UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

May/June 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Given that $x = 4(3^{-y})$, express y in terms of x. [3]

2 Solve the inequality $2x > |x - 1|$. [4]

3 The parametric equations of a curve are

$$x = 2\theta + \sin 2\theta, \quad y = 1 - \cos 2\theta.$$

Show that $\dfrac{dy}{dx} = \tan\theta$. [5]

4 **(i)** Express $7\cos\theta + 24\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$7\cos\theta + 24\sin\theta = 15,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [4]

5 In a certain industrial process, a substance is being produced in a container. The mass of the substance in the container t minutes after the start of the process is x grams. At any time, the rate of formation of the substance is proportional to its mass. Also, throughout the process, the substance is removed from the container at a constant rate of 25 grams per minute. When $t = 0$, $x = 1000$ and $\dfrac{dx}{dt} = 75$.

(i) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.1(x - 250).$$ [2]

(ii) Solve this differential equation, obtaining an expression for x in terms of t. [6]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$2\cot x = 1 + e^x,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 1.0. [2]

(iii) Show that this root also satisfies the equation

$$x = \tan^{-1}\left(\frac{2}{1 + e^x}\right).$$ [1]

(iv) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{2}{1 + e^{x_n}}\right),$$

with initial value $x_1 = 0.7$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The complex number $2 + \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

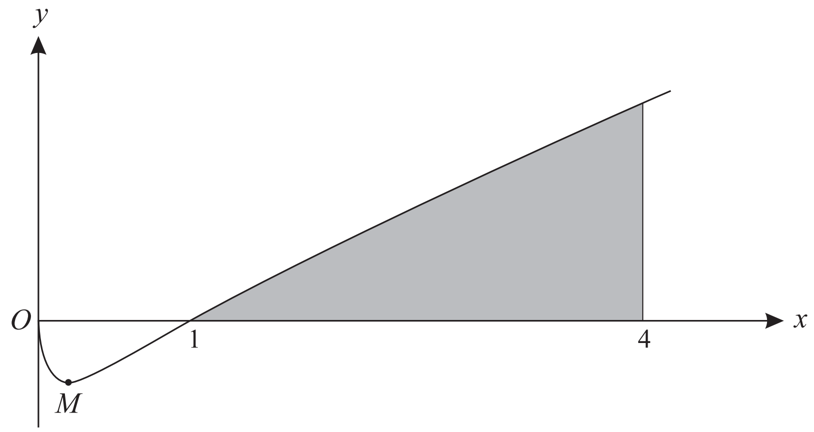
(i) Show, on a sketch of an Argand diagram with origin O, the points A, B and C representing the complex numbers u, u^* and $u + u^*$ respectively. Describe in geometrical terms the relationship between the four points O, A, B and C. [4]

(ii) Express $\dfrac{u}{u^*}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\dfrac{u}{u^*}$, or otherwise, prove that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = 2\tan^{-1}\left(\tfrac{1}{2}\right).$$ [2]

8



The diagram shows a sketch of the curve $y = x^{\frac{1}{2}} \ln x$ and its minimum point M. The curve cuts the x-axis at the point $(1, 0)$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Use integration by parts to find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 4$. Give your answer correct to 2 decimal places. [5]

9 **(i)** Express $\dfrac{10}{(2 - x)(1 + x^2)}$ in partial fractions. [5]

(ii) Hence, given that $|x| < 1$, obtain the expansion of $\dfrac{10}{(2 - x)(1 + x^2)}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [5]

10 The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ -1 \\ -4 \end{pmatrix}.$$

The line l passes through A and is parallel to OB. The point N is the foot of the perpendicular from B to l.

(i) State a vector equation for the line l. [1]

(ii) Find the position vector of N and show that $BN = 3$. [6]

(iii) Find the equation of the plane containing A, B and N, giving your answer in the form $ax + by + cz = d$. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2007**

1 hour 45 minutes

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List of Formulae (MF9)

6942525831

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1 Expand $(2 + 3x)^{-2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 The polynomial $x^3 - 2x + a$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

(ii) When a has this value, find the quadratic factor of $\mathrm{p}(x)$. [2]

3 The equation of a curve is $y = x \sin 2x$, where x is in radians. Find the equation of the tangent to the curve at the point where $x = \frac{1}{4}\pi$. [4]

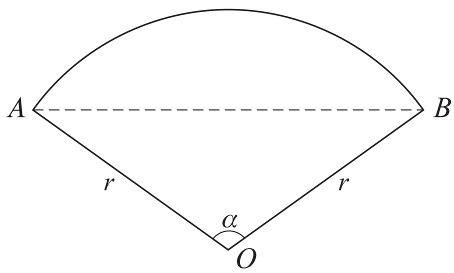
4 Using the substitution $u = 3^x$, or otherwise, solve, correct to 3 significant figures, the equation

$$3^x = 2 + 3^{-x}.$$ [6]

5 **(i)** Express $\cos\theta + (\sqrