b$. [2]

It is given that f is a one-one function.

(b) State the smallest possible value of c. [1]

It is now given that $c = 5$.

(c) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(d) Find an expression for $gf(x)$ and state the range of gf. [3]

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10 **(a)** The coordinates of two points A and B are $(-7, 3)$ and $(5, 11)$ respectively.

Show that the equation of the perpendicular bisector of AB is $3x + 2y = 11$. [4]

(b) A circle passes through A and B and its centre lies on the line $12x - 5y = 70$.

Find an equation of the circle. [5]

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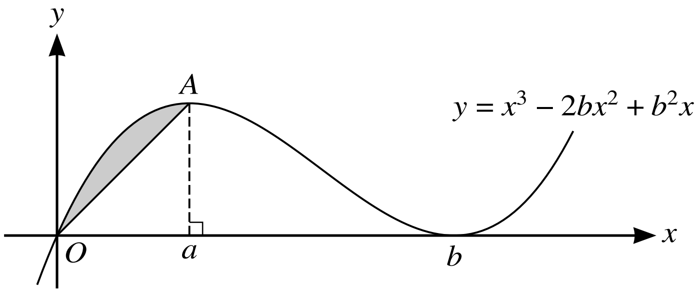

The diagram shows part of the curve with equation $y = x^3 - 2bx^2 + b^2x$ and the line OA, where A is the maximum point on the curve. The x-coordinate of A is a and the curve has a minimum point at $(b, 0)$, where a and b are positive constants.

(a) Show that $b = 3a$. [4]

(b) Show that the area of the shaded region between the line and the curve is ka^4, where k is a fraction to be found. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7942312992

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $\left(\frac{1}{2}, 4\right)$.

Find the equation of the curve. [4]

2 The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression. [5]

3 **(a)** Find the first three terms in the expansion of $(3-2x)^5$ in ascending powers of x. [3]

(b) Hence find the coefficient of x^2 in the expansion of $(4+x)^2(3-2x)^5$. [3]

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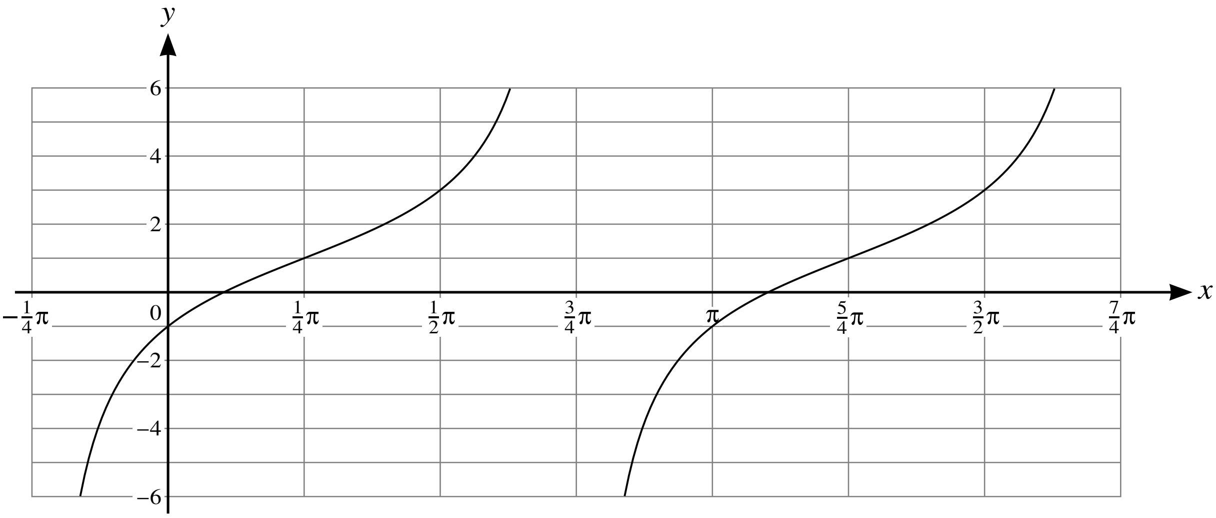


The diagram shows part of the graph of $y = a\tan(x - b) + c$.

Given that $0 < b < \pi$, state the values of the constants a, b and c. [3]

5 The fifth, sixth and seventh terms of a geometric progression are $8k$, -12 and $2k$ respectively.

Given that k is negative, find the sum to infinity of the progression. [4]

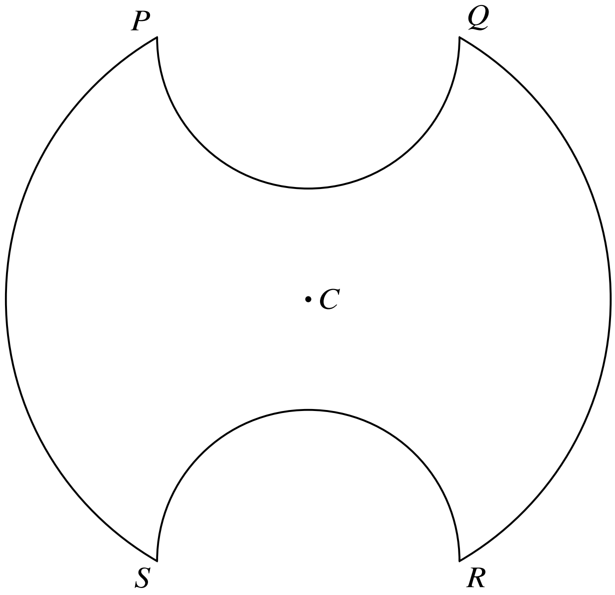
6 The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k. [5]

7 **(a)** Prove the identity $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} \equiv 1 - \tan^2\theta$. [2]

(b) Hence solve the equation $\dfrac{1 - 2\sin^2\theta}{1 - \sin^2\theta} = 2\tan^4\theta$ for $0° \leqslant \theta \leqslant 180°$. [3]

8



The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C. The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

(a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

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(b) Find the exact perimeter of the plate. [3]

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(c) Show that the area of the plate is $\left(\frac{20}{3}\pi + 8\sqrt{3}\right)\text{cm}^2$. [5]

9 Functions f and g are defined as follows:

$$f(x) = (x-2)^2 - 4 \text{ for } x \geqslant 2,$$
$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

(a) State the range of f. [1]

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(b) Find $f^{-1}(x)$. [2]

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(c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]

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(d) Given instead that $\mathrm{ggf}^{-1}(12) = 62$, find the possible values of a. [5]

10 The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

(a) Find the x-coordinates of the points A and B where the circle intersects the x-axis. [2]

(b) Find the point of intersection of the tangents to the circle at A and B. [6]

11 The equation of a curve is $y = 2\sqrt{3x+4} - x$.

(a) Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

(b) Find the coordinates of the stationary point. [3]

(c) Determine the nature of the stationary point. [2]

(d) Find the exact area of the region bounded by the curve, the x-axis and the lines $x = 0$ and $x = 4$. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

1 **(a)** Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

2 **(a)** The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

3 The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2, k)$ $B\,(2.9, 2.8025)$ $C\,(2.99, 2.9800)$ $D\,(2.999, 2.9980)$ $E\,(3, 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

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(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

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The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

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4 The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

5 The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

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(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

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6 Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

7 The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

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(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

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8 The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b + 3$ respectively.

(a) Find the values of a and b. [5]

(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

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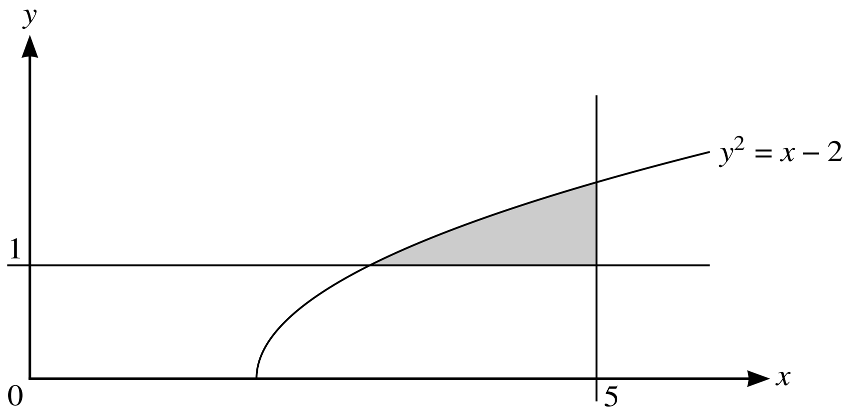

The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

10 **(a)** Prove the identity $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} \equiv \frac{4\tan x}{\cos x}$. [4]

(b) Hence solve the equation $\dfrac{1+\sin x}{1-\sin x} - \dfrac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

11 The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

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(b) Find the equation of the curve. [4]

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(c) Find $\frac{d^2y}{dx^2}$. [2]

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(d) Determine the nature of the stationary point at (2, −3.5). [2]

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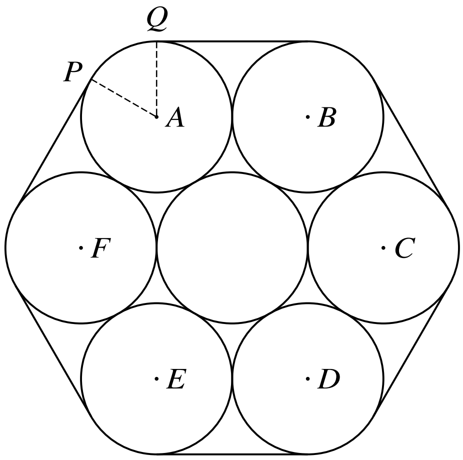

The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

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(b) Find the length of the rope. [4]

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(c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$. [2]

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(d) Find the area of the complete region enclosed by the rope. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 6 4 1 2 2 4 5 0 5 1 *

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

JC21 06_9709_13/RP

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1 A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$.

Find $f(x)$. [3]

2 The function f is defined by $f(x) = \frac{1}{3}(2x - 1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a. [4]

3 A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m, and the corresponding coordinates of the points at which the line touches the curve. [6]

4 **(a)** Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k.$$ [2]

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(b) Hence express $\cos x$ in terms of k. [2]

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(c) Hence solve the equation $\dfrac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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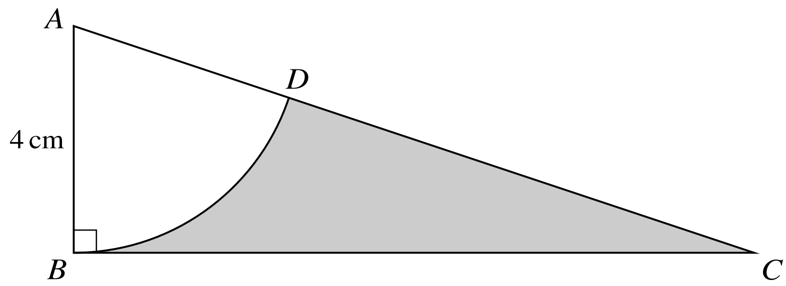

The diagram shows a triangle ABC, in which angle $ABC = 90°$ and $AB = 4$ cm. The sector ABD is part of a circle with centre A. The area of the sector is $10\,\text{cm}^2$.

(a) Find angle BAD in radians. [2]

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(b) Find the perimeter of the shaded region. [4]

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6 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$
$$g(x) = x^2 + 4x + 13.$$

(a) By first expressing each of f(x) and g(x) in completed square form, express g(x) in the form $f(x + p) + q$, where p and q are constants. [4]

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(b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

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7 **(a)** Write down the first four terms of the expansion, in ascending powers of x, of $(a - x)^6$. [2]

(b) Given that the coefficient of x^2 in the expansion of $\left(1 + \frac{2}{ax}\right)(a - x)^6$ is -20, find in exact form the possible values of the constant a. [5]

8 Functions f and g are defined as follows:

$$\mathrm{f} : x \mapsto x^2 - 1 \text{ for } x < 0,$$

$$\mathrm{g} : x \mapsto \frac{1}{2x+1} \text{ for } x < -\tfrac{1}{2}.$$

(a) Solve the equation $\mathrm{fg}(x) = 3$. [4]

(b) Find an expression for $(\mathrm{fg})^{-1}(x)$. [3]

9 **(a)** A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio. [3]

(b) An arithmetic progression P has first term a and common difference d. An arithmetic progression Q has first term $2(a+1)$ and common difference $(d+1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d. [6]

10 Points $A(-2, 3)$, $B(3, 0)$ and $C(6, 5)$ lie on the circumference of a circle with centre D.

(a) Show that angle $ABC = 90°$. [2]

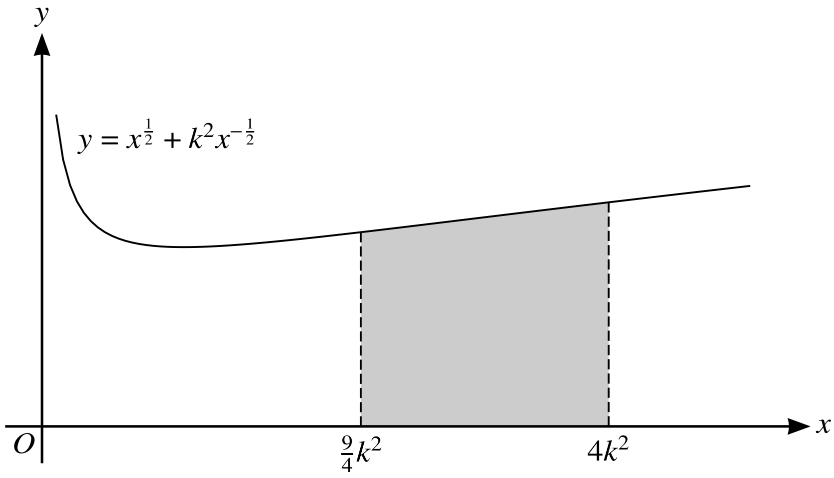
(b) Hence state the coordinates of D. [1]

(c) Find an equation of the circle. [2]

The point E lies on the circumference of the circle such that BE is a diameter.

(d) Find an equation of the tangent to the circle at E. [5]

11



The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

(a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k. [4]

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The tangent at the point on the curve where $x = 4k^2$ intersects the y-axis at P.

(b) Find the y-coordinate of P in terms of k. [4]

The shaded region is bounded by the curve, the x-axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

(c) Find the area of the shaded region in terms of k. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International AS & A Level

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* 9 2 0 4 7 5 9 1 2 3 *

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **4** printed pages and **1** insert.

JC16 11_9709_11/2R

CAMBRIDGE International Examinations

1 **(i)** Express $x^2 + 6x + 2$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(ii) Hence, or otherwise, find the set of values of x for which $x^2 + 6x + 2 > 9$. [2]

2 Find the term independent of x in the expansion of $\left(2x + \frac{1}{2x^3}\right)^8$. [4]

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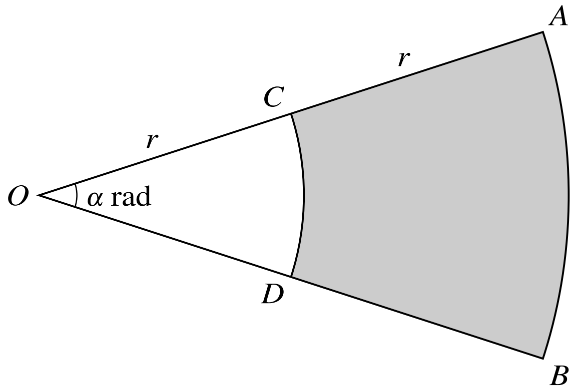


In the diagram OCA and ODB are radii of a circle with centre O and radius $2r$ cm. Angle $AOB = \alpha$ radians. CD and AB are arcs of circles with centre O and radii r cm and $2r$ cm respectively. The perimeter of the shaded region $ABDC$ is $4.4r$ cm.

(i) Find the value of α. [2]

(ii) It is given that the area of the shaded region is $30\,\text{cm}^2$. Find the value of r. [3]

4 C is the mid-point of the line joining $A\,(14, -7)$ to $B\,(-6, 3)$. The line through C perpendicular to AB crosses the y-axis at D.

(i) Find the equation of the line CD, giving your answer in the form $y = mx + c$. [4]

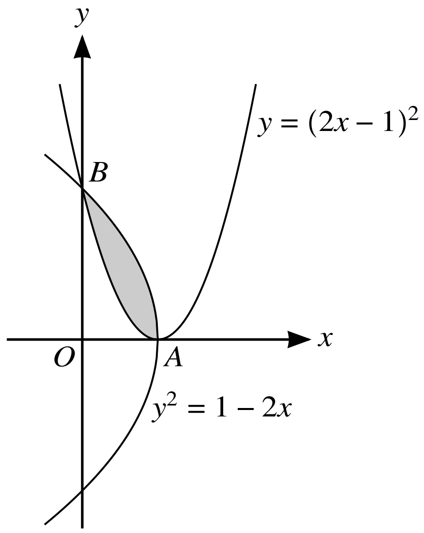
(ii) Find the distance AD. [2]

5 The sum of the 1st and 2nd terms of a geometric progression is 50 and the sum of the 2nd and 3rd terms is 30. Find the sum to infinity. [6]

6 **(i)** Show that $\cos^4 x \equiv 1 - 2\sin^2 x + \sin^4 x$. [1]

(ii) Hence, or otherwise, solve the equation $8\sin^4 x + \cos^4 x = 2\cos^2 x$ for $0° \leqslant x \leqslant 360°$. [5]

7



The diagram shows parts of the curves $y = (2x - 1)^2$ and $y^2 = 1 - 2x$, intersecting at points A and B.

(i) State the coordinates of A. [1]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

8 The functions f and g are defined by

$$\mathrm{f}(x) = \frac{4}{x} - 2 \quad \text{for } x > 0,$$

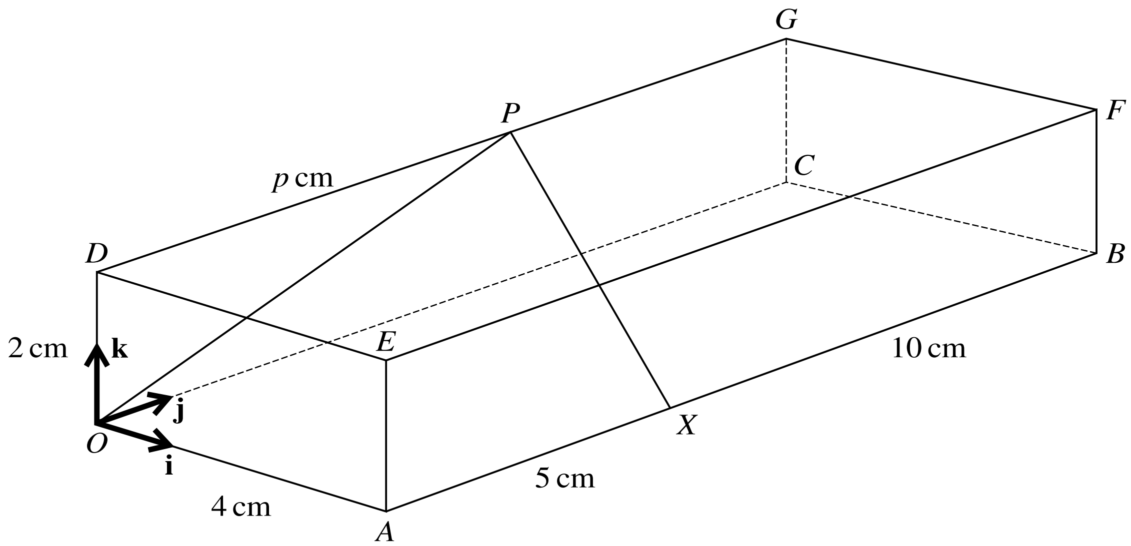
$$\mathrm{g}(x) = \frac{4}{5x + 2} \quad \text{for } x \geqslant 0.$$

(i) Find and simplify an expression for $\mathrm{fg}(x)$ and state the range of fg. [3]

(ii) Find an expression for $\mathrm{g}^{-1}(x)$ and find the domain of g^{-1}. [5]

[Questions 9, 10 and 11 are printed on the next page.]

9



The diagram shows a cuboid $OABCDEFG$ with a horizontal base $OABC$ in which $OA = 4$ cm and $AB = 15$ cm. The height OD of the cuboid is 2 cm. The point X on AB is such that $AX = 5$ cm and the point P on DG is such that $DP = p$ cm, where p is a constant. Unit vectors **i**, **j** and **k** are parallel to OA, OC and OD respectively.

(i) Find the possible values of p such that angle $OPX = 90°$. [4]

(ii) For the case where $p = 9$, find the unit vector in the direction of $\overrightarrow{XP}$. [2]

(iii) A point Q lies on the face $CBFG$ and is such that XQ is parallel to AG. Find $\overrightarrow{XQ}$. [3]

10 A curve has equation $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 3x^{\frac{1}{2}} - 2x^{-\frac{1}{2}}$. The point A is the only point on the curve at which the gradient is -1.

(i) Find the x-coordinate of A. [3]

(ii) Given that the curve also passes through the point $(4, 10)$, find the y-coordinate of A, giving your answer as a fraction. [6]

11 The point $P\,(3, 5)$ lies on the curve $y = \dfrac{1}{x-1} - \dfrac{9}{x-5}$.

(i) Find the x-coordinate of the point where the normal to the curve at P intersects the x-axis. [5]

(ii) Find the x-coordinate of each of the stationary points on the curve and determine the nature of each stationary point, justifying your answers. [6]

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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **3** printed pages, **1** blank page and **1** insert.

JC16 11_9709_12/RP

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1 A curve is such that $\frac{dy}{dx} = \frac{8}{\sqrt{(4x+1)}}$. The point (2, 5) lies on the curve. Find the equation of the curve. [4]

2 **(i)** Express the equation $\sin 2x + 3\cos 2x = 3(\sin 2x - \cos 2x)$ in the form $\tan 2x = k$, where k is a constant. [2]

(ii) Hence solve the equation for $-90° \leqslant x \leqslant 90°$. [3]

3 A curve has equation $y = 2x^2 - 6x + 5$.

(i) Find the set of values of x for which $y > 13$. [3]

(ii) Find the value of the constant k for which the line $y = 2x + k$ is a tangent to the curve. [3]

4 In the expansion of $(3 - 2x)\left(1 + \frac{x}{2}\right)^n$, the coefficient of x is 7. Find the value of the constant n and hence find the coefficient of x^2. [6]

5 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, intersects the x- and y-axes at the points A and B respectively. The mid-point of AB lies on the line $2x + y = 10$ and the distance $AB = 10$. Find the values of a and b. [6]

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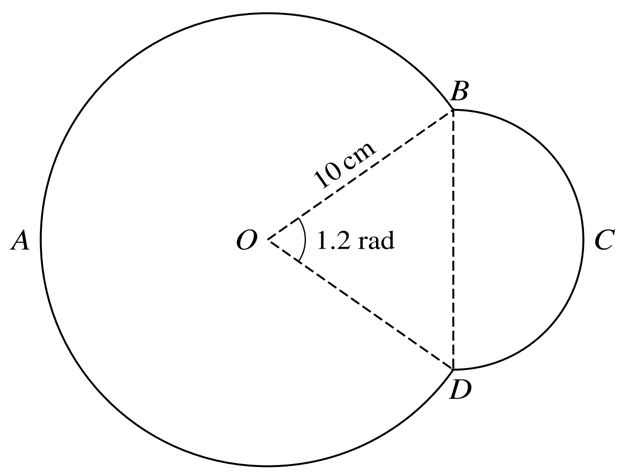

The diagram shows a metal plate $ABCD$ made from two parts. The part BCD is a semicircle. The part DAB is a segment of a circle with centre O and radius 10 cm. Angle BOD is 1.2 radians.

(i) Show that the radius of the semicircle is 5.646 cm, correct to 3 decimal places. [2]

(ii) Find the perimeter of the metal plate. [3]

(iii) Find the area of the metal plate. [3]

7 The equation of a curve is $y = 2 + \dfrac{3}{2x - 1}$.

(i) Obtain an expression for $\dfrac{dy}{dx}$. [2]

(ii) Explain why the curve has no stationary points. [1]

At the point P on the curve, $x = 2$.

(iii) Show that the normal to the curve at P passes through the origin. [4]

(iv) A point moves along the curve in such a way that its x-coordinate is decreasing at a constant rate of 0.06 units per second. Find the rate of change of the y-coordinate as the point passes through P. [2]

8 **(a)** A cyclist completes a long-distance charity event across Africa. The total distance is 3050 km. He starts the event on May 1st and cycles 200 km on that day. On each subsequent day he reduces the distance cycled by 5 km.

(i) How far will he travel on May 15th? [2]

(ii) On what date will he finish the event? [3]

(b) A geometric progression is such that the third term is 8 times the sixth term, and the sum of the first six terms is $31\frac{1}{2}$. Find

(i) the first term of the progression, [4]

(ii) the sum to infinity of the progression. [1]

9 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -2 \\ 3 \\ 6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ 5 \end{pmatrix}.$$

(i) Use a scalar product to find angle AOB. [4]

(ii) Find the vector which is in the same direction as $\overrightarrow{AC}$ and of magnitude 15 units. [3]

(iii) Find the value of the constant p for which $p\overrightarrow{OA} + \overrightarrow{OC}$ is perpendicular to $\overrightarrow{OB}$. [3]

10 A function f is defined by $\mathrm{f} : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant \pi$.

(i) Find the range of f. [2]

(ii) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(iii) Solve the equation $\mathrm{f}(x) = 6$, giving answers in terms of π. [3]

The function g is defined by $\mathrm{g} : x \mapsto 5 - 2\sin 2x$ for $0 \leqslant x \leqslant k$, where k is a constant.

(iv) State the largest value of k for which g has an inverse. [1]

(v) For this value of k, find an expression for $\mathrm{g}^{-1}(x)$. [3]

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MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **4** printed pages and **1** insert.

1 Find the set of values of k for which the curve $y = kx^2 - 3x$ and the line $y = x - k$ do not meet. [3]

2 The coefficient of x^3 in the expansion of $(1 - 3x)^6 + (1 + ax)^5$ is 100. Find the value of the constant a. [4]

3 Showing all necessary working, solve the equation $6\sin^2 x - 5\cos^2 x = 2\sin^2 x + \cos^2 x$ for $0° \leqslant x \leqslant 360°$. [4]

4 The function f is such that $f(x) = x^3 - 3x^2 - 9x + 2$ for $x > n$, where n is an integer. It is given that f is an increasing function. Find the least possible value of n. [4]

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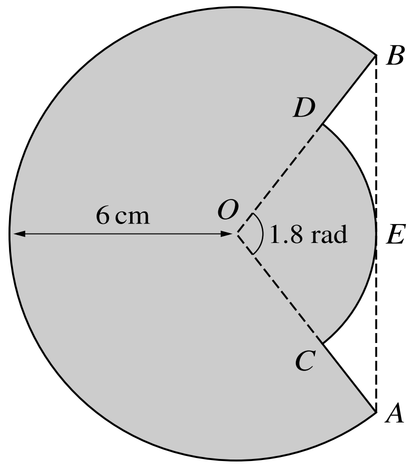

The diagram shows a major arc AB of a circle with centre O and radius 6 cm. Points C and D on OA and OB respectively are such that the line AB is a tangent at E to the arc CED of a smaller circle also with centre O. Angle COD = 1.8 radians.

(i) Show that the radius of the arc CED is 3.73 cm, correct to 3 significant figures. [2]

(ii) Find the area of the shaded region. [4]

6 Three points, A, B and C, are such that B is the mid-point of AC. The coordinates of A are $(2, m)$ and the coordinates of B are $(n, -6)$, where m and n are constants.

(i) Find the coordinates of C in terms of m and n. [2]

The line $y = x + 1$ passes through C and is perpendicular to AB.

(ii) Find the values of m and n. [5]

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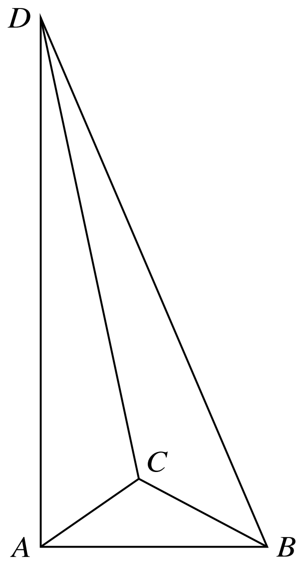


The diagram shows a triangular pyramid $ABCD$. It is given that

$$\overrightarrow{AB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}, \quad \overrightarrow{AC} = \mathbf{i} - 2\mathbf{j} - \mathbf{k} \quad \text{and} \quad \overrightarrow{AD} = \mathbf{i} + 4\mathbf{j} - 7\mathbf{k}.$$

(i) Verify, showing all necessary working, that each of the angles DAB, DAC and CAB is 90°. [3]

(ii) Find the exact value of the area of the triangle ABC, and hence find the exact value of the volume of the pyramid. [4]

[The volume V of a pyramid of base area A and vertical height h is given by $V = \frac{1}{3}Ah$.]

8 **(i)** Express $4x^2 + 12x + 10$ in the form $(ax + b)^2 + c$, where a, b and c are constants. [3]

(ii) Functions f and g are both defined for $x > 0$. It is given that $\mathrm{f}(x) = x^2 + 1$ and $\mathrm{fg}(x) = 4x^2 + 12x + 10$. Find $\mathrm{g}(x)$. [1]

(iii) Find $(\mathrm{fg})^{-1}(x)$ and give the domain of $(\mathrm{fg})^{-1}$. [4]

9 **(a)** Two convergent geometric progressions, P and Q, have the same sum to infinity. The first and second terms of P are 6 and $6r$ respectively. The first and second terms of Q are 12 and $-12r$ respectively. Find the value of the common sum to infinity. [3]

(b) The first term of an arithmetic progression is $\cos\theta$ and the second term is $\cos\theta + \sin^2\theta$, where $0 \leqslant \theta \leqslant \pi$. The sum of the first 13 terms is 52. Find the possible values of θ. [5]

[Questions 10 and 11 are printed on the next page.]

10 A curve is such that $\dfrac{dy}{dx} = \dfrac{2}{a}x^{-\frac{1}{2}} + ax^{-\frac{3}{2}}$, where a is a positive constant. The point $A\,(a^2, 3)$ lies on the curve. Find, in terms of a,

(i) the equation of the tangent to the curve at A, simplifying your answer, [3]

(ii) the equation of the curve. [4]

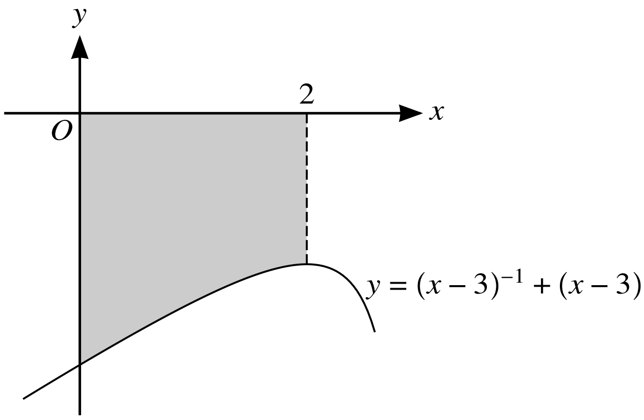
It is now given that $B\,(16, 8)$ also lies on the curve.

(iii) Find the value of a and, using this value, find the distance AB. [5]

11 A curve has equation $y = (kx - 3)^{-1} + (kx - 3)$, where k is a non-zero constant.

(i) Find the x-coordinates of the stationary points in terms of k, and determine the nature of each stationary point, justifying your answers. [7]

(ii)



The diagram shows part of the curve for the case when $k = 1$. Showing all necessary working, find the volume obtained when the region between the curve, the x-axis, the y-axis and the line $x = 2$, shown shaded in the diagram, is rotated through 360° about the x-axis. [5]

Cambridge International Examinations
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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC17 11_9709_11/RP

[Turn over

1 A curve has equation $y = 2x^{\frac{3}{2}} - 3x - 4x^{\frac{1}{2}} + 4$. Find the equation of the tangent to the curve at the point $(4, 0)$. [4]

2 A function f is defined by $\mathrm{f} : x \mapsto x^3 - x^2 - 8x + 5$ for $x < a$. It is given that f is an increasing function. Find the largest possible value of the constant a. [4]

3 **(a)** A geometric progression has first term $3a$ and common ratio r. A second geometric progression has first term a and common ratio $-2r$. The two progressions have the same sum to infinity. Find the value of r. [3]

(b) The first two terms of an arithmetic progression are 15 and 19 respectively. The first two terms of a second arithmetic progression are 420 and 415 respectively. The two progressions have the same sum of the first n terms. Find the value of n. [3]

4 Machines in a factory make cardboard cones of base radius r cm and vertical height h cm. The volume, V cm^3, of such a cone is given by $V = \frac{1}{3}\pi r^2 h$. The machines produce cones for which $h + r = 18$.

(i) Show that $V = 6\pi r^2 - \frac{1}{3}\pi r^3$. [1]

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(ii) Given that r can vary, find the non-zero value of r for which V has a stationary value and show that the stationary value is a maximum. [4]

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(iii) Find the maximum volume of a cone that can be made by these machines. [1]

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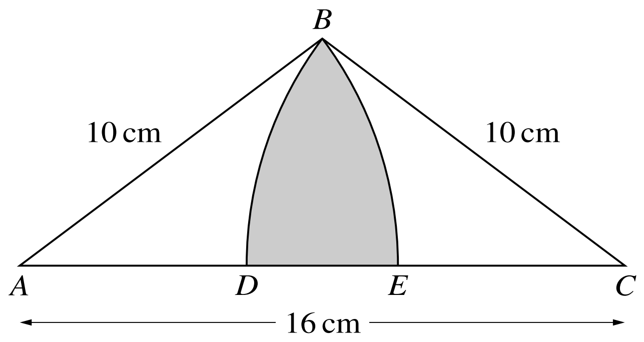

The diagram shows an isosceles triangle ABC in which $AC = 16$ cm and $AB = BC = 10$ cm. The circular arcs BE and BD have centres at A and C respectively, where D and E lie on AC.

(i) Show that angle $BAC = 0.6435$ radians, correct to 4 decimal places. [1]

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(ii) Find the area of the shaded region. [5]

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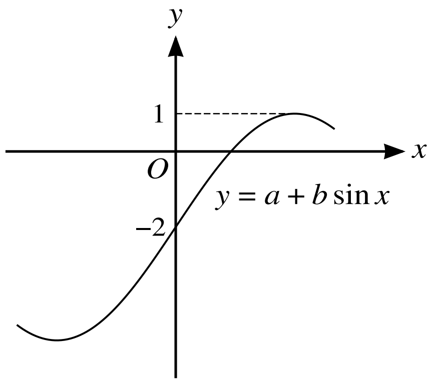
6 The points $A\,(1,\ 1)$ and $B\,(5,\ 9)$ lie on the curve $6y = 5x^2 - 18x + 19$.

(i) Show that the equation of the perpendicular bisector of AB is $2y = 13 - x$. [4]

The perpendicular bisector of AB meets the curve at C and D.

(ii) Find, by calculation, the distance CD, giving your answer in the form $\sqrt{\left(\frac{p}{q}\right)}$, where p and q are integers. [5]

7 **(a)**



The diagram shows part of the graph of $y = a + b\sin x$. Find the values of the constants a and b. [2]

(b) **(i)** Show that the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

may be expressed as $3\cos^2\theta - 2\cos\theta - 1 = 0$. [3]

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(ii) Hence solve the equation

$$(\sin\theta + 2\cos\theta)(1 + \sin\theta - \cos\theta) = \sin\theta(1 + \cos\theta)$$

for $-180° \leqslant \theta \leqslant 180°$. [4]

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8 **(a)** Relative to an origin O, the position vectors of two points P and Q are **p** and **q** respectively. The point R is such that PQR is a straight line with Q the mid-point of PR. Find the position vector of R in terms of **p** and **q**, simplifying your answer. [3]

(b) The vector $6\mathbf{i} + a\mathbf{j} + b\mathbf{k}$ has magnitude 21 and is perpendicular to $3\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$. Find the possible values of a and b, showing all necessary working. [6]

9 Functions f and g are defined for $x > 3$ by

$$\mathrm{f} : x \mapsto \frac{1}{x^2 - 9},$$
$$\mathrm{g} : x \mapsto 2x - 3.$$

(i) Find and simplify an expression for $\mathrm{gg}(x)$. [2]

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(ii) Find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

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(iii) Solve the equation $fg(x) = \frac{1}{7}$. [4]

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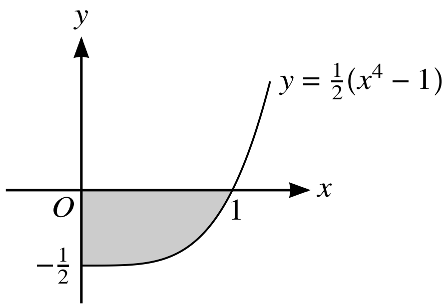

The diagram shows part of the curve $y = \frac{1}{2}(x^4 - 1)$, defined for $x \geqslant 0$.

(i) Find, showing all necessary working, the area of the shaded region. [3]

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(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [4]

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(iii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the y-axis. [5]

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Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3053484001

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC17 11_9709_12/RP

CAMBRIDGE International Examinations

1 Find the term independent of x in the expansion of $\left(2x - \frac{1}{4x^2}\right)^9$. [4]

2 A function f is defined by $\mathrm{f} : x \mapsto 4 - 5x$ for $x \in \mathbb{R}$.

(i) Find an expression for $\mathrm{f}^{-1}(x)$ and find the point of intersection of the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [3]

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(ii) Sketch, on the same diagram, the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

3 **(a)** Each year, the value of a certain rare stamp increases by 5% of its value at the beginning of the year. A collector bought the stamp for \$10 000 at the beginning of 2005. Find its value at the beginning of 2015 correct to the nearest \$100. [2]

(b) The sum of the first n terms of an arithmetic progression is $\frac{1}{2}n(3n + 7)$. Find the 1st term and the common difference of the progression. [4]

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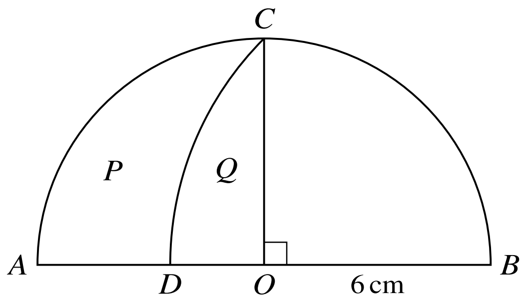

The diagram shows a semicircle with centre O and radius 6 cm. The radius OC is perpendicular to the diameter AB. The point D lies on AB, and DC is an arc of a circle with centre B.

(i) Calculate the length of the arc DC. [3]

(ii) Find the value of

$$\frac{\text{area of region } P}{\text{area of region } Q},$$

giving your answer correct to 3 significant figures. [4]

5 **(i)** Show that the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ can be expressed as

$$2\cos^2 2x + 3\cos 2x + 1 = 0.$$ [3]

(ii) Hence solve the equation $\cos 2x(\tan^2 2x + 3) + 3 = 0$ for $0° \leqslant x \leqslant 180°$. [4]

6 **(a)** The function f, defined by $f : x \mapsto a + b\sin x$ for $x \in \mathbb{R}$, is such that $f\left(\frac{1}{6}\pi\right) = 4$ and $f\left(\frac{1}{2}\pi\right) = 3$.

(i) Find the values of the constants a and b. [3]

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(ii) Evaluate ff(0). [2]

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(b) The function g is defined by $\mathrm{g} : x \mapsto c + d \sin x$ for $x \in \mathbb{R}$. The range of g is given by $-4 \leqslant \mathrm{g}(x) \leqslant 10$. Find the values of the constants c and d. [3]

7 Points A and B lie on the curve $y = x^2 - 4x + 7$. Point A has coordinates $(4, 7)$ and B is the stationary point of the curve. The equation of a line L is $y = mx - 2$, where m is a constant.

(i) In the case where L passes through the mid-point of AB, find the value of m. [4]

(ii) Find the set of values of m for which L does not meet the curve. [4]

8 A curve is such that $\frac{dy}{dx} = -x^2 + 5x - 4$.

(i) Find the x-coordinate of each of the stationary points of the curve. [2]

(ii) Obtain an expression for $\frac{d^2y}{dx^2}$ and hence or otherwise find the nature of each of the stationary points. [3]

(iii) Given that the curve passes through the point $(6, 2)$, find the equation of the curve. [4]

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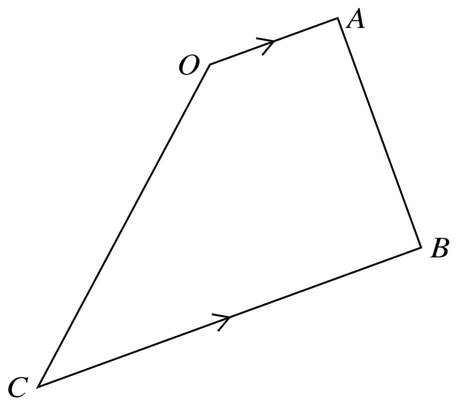

The diagram shows a trapezium $OABC$ in which OA is parallel to CB. The position vectors of A and B relative to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ 1 \\ 1 \end{pmatrix}$.

(i) Show that angle OAB is 90°. [3]

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The magnitude of $\overrightarrow{CB}$ is three times the magnitude of $\overrightarrow{OA}$.

(ii) Find the position vector of C. [3]

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(iii) Find the exact area of the trapezium $OABC$, giving your answer in the form $a\sqrt{b}$, where a and b are integers. [3]

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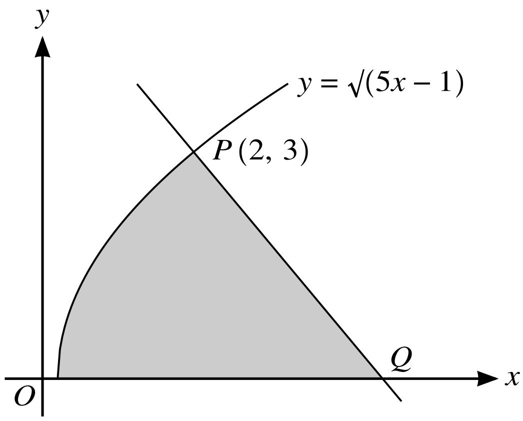

The diagram shows part of the curve $y = \sqrt{(5x - 1)}$ and the normal to the curve at the point $P\,(2, 3)$. This normal meets the x-axis at Q.

(i) Find the equation of the normal at P. [4]

(ii) Find, showing all necessary working, the area of the shaded region. [7]

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Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 7 2 3 2 0 4 4 4 6 5 *

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC17 11_9709_13/RP

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[Turn over

1 An arithmetic progression has first term -12 and common difference 6. The sum of the first n terms exceeds 3000. Calculate the least possible value of n. [4]

2 Find the set of values of a for which the curve $y = -\frac{2}{x}$ and the straight line $y = ax + 3a$ meet at two distinct points. [4]

3 **(i)** Find the term independent of x in the expansion of $\left(\frac{2}{x} - 3x\right)^6$. [2]

(ii) Find the value of a for which there is no term independent of x in the expansion of

$$(1 + ax^2)\left(\frac{2}{x} - 3x\right)^6.$$ [3]

4 The function f is such that $f(x) = (2x - 1)^{\frac{3}{2}} - 6x$ for $\frac{1}{2} < x < k$, where k is a constant. Find the largest value of k for which f is a decreasing function. [5]

5 **(i)** Show that the equation $\dfrac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ may be expressed as $5\cos^2\theta - \cos\theta - 4 = 0$. [3]

(ii) Hence solve the equation $\dfrac{\cos\theta + 4}{\sin\theta + 1} + 5\sin\theta - 5 = 0$ for $0° \leqslant \theta \leqslant 360°$. [4]

6 The functions f and g are defined by

$$f(x) = \frac{2}{x^2 - 1} \text{ for } x < -1,$$
$$g(x) = x^2 + 1 \text{ for } x > 0.$$

(i) Find an expression for $f^{-1}(x)$. [3]

(ii) Solve the equation $gf(x) = 5$. [4]

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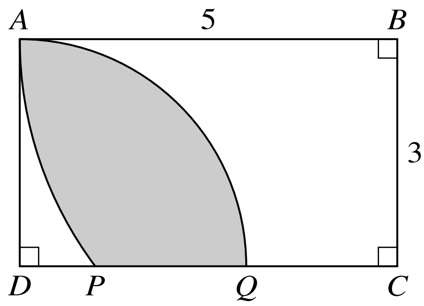

The diagram shows a rectangle $ABCD$ in which $AB = 5$ units and $BC = 3$ units. Point P lies on DC and AP is an arc of a circle with centre B. Point Q lies on DC and AQ is an arc of a circle with centre D.

(i) Show that angle $ABP = 0.6435$ radians, correct to 4 decimal places. [1]

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(ii) Calculate the areas of the sectors BAP and DAQ. [3]

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(iii) Calculate the area of the shaded region. [3]

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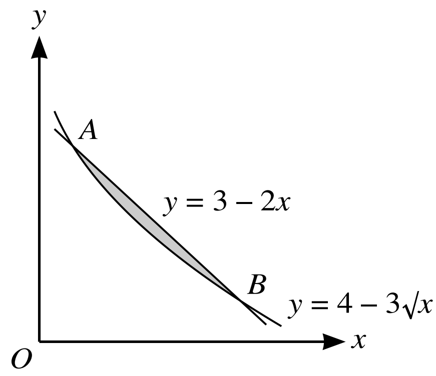


The diagram shows parts of the graphs of $y = 3 - 2x$ and $y = 4 - 3\sqrt{x}$ intersecting at points A and B.

(i) Find by calculation the x-coordinates of A and B. [3]

(ii) Find, showing all necessary working, the area of the shaded region. [5]

9 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 8 \\ -6 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -10 \\ 3 \\ -13 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}.$$

A fourth point, D, is such that the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$ are the first, second and third terms respectively of a geometric progression.

(i) Find the magnitudes $|\overrightarrow{AB}|$, $|\overrightarrow{BC}|$ and $|\overrightarrow{CD}|$. [5]

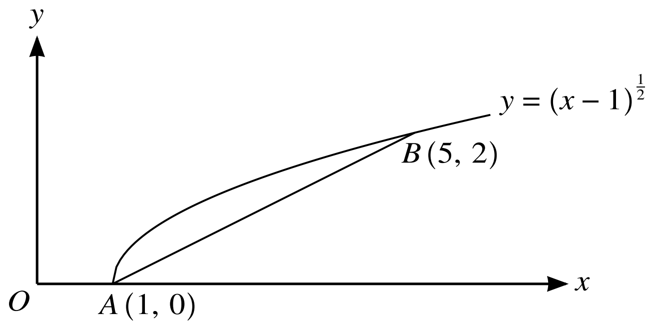
(ii) Given that D is a point lying on the line through B and C, find the two possible position vectors of the point D. [4]

10 A curve has equation $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = ax^2 + bx$, where a and b are positive constants.

(i) Find, in terms of a and b, the non-zero value of x for which the curve has a stationary point and determine, showing all necessary working, the nature of the stationary point. [3]

(ii) It is now given that the curve has a stationary point at $(-2, -3)$ and that the gradient of the curve at $x = 1$ is 9. Find $f(x)$. [6]

11



The diagram shows the curve $y = (x - 1)^{\frac{1}{2}}$ and points $A\,(1,\ 0)$ and $B\,(5,\ 2)$ lying on the curve.

(i) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [2]

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(ii) Find, showing all necessary working, the equation of the tangent to the curve which is parallel to AB. [5]

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(iii) Find the perpendicular distance between the line AB and the tangent parallel to AB. Give your answer correct to 2 decimal places. [3]

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CANDIDATE NAME

CENTRE NUMBER

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0906973490

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **20** printed pages.

JC18 11_9709_11/RP

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International Examinations

[Turn over

1 Showing all necessary working, solve the equation $4x - 11x^{\frac{1}{2}} + 6 = 0$. [3]

2 A line has equation $y = x + 1$ and a curve has equation $y = x^2 + bx + 5$. Find the set of values of the constant b for which the line meets the curve. [4]

3 Two points A and B have coordinates $(3a, -a)$ and $(-a, 2a)$ respectively, where a is a positive constant.

(i) Find the equation of the line through the origin parallel to AB. [2]

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(ii) The length of the line AB is $3\frac{1}{3}$ units. Find the value of a. [3]

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4 The first term of a series is 6 and the second term is 2.

(i) For the case where the series is an arithmetic progression, find the sum of the first 80 terms. [3]

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(ii) For the case where the series is a geometric progression, find the sum to infinity. [2]

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5 **(i)** Show that the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

may be expressed as $9\cos^2\theta - 22\cos\theta + 4 = 0$. [3]

(ii) Hence solve the equation

$$\frac{\cos\theta - 4}{\sin\theta} - \frac{4\sin\theta}{5\cos\theta - 2} = 0$$

for $0° \leqslant \theta \leqslant 360°$. [3]

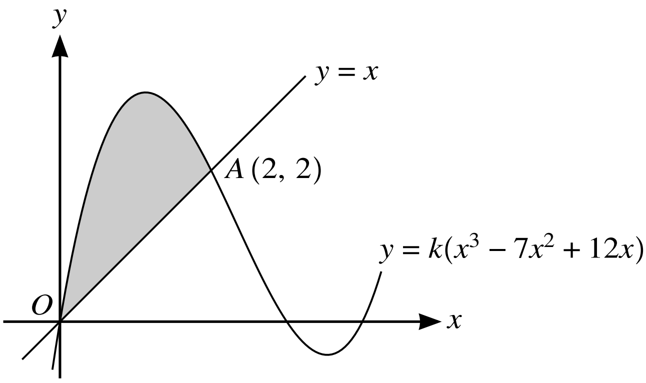
6 A curve has a stationary point at $(3, 9\frac{1}{2})$ and has an equation for which $\frac{dy}{dx} = ax^2 + a^2x$, where a is a non-zero constant.

(i) Find the value of a. [2]

(ii) Find the equation of the curve. [4]

(iii) Determine, showing all necessary working, the nature of the stationary point. [2]

7



The diagram shows part of the curve with equation $y = k(x^3 - 7x^2 + 12x)$ for some constant k. The curve intersects the line $y = x$ at the origin O and at the point $A\ (2,\ 2)$.

(i) Find the value of k. [1]

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(ii) Verify that the curve meets the line $y = x$ again when $x = 5$. [2]

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(iii) Find, showing all necessary working, the area of the shaded region. [5]

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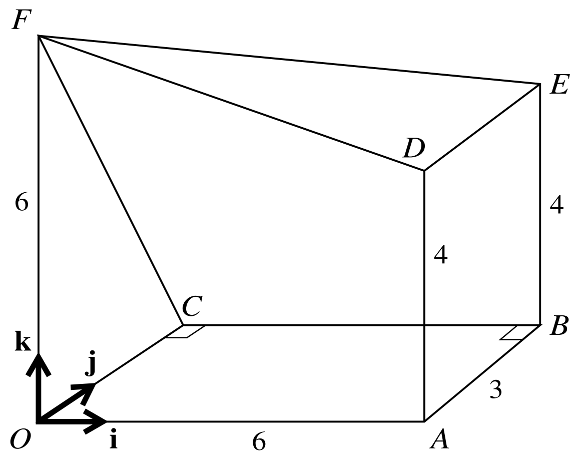

The diagram shows a solid figure *OABCDEF* having a horizontal rectangular base *OABC* with $OA = 6$ units and $AB = 3$ units. The vertical edges *OF*, *AD* and *BE* have lengths 6 units, 4 units and 4 units respectively. Unit vectors **i**, **j** and **k** are parallel to *OA*, *OC* and *OF* respectively.

(i) Find $\overrightarrow{DF}$. [1]

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(ii) Find the unit vector in the direction of $\overrightarrow{EF}$. [3]

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(iii) Use a scalar product to find angle EFD. [4]

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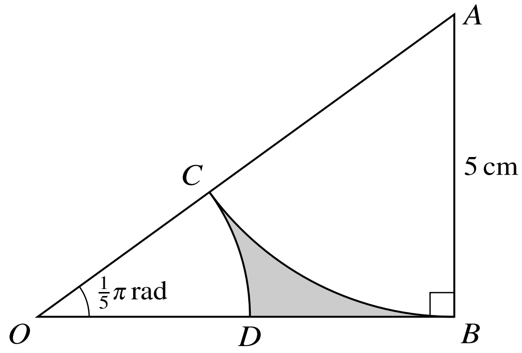

The diagram shows a triangle OAB in which angle ABO is a right angle, angle $AOB = \frac{1}{5}\pi$ radians and $AB = 5$ cm. The arc BC is part of a circle with centre A and meets OA at C. The arc CD is part of a circle with centre O and meets OB at D. Find the area of the shaded region. [8]

10 A curve has equation $y = \frac{1}{2}(4x - 3)^{-1}$. The point A on the curve has coordinates $\left(1, \frac{1}{2}\right)$.

(i) (a) Find and simplify the equation of the normal through A. [5]

(b) Find the x-coordinate of the point where this normal meets the curve again. [3]

(ii) A point is moving along the curve in such a way that as it passes through A its x-coordinate is decreasing at the rate of 0.3 units per second. Find the rate of change of its y-coordinate at A. [2]

11 **(a)** The one-one function f is defined by $f(x) = (x - 3)^2 - 1$ for $x < a$, where a is a constant.

(i) State the greatest possible value of a. [1]

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(ii) It is given that a takes this greatest possible value. State the range of f and find an expression for $f^{-1}(x)$. [3]

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(b) The function g is defined by $g(x) = (x-3)^2$ for $x \geqslant 0$.

(i) Show that $gg(2x)$ can be expressed in the form $(2x-3)^4 + b(2x-3)^2 + c$, where b and c are constants to be found. [2]

(ii) Hence expand $gg(2x)$ completely, simplifying your answer. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International Examinations
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CANDIDATE NAME	

CENTRE NUMBER						CANDIDATE NUMBER					

9945076812

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC18 11_9709_12/RP

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[Turn over

1 Find the coefficient of $\frac{1}{x^2}$ in the expansion of $\left(3x + \frac{2}{3x^2}\right)^7$. [4]

2 Showing all necessary working, find $\int_1^4 \left(\sqrt{x} + \frac{2}{\sqrt{x}}\right) \mathrm{d}x$. [4]

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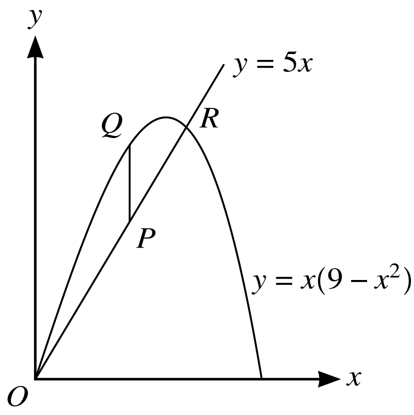

The diagram shows part of the curve $y = x(9 - x^2)$ and the line $y = 5x$, intersecting at the origin O and the point R. Point P lies on the line $y = 5x$ between O and R and the x-coordinate of P is t. Point Q lies on the curve and PQ is parallel to the y-axis.

(i) Express the length of PQ in terms of t, simplifying your answer. [2]

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(ii) Given that t can vary, find the maximum value of the length of PQ. [3]

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4 Functions f and g are defined by

$$f : x \mapsto 2 - 3\cos x \quad \text{for } 0 \leqslant x \leqslant 2\pi,$$

$$g : x \mapsto \tfrac{1}{2}x \quad \text{for } 0 \leqslant x \leqslant 2\pi.$$

(i) Solve the equation $fg(x) = 1$. [3]

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(ii) Sketch the graph of $y = f(x)$. [3]

5 The first three terms of an arithmetic progression are 4, x and y respectively. The first three terms of a geometric progression are x, y and 18 respectively. It is given that both x and y are positive.

(i) Find the value of x and the value of y. [4]

(ii) Find the fourth term of each progression. [3]

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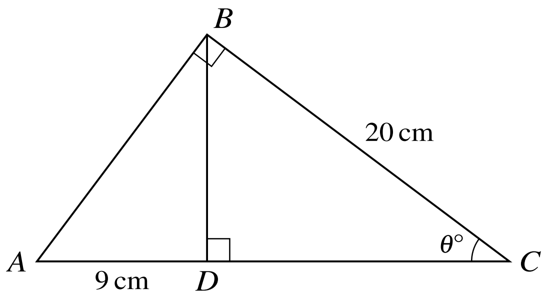

The diagram shows a triangle ABC in which $BC = 20$ cm and angle $ABC = 90°$. The perpendicular from B to AC meets AC at D and $AD = 9$ cm. Angle $BCA = \theta°$.

(i) By expressing the length of BD in terms of θ in each of the triangles ABD and DBC, show that $20\sin^2\theta = 9\cos\theta$. [4]

(ii) Hence, showing all necessary working, calculate θ. [3]

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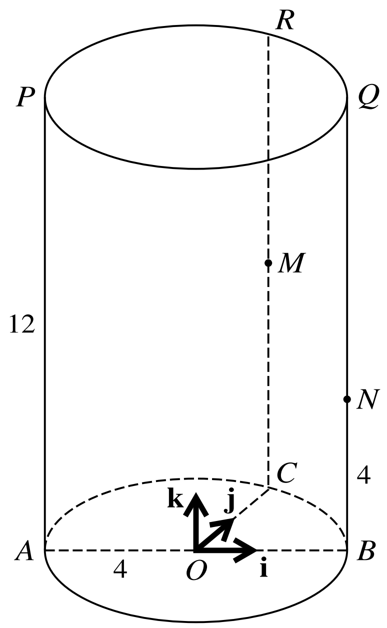

The diagram shows a solid cylinder standing on a horizontal circular base with centre O and radius 4 units. Points A, B and C lie on the circumference of the base such that AB is a diameter and angle $BOC = 90°$. Points P, Q and R lie on the upper surface of the cylinder vertically above A, B and C respectively. The height of the cylinder is 12 units. The mid-point of CR is M and N lies on BQ with $BN = 4$ units.

Unit vectors **i** and **j** are parallel to OB and OC respectively and the unit vector **k** is vertically upwards.

Evaluate $\overrightarrow{PN}.\overrightarrow{PM}$ and hence find angle MPN. [7]

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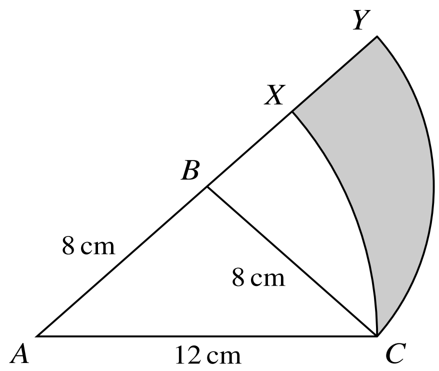

The diagram shows an isosceles triangle ACB in which $AB = BC = 8$ cm and $AC = 12$ cm. The arc XC is part of a circle with centre A and radius 12 cm, and the arc YC is part of a circle with centre B and radius 8 cm. The points A, B, X and Y lie on a straight line.

(i) Show that angle $CBY = 1.445$ radians, correct to 4 significant figures. [3]

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(ii) Find the perimeter of the shaded region. [4]

9 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $2x^2 - 12x + 7$ in the form $2(x + a)^2 + b$, where a and b are constants. [2]

(ii) State the range of f. [1]

The function g is defined by $g : x \mapsto 2x^2 - 12x + 7$ for $x \leqslant k$.

(iii) State the largest value of k for which g has an inverse. [1]

(iv) Given that g has an inverse, find an expression for $g^{-1}(x)$. [3]

10 The equation of a curve is $y = 2x + \dfrac{12}{x}$ and the equation of a line is $y + x = k$, where k is a constant.

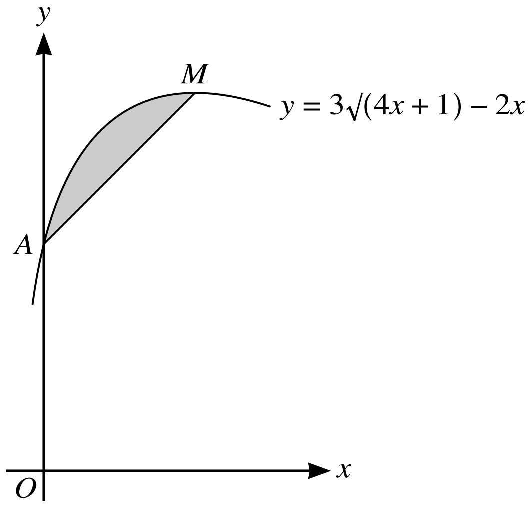
(i) Find the set of values of k for which the line does not meet the curve. [3]

In the case where $k = 15$, the curve intersects the line at points A and B.

(ii) Find the coordinates of A and B. [3]

(iii) Find the equation of the perpendicular bisector of the line joining A and B. [3]

11



The diagram shows part of the curve $y = 3\sqrt{(4x + 1)} - 2x$. The curve crosses the y-axis at A and the stationary point on the curve is M.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y\,dx$. [5]

(ii) Find the coordinates of M. [3]

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(iii) Find, showing all necessary working, the area of the shaded region. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

3414314086

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

CAMBRIDGE International Examinations

1 Find the coefficient of $\frac{1}{x^3}$ in the expansion of $\left(x - \frac{2}{x}\right)^7$. [3]

2 The function f is defined by $f(x) = x^3 + 2x^2 - 4x + 7$ for $x \geqslant -2$. Determine, showing all necessary working, whether f is an increasing function, a decreasing function or neither. [4]

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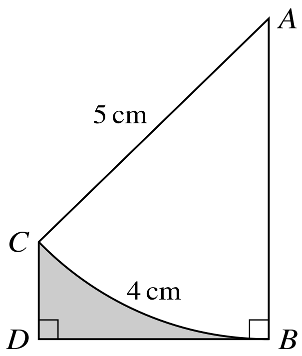

The diagram shows an arc BC of a circle with centre A and radius 5 cm. The length of the arc BC is 4 cm. The point D is such that the line BD is perpendicular to BA and DC is parallel to BA.

(i) Find angle BAC in radians. [1]

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(ii) Find the area of the shaded region BDC. [5]

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4 Two points A and B have coordinates $(-1, 1)$ and $(3, 4)$ respectively. The line BC is perpendicular to AB and intersects the x-axis at C.

(i) Find the equation of BC and the x-coordinate of C. [4]

(ii) Find the distance AC, giving your answer correct to 3 decimal places. [2]

5 In an arithmetic progression the first term is a and the common difference is 3. The nth term is 94 and the sum of the first n terms is 1420. Find n and a. [6]

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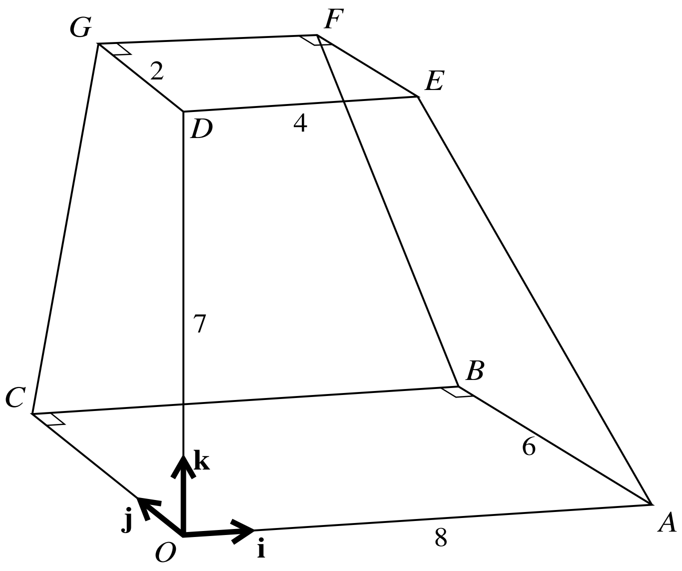

The diagram shows a solid figure *OABCDEFG* with a horizontal rectangular base *OABC* in which $OA = 8$ units and $AB = 6$ units. The rectangle *DEFG* lies in a horizontal plane and is such that *D* is 7 units vertically above *O* and *DE* is parallel to *OA*. The sides *DE* and *DG* have lengths 4 units and 2 units respectively. Unit vectors **i**, **j** and **k** are parallel to *OA*, *OC* and *OD* respectively. Use a scalar product to find angle *OBF*, giving your answer in the form $\cos^{-1}\left(\frac{a}{b}\right)$, where *a* and *b* are integers. [6]

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7 **(i)** Show that $\dfrac{\tan\theta + 1}{1 + \cos\theta} + \dfrac{\tan\theta - 1}{1 - \cos\theta} \equiv \dfrac{2(\tan\theta - \cos\theta)}{\sin^2\theta}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$\frac{\tan\theta + 1}{1 + \cos\theta} + \frac{\tan\theta - 1}{1 - \cos\theta} = 0$$

for $0° < \theta < 90°$. [4]

8 A curve passes through (0, 11) and has an equation for which $\frac{dy}{dx} = ax^2 + bx - 4$, where a and b are constants.

(i) Find the equation of the curve in terms of a and b. [3]

(ii) It is now given that the curve has a stationary point at $(2, 3)$. Find the values of a and b. [5]

9 A curve has equation $y = 2x^2 - 3x + 1$ and a line has equation $y = kx + k^2$, where k is a constant.

(i) Show that, for all values of k, the curve and the line meet. [4]

(ii) State the value of k for which the line is a tangent to the curve and find the coordinates of the point where the line touches the curve. [4]

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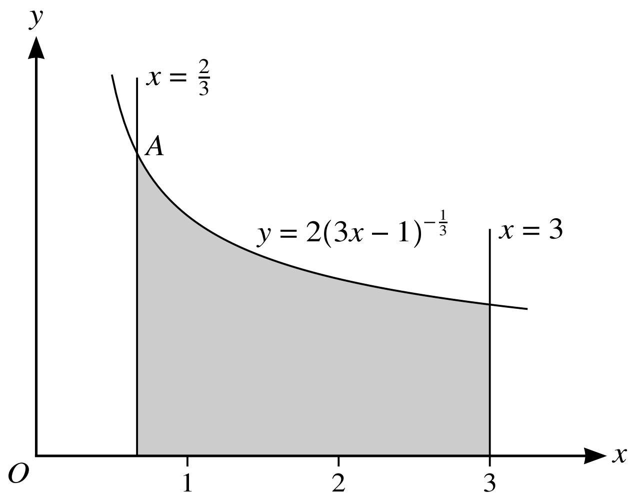

The diagram shows part of the curve $y = 2(3x - 1)^{-\frac{1}{3}}$ and the lines $x = \frac{2}{3}$ and $x = 3$. The curve and the line $x = \frac{2}{3}$ intersect at the point A.

(i) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [5]

(ii) Find the equation of the normal to the curve at A, giving your answer in the form $y = mx + c$. [5]

11 **(i)** Express $2x^2 - 12x + 11$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

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The function f is defined by $\mathrm{f}(x) = 2x^2 - 12x + 11$ for $x \leqslant k$.

(ii) State the largest value of the constant k for which f is a one-one function. [1]

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(iii) For this value of k find an expression for $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

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The function g is defined by $g(x) = x + 3$ for $x \leqslant p$.

(iv) With k now taking the value 1, find the largest value of the constant p which allows the composite function fg to be formed, and find an expression for $fg(x)$ whenever this composite function exists. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2590531708

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

Cambridge Assessment International Education

[Turn over

1 Find the term independent of x in the expansion of $\left(2x + \frac{1}{4x^2}\right)^6$. [3]

2 An increasing function, f, is defined for $x > n$, where n is an integer. It is given that $f'(x) = x^2 - 6x + 8$. Find the least possible value of n. [3]

3 The line $y = ax + b$ is a tangent to the curve $y = 2x^3 - 5x^2 - 3x + c$ at the point $(2, 6)$. Find the values of the constants a, b and c. [5]

4 A runner who is training for a long-distance race plans to run increasing distances each day for 21 days. She will run x km on day 1, and on each subsequent day she will increase the distance by 10% of the previous day's distance. On day 21 she will run 20 km.

(i) Find the distance she must run on day 1 in order to achieve this. Give your answer in km correct to 1 decimal place. [3]

(ii) Find the total distance she runs over the 21 days. [2]

5 **(i)** Given that $4\tan x + 3\cos x + \dfrac{1}{\cos x} = 0$, show, without using a calculator, that $\sin x = -\frac{2}{3}$. [3]

(ii) Hence, showing all necessary working, solve the equation

$$4\tan(2x - 20°) + 3\cos(2x - 20°) + \frac{1}{\cos(2x - 20°)} = 0$$

for $0° \leqslant x \leqslant 180°$. [4]

6 A straight line has gradient m and passes through the point $(0, -2)$. Find the two values of m for which the line is a tangent to the curve $y = x^2 - 2x + 7$ and, for each value of m, find the coordinates of the point where the line touches the curve. [7]

7 Functions f and g are defined by

$$\mathrm{f} : x \mapsto \frac{3}{2x+1} \quad \text{for } x > 0,$$

$$\mathrm{g} : x \mapsto \frac{1}{x} + 2 \quad \text{for } x > 0.$$

(i) Find the range of f and the range of g. [3]

(ii) Find an expression for $\mathrm{fg}(x)$, giving your answer in the form $\dfrac{ax}{bx+c}$, where a, b and c are integers. [2]

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(iii) Find an expression for $(\mathrm{fg})^{-1}(x)$, giving your answer in the same form as for part **(ii)**. [3]

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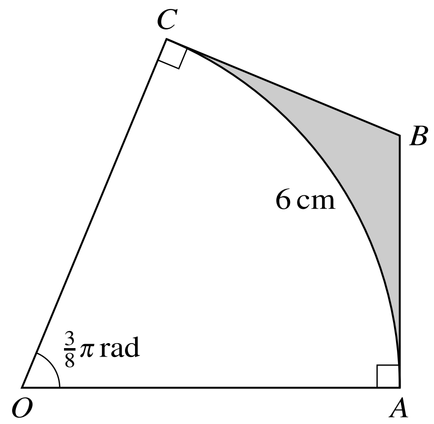

The diagram shows a sector OAC of a circle with centre O. Tangents AB and CB to the circle meet at B. The arc AC is of length 6 cm and angle $AOC = \frac{3}{8}\pi$ radians.

(i) Find the length of OA correct to 4 significant figures. [2]

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(ii) Find the perimeter of the shaded region. [2]

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(iii) Find the area of the shaded region. [4]

9 A curve for which $\frac{dy}{dx} = (5x - 1)^{\frac{1}{2}} - 2$ passes through the point (2, 3).

(i) Find the equation of the curve. [4]

(ii) Find $\frac{d^2y}{dx^2}$. [2]

(iii) Find the coordinates of the stationary point on the curve and, showing all necessary working, determine the nature of this stationary point. [4]

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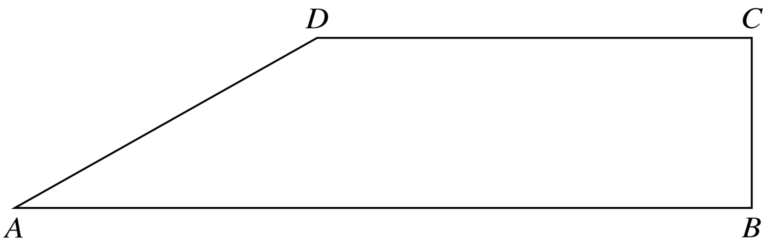

Relative to an origin O, the position vectors of the points A, B, C and D, shown in the diagram, are given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -2 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}.$$

(i) Show that AB is perpendicular to BC. [3]

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(ii) Show that $ABCD$ is a trapezium. [3]

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(iii) Find the area of $ABCD$, giving your answer correct to 2 decimal places. [3]

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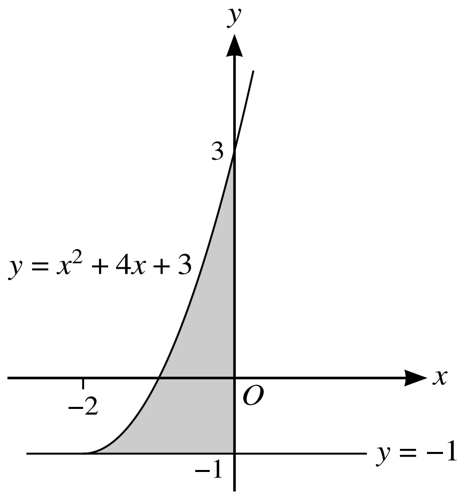

The diagram shows a shaded region bounded by the y-axis, the line $y = -1$ and the part of the curve $y = x^2 + 4x + 3$ for which $x \geqslant -2$.

(i) Express $y = x^2 + 4x + 3$ in the form $y = (x + a)^2 + b$, where a and b are constants. Hence, for $x \geqslant -2$, express x in terms of y. [4]

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(ii) Hence, showing all necessary working, find the volume obtained when the shaded region is rotated through 360° about the ***y*-axis**. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME	

CENTRE NUMBER							CANDIDATE NUMBER					

9810374883

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

Cambridge Assessment International Education

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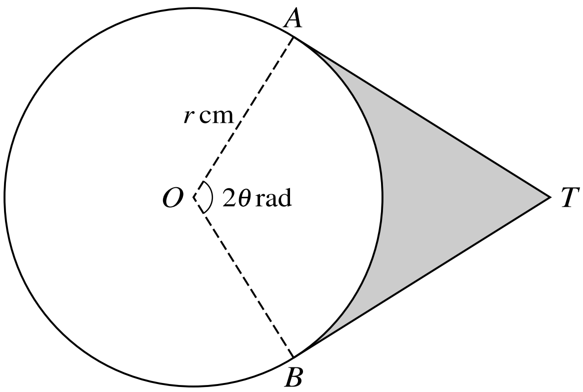
1 The coefficient of x^2 in the expansion of $(4+ax)\left(1+\frac{x}{2}\right)^6$ is 3. Find the value of the constant a. [4]

2 The point M is the mid-point of the line joining the points $(3, 7)$ and $(-1, 1)$. Find the equation of the line through M which is parallel to the line $\frac{x}{3} + \frac{y}{2} = 1$. [4]

3 A curve is such that $\frac{dy}{dx} = \frac{k}{\sqrt{x}}$, where k is a constant. The points $P(1, -1)$ and $Q(4, 4)$ lie on the curve. Find the equation of the curve. [4]

4



The diagram shows a circle with centre O and radius r cm. Points A and B lie on the circle and angle $AOB = 2\theta$ radians. The tangents to the circle at A and B meet at T.

(i) Express the perimeter of the shaded region in terms of r and θ. [3]

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(ii) In the case where $r = 5$ and $\theta = 1.2$, find the area of the shaded region. [4]

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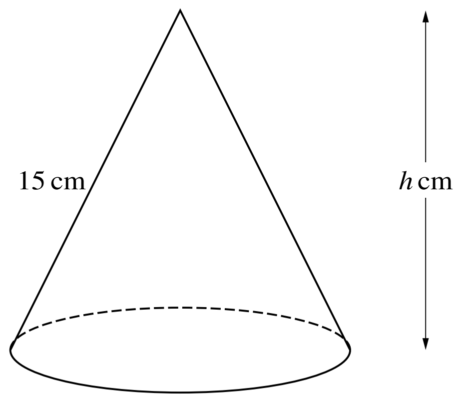

The diagram shows a solid cone which has a slant height of 15 cm and a vertical height of h cm.

(i) Show that the volume, V cm^3, of the cone is given by $V = \frac{1}{3}\pi(225h - h^3)$. [2]

[The volume of a cone of radius r and vertical height h is $\frac{1}{3}\pi r^2 h$.]

(ii) Given that h can vary, find the value of h for which V has a stationary value. Determine, showing all necessary working, the nature of this stationary value. [5]

6 **(a)** Given that $x > 0$, find the two smallest values of x, in radians, for which $3\tan(2x + 1) = 1$. Show all necessary working. [4]

(b) The function $f : x \mapsto 3\cos^2 x - 2\sin^2 x$ is defined for $0 \leqslant x \leqslant \pi$.

(i) Express $f(x)$ in the form $a\cos^2 x + b$, where a and b are constants. [1]

(ii) Find the range of f. [2]

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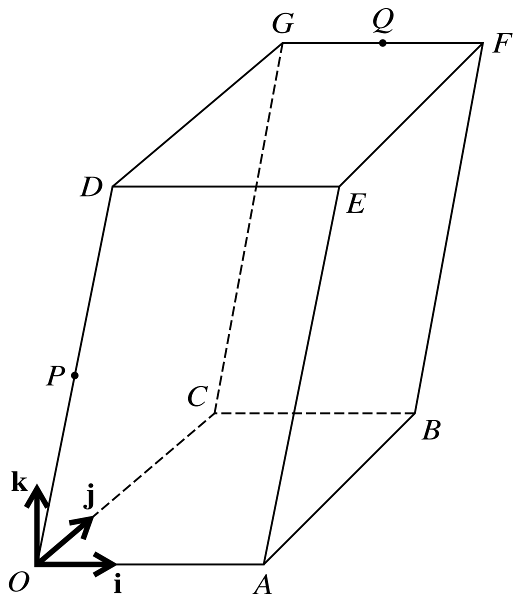

The diagram shows a three-dimensional shape $OABCDEFG$. The base $OABC$ and the upper surface $DEFG$ are identical horizontal rectangles. The parallelograms $OAED$ and $CBFG$ both lie in vertical planes. Points P and Q are the mid-points of OD and GF respectively. Unit vectors $\mathbf{i}$ and $\mathbf{j}$ are parallel to $\overrightarrow{OA}$ and $\overrightarrow{OC}$ respectively and the unit vector $\mathbf{k}$ is vertically upwards. The position vectors of A, C and D are given by $\overrightarrow{OA} = 6\mathbf{i}$, $\overrightarrow{OC} = 8\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 10\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{PB}$ and $\overrightarrow{PQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [4]

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(ii) Determine whether P is nearer to Q or to B. [2]

(iii) Use a scalar product to find angle BPQ. [3]

8 **(a)** Over a 21-day period an athlete prepares for a marathon by increasing the distance she runs each day by 1.2 km. On the first day she runs 13 km.

(i) Find the distance she runs on the last day of the 21-day period. [1]

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(ii) Find the total distance she runs in the 21-day period. [2]

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(b) The first, second and third terms of a geometric progression are x, $x - 3$ and $x - 5$ respectively.

(i) Find the value of x. [2]

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(ii) Find the fourth term of the progression. [2]

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(iii) Find the sum to infinity of the progression. [2]

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9 Functions f and g are defined by

$$f(x) = 2x^2 + 8x + 1 \quad \text{for } x \in \mathbb{R},$$
$$g(x) = 2x - k \quad \text{for } x \in \mathbb{R},$$

where k is a constant.

(i) Find the value of k for which the line $y = g(x)$ is a tangent to the curve $y = f(x)$. [3]

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(ii) In the case where $k = -9$, find the set of values of x for which $f(x) < g(x)$. [3]

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(iii) In the case where $k = -1$, find $g^{-1}f(x)$ and solve the equation $g^{-1}f(x) = 0$. [3]

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(iv) Express $f(x)$ in the form $2(x + a)^2 + b$, where a and b are constants, and hence state the least value of $f(x)$. [3]

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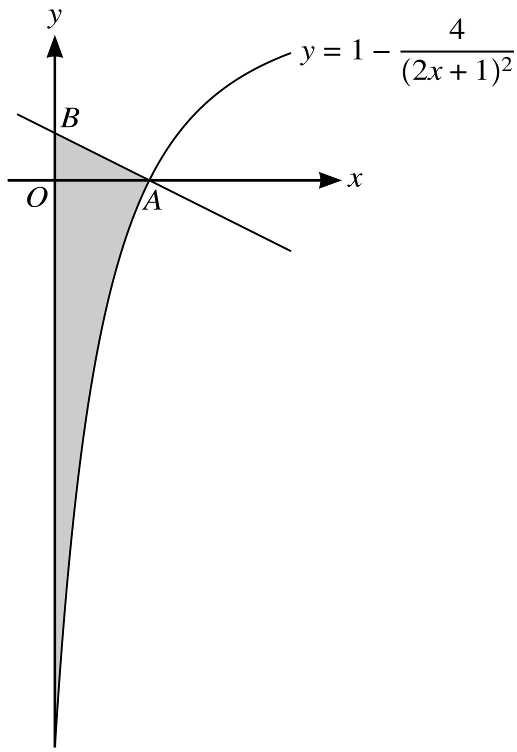

The diagram shows part of the curve $y = 1 - \frac{4}{(2x+1)^2}$. The curve intersects the x-axis at A. The normal to the curve at A intersects the y-axis at B.

(i) Obtain expressions for $\frac{dy}{dx}$ and $\int y\,dx$. [4]

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(ii) Find the coordinates of B. [4]

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(iii) Find, showing all necessary working, the area of the shaded region. [4]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME	

CENTRE NUMBER							CANDIDATE NUMBER					

3117579859

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

JC19 11_9709_13/RP

Cambridge Assessment International Education

[Turn over

1 **(i)** Expand $(1 + y)^6$ in ascending powers of y as far as the term in y^2. [1]

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(ii) In the expansion of $\left(1 + (px - 2x^2)\right)^6$ the coefficient of x^2 is 48. Find the value of the positive constant p. [3]

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2 The function g is defined by $g(x) = x^2 - 6x + 7$ for $x > 4$. By first completing the square, find an expression for $g^{-1}(x)$ and state the domain of g^{-1}. [5]

3 The equation of a curve is $y = x^3 + x^2 - 8x + 7$. The curve has no stationary points in the interval $a < x < b$. Find the least possible value of a and the greatest possible value of b. [4]

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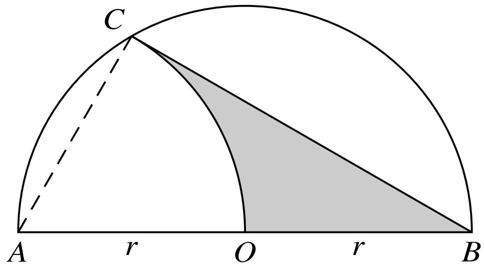

The diagram shows a semicircle ACB with centre O and radius r. Arc OC is part of a circle with centre A.

(i) Express angle CAO in radians in terms of π. [1]

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(ii) Find the area of the shaded region in terms of r, π and $\sqrt{3}$, simplifying your answer. [4]

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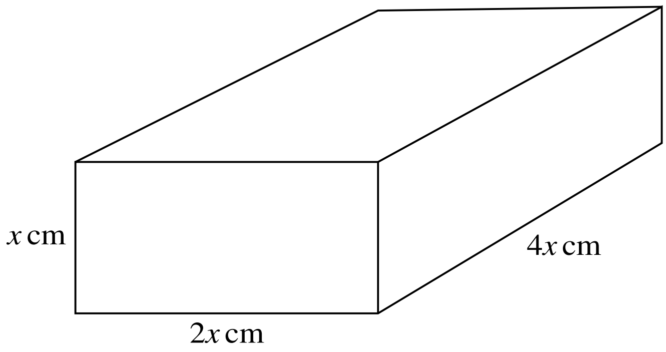

The dimensions of a cuboid are x cm, $2x$ cm and $4x$ cm, as shown in the diagram.

(i) Show that the surface area $S\text{ cm}^2$ and the volume $V\text{ cm}^3$ are connected by the relation

$$S = 7V^{\frac{2}{3}}.$$ [3]

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(ii) When the volume of the cuboid is $1000\,\text{cm}^3$ the surface area is increasing at $2\,\text{cm}^2\,\text{s}^{-1}$. Find the rate of increase of the volume at this instant. [4]

6 A line has equation $y = 3kx - 2k$ and a curve has equation $y = x^2 - kx + 2$, where k is a constant.

(i) Find the set of values of k for which the line and curve meet at two distinct points. [4]

(ii) For each of two particular values of k, the line is a tangent to the curve. Show that these two tangents meet on the x-axis. [3]

7 **(i)** Show that the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ can be expressed as $3x^2 - 4x + 1 = 0$, where $x = \cos^2\theta$. [2]

(ii) Hence solve the equation $3\cos^4\theta + 4\sin^2\theta - 3 = 0$ for $0° \leqslant \theta \leqslant 180°$. [5]

8 A function f is defined for $x > \frac{1}{2}$ and is such that $f'(x) = 3(2x-1)^{\frac{1}{2}} - 6$.

(i) Find the set of values of x for which f is decreasing. [4]

(ii) It is now given that $f(1) = -3$. Find $f(x)$. [4]

9 The first, second and third terms of a geometric progression are $3k$, $5k - 6$ and $6k - 4$, respectively.

(i) Show that k satisfies the equation $7k^2 - 48k + 36 = 0$. [2]

(ii) Find, showing all necessary working, the exact values of the common ratio corresponding to each of the possible values of k. [4]

(iii) One of these ratios gives a progression which is convergent. Find the sum to infinity. [2]

10 Relative to an origin O, the position vectors of the points A, B and X are given by

$$\overrightarrow{OA} = \begin{pmatrix} -8 \\ -4 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 10 \\ 2 \\ 11 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OX} = \begin{pmatrix} -2 \\ -2 \\ 5 \end{pmatrix}.$$

(i) Find $\overrightarrow{AX}$ and show that AXB is a straight line. [3]

The position vector of a point C is given by $\overrightarrow{OC} = \begin{pmatrix} 1 \\ -8 \\ 3 \end{pmatrix}$.

(ii) Show that CX is perpendicular to AX. [3]

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(iii) Find the area of triangle ABC. [3]

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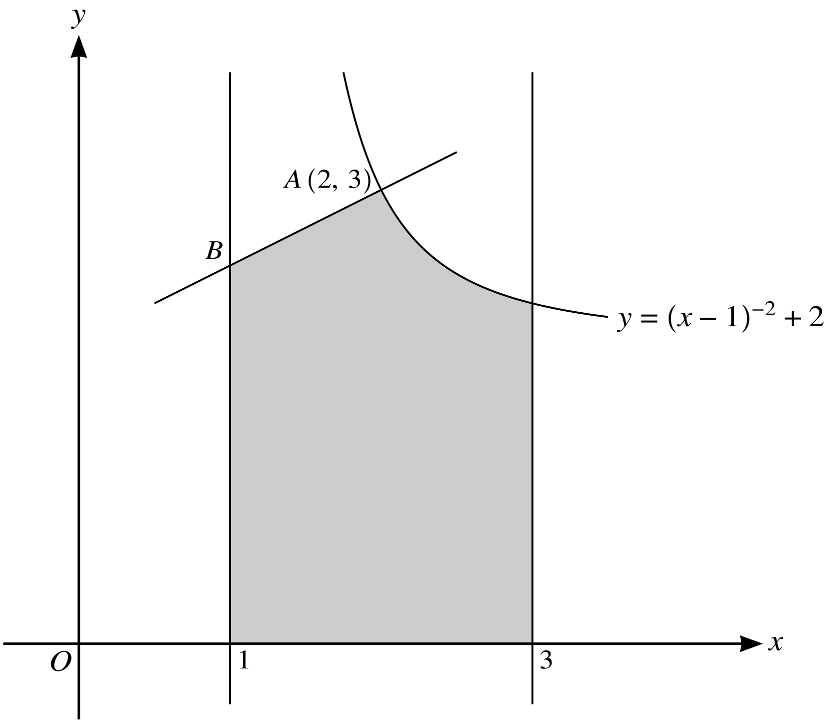

The diagram shows part of the curve $y = (x - 1)^{-2} + 2$, and the lines $x = 1$ and $x = 3$. The point A on the curve has coordinates $(2, 3)$. The normal to the curve at A crosses the line $x = 1$ at B.

(i) Show that the normal AB has equation $y = \frac{1}{2}x + 2$. [3]

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(ii) Find, showing all necessary working, the volume of revolution obtained when the shaded region is rotated through 360° about the x-axis. [8]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 2 0 2 8 6 7 5 9 3 7 *

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 11_9709_11/RP

1 Find the set of values of m for which the line with equation $y = mx - 3$ and the curve with equation $y = 2x^2 + 5$ do not meet. [3]

2 The equation of a curve is such that $\frac{dy}{dx} = \frac{1}{(x-3)^2} + x$. It is given that the curve passes through the point (2, 7).

Find the equation of the curve. [4]

3 Air is being pumped into a balloon in the shape of a sphere so that its volume is increasing at a constant rate of $50\,\text{cm}^3\,\text{s}^{-1}$.

Find the rate at which the radius of the balloon is increasing when the radius is 10 cm. [3]

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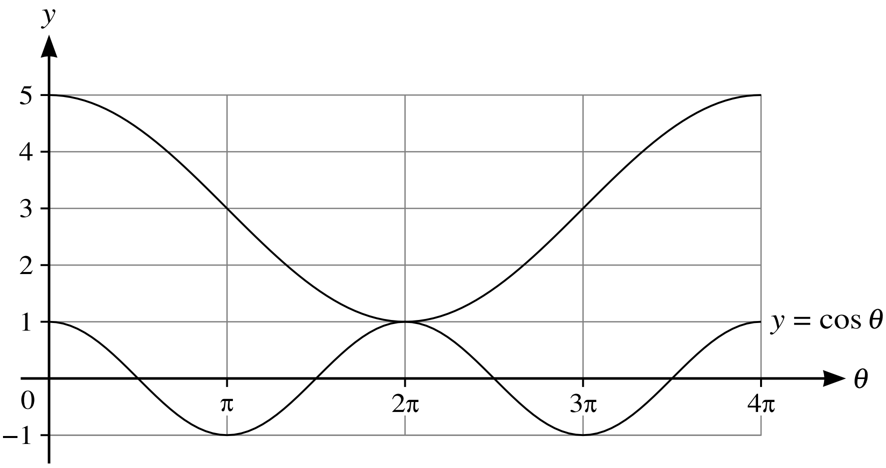

In the diagram, the lower curve has equation $y = \cos\theta$. The upper curve shows the result of applying a combination of transformations to $y = \cos\theta$.

Find, in terms of a cosine function, the equation of the upper curve. [3]

5 In the expansion of $\left(2x^2 + \frac{a}{x}\right)^6$, the coefficients of x^6 and x^3 are equal.

(a) Find the value of the non-zero constant a. [4]

(b) Find the coefficient of x^6 in the expansion of $(1 - x^3)\left(2x^2 + \frac{a}{x}\right)^6$. [1]

6 The equation of a curve is $y = 2 + \sqrt{25 - x^2}$.

Find the coordinates of the point on the curve at which the gradient is $\frac{4}{3}$. [5]

7 **(a)** Show that $\dfrac{\sin\theta}{1-\sin\theta} - \dfrac{\sin\theta}{1+\sin\theta} \equiv 2\tan^2\theta$. [3]

(b) Hence solve the equation $\frac{\sin\theta}{1-\sin\theta} - \frac{\sin\theta}{1+\sin\theta} = 8$, for $0° < \theta < 180°$. [3]

8 A geometric progression has first term a, common ratio r and sum to infinity S. A second geometric progression has first term a, common ratio R and sum to infinity $2S$.

(a) Show that $r = 2R - 1$. [3]

It is now given that the 3rd term of the first progression is equal to the 2nd term of the second progression.

(b) Express S in terms of a. [4]

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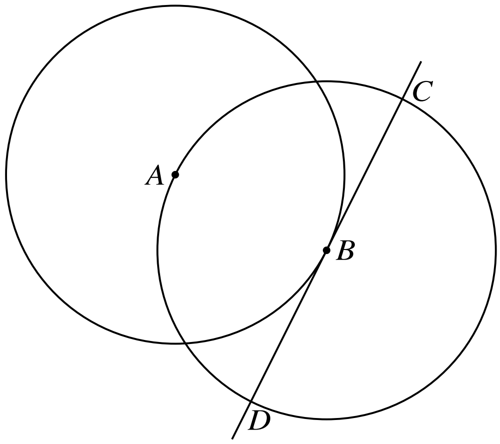

The diagram shows a circle with centre A passing through the point B. A second circle has centre B and passes through A. The tangent at B to the first circle intersects the second circle at C and D.

The coordinates of A are $(-1, 4)$ and the coordinates of B are $(3, 2)$.

(a) Find the equation of the tangent CBD. [2]

(b) Find an equation of the circle with centre B. [3]

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(c) Find, by calculation, the x-coordinates of C and D. [3]

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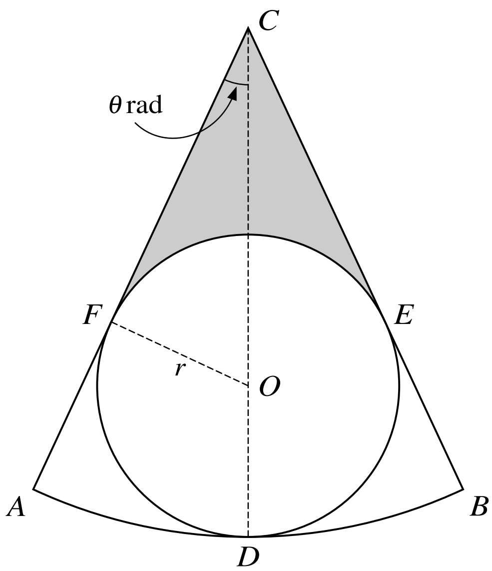

The diagram shows a sector CAB which is part of a circle with centre C. A circle with centre O and radius r lies within the sector and touches it at D, E and F, where COD is a straight line and angle ACD is θ radians.

(a) Find CD in terms of r and $\sin\theta$. [3]

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It is now given that $r = 4$ and $\theta = \frac{1}{6}\pi$.

(b) Find the perimeter of sector CAB in terms of π. [3]

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(c) Find the area of the shaded region in terms of π and $\sqrt{3}$. [4]

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11 The functions f and g are defined by

$$f(x) = x^2 + 3 \quad \text{for } x > 0,$$
$$g(x) = 2x + 1 \quad \text{for } x > -\tfrac{1}{2}.$$

(a) Find an expression for $fg(x)$. [1]

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(b) Find an expression for $(fg)^{-1}(x)$ and state the domain of $(fg)^{-1}$. [4]

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(c) Solve the equation $fg(x) - 3 = gf(x)$. [4]

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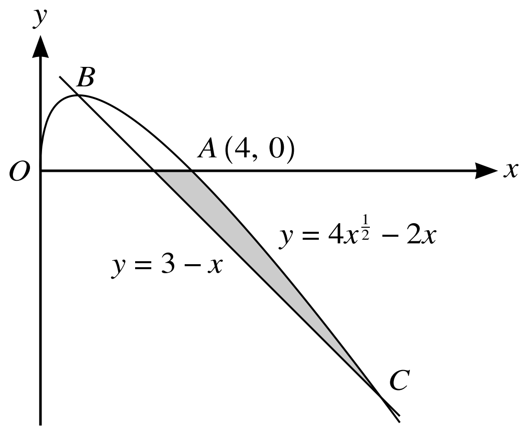

The diagram shows a curve with equation $y = 4x^{\frac{1}{2}} - 2x$ for $x \geqslant 0$, and a straight line with equation $y = 3 - x$. The curve crosses the x-axis at $A\,(4, 0)$ and crosses the straight line at B and C.

(a) Find, by calculation, the x-coordinates of B and C. [4]

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(b) Show that B is a stationary point on the curve. [2]

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(c) Find the area of the shaded region. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International AS & A Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 11_9709_12/2R

[Turn over

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1 The coefficient of x^3 in the expansion of $(1 + kx)(1 - 2x)^5$ is 20.

Find the value of the constant k. [4]

2 The first, second and third terms of a geometric progression are $2p + 6$, $-2p$ and $p + 2$ respectively, where p is positive.

Find the sum to infinity of the progression. [5]

3 The equation of a curve is $y = 2x^2 + m(2x + 1)$, where m is a constant, and the equation of a line is $y = 6x + 4$.

Show that, for all values of m, the line intersects the curve at two distinct points. [5]

4 The sum, S_n, of the first n terms of an arithmetic progression is given by

$$S_n = n^2 + 4n.$$

The kth term in the progression is greater than 200.

Find the smallest possible value of k. [5]

5 Functions f and g are defined by

$$f(x) = 4x - 2, \quad \text{for } x \in \mathbb{R},$$
$$g(x) = \frac{4}{x+1}, \quad \text{for } x \in \mathbb{R},\ x \neq -1.$$

(a) Find the value of fg(7). [1]

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(b) Find the values of x for which $f^{-1}(x) = g^{-1}(x)$. [5]

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6 **(a)** Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) \equiv \frac{1}{\tan x}$. [4]

(b) Hence solve the equation $\left(\frac{1}{\cos x} - \tan x\right)\left(\frac{1}{\sin x} + 1\right) = 2\tan^2 x$ for $0° \leqslant x \leqslant 180°$. [2]

7 The point (4, 7) lies on the curve $y = \mathrm{f}(x)$ and it is given that $\mathrm{f}'(x) = 6x^{-\frac{1}{2}} - 4x^{-\frac{3}{2}}$.

(a) A point moves along the curve in such a way that the x-coordinate is increasing at a constant rate of 0.12 units per second.

Find the rate of increase of the y-coordinate when $x = 4$. [3]

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(b) Find the equation of the curve. [4]

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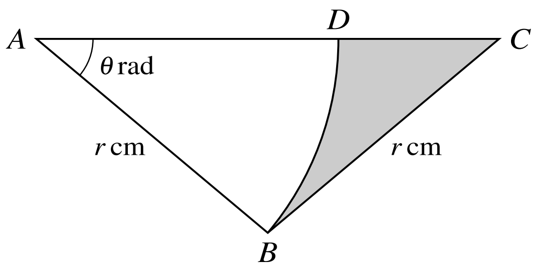

In the diagram, ABC is an isosceles triangle with $AB = BC = r$ cm and angle $BAC = \theta$ radians. The point D lies on AC and ABD is a sector of a circle with centre A.

(a) Express the area of the shaded region in terms of r and θ. [3]

(b) In the case where $r = 10$ and $\theta = 0.6$, find the perimeter of the shaded region. [4]

9 A circle has centre at the point $B(5, 1)$. The point $A(-1, -2)$ lies on the circle.

(a) Find the equation of the circle. [3]

Point C is such that AC is a diameter of the circle. Point D has coordinates $(5, 16)$.

(b) Show that DC is a tangent to the circle. [4]

The other tangent from D to the circle touches the circle at E.

(c) Find the coordinates of E. [2]

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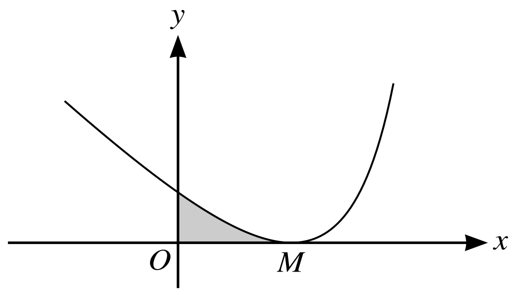

The diagram shows part of the curve $y = \dfrac{2}{(3-2x)^2} - x$ and its minimum point M, which lies on the x-axis.

(a) Find expressions for $\dfrac{dy}{dx}$, $\dfrac{d^2y}{dx^2}$ and $\int y\,dx$. [6]

(b) Find, by calculation, the x-coordinate of M. [2]

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(c) Find the area of the shaded region bounded by the curve and the coordinate axes. [2]

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11 A curve has equation $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$.

(a) State the greatest and least values of y. [2]

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(b) Sketch the graph of $y = 3\cos 2x + 2$ for $0 \leqslant x \leqslant \pi$. [2]

(c) By considering the straight line $y = kx$, where k is a constant, state the number of solutions of the equation $3\cos 2x + 2 = kx$ for $0 \leqslant x \leqslant \pi$ in each of the following cases.

(i) $k = -3$ [1]

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(ii) $k = 1$ [1]

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(iii) $k = 3$ [1]

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Functions f, g and h are defined for $x \in \mathbb{R}$ by

$$f(x) = 3\cos 2x + 2,$$
$$g(x) = f(2x) + 4,$$
$$h(x) = 2f\left(x + \tfrac{1}{2}\pi\right).$$

(d) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = g(x)$. [2]

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(e) Describe fully a sequence of transformations that maps the graph of $y = f(x)$ on to $y = h(x)$. [2]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME										
CENTRE NUMBER						CANDIDATE NUMBER				

* 8 6 7 7 4 6 7 3 9 8 *

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

JC20 11_9709_13/RP

1 **(a)** Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

2 The function f is defined by $f(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\displaystyle\int_1^{\infty} f(x)\,dx$. [3]

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(b) The equation of a curve is such that $\dfrac{dy}{dx} = f(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

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3 Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

4 A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection. [5]

5 In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

6 The function f is defined by $f(x) = \dfrac{2x}{3x - 1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

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(b) Show that $\dfrac{2}{3} + \dfrac{2}{3(3x - 1)}$ can be expressed as $\dfrac{2x}{3x - 1}$. [2]

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(c) State the range of f. [1]

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7 The first and second terms of an arithmetic progression are $\dfrac{1}{\cos^2\theta}$ and $-\dfrac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\dfrac{1}{\cos^4\theta}$. [4]

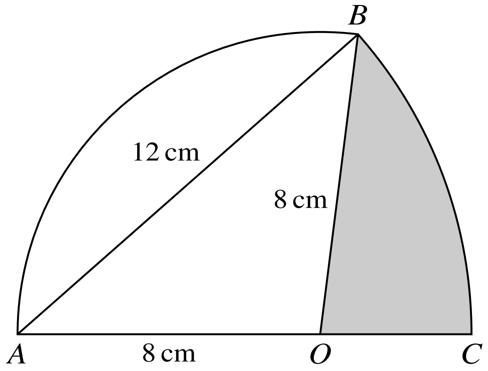
(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

8 The equation of a curve is $y = 2x + 1 + \frac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

9



In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

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(b) Find the area of the shaded region. [4]

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(c) Find the perimeter of the shaded region. [3]

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10 A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k} x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2} \right) \mathrm{d}x = \frac{13}{12}$.

Find the value of k. [5]

11 A circle with centre C has equation $(x-8)^2 + (y-4)^2 = 100$.

(a) Show that the point $T\,(-6, 6)$ is outside the circle. [3]

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Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

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The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

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(d) Find the x-coordinates of A and B. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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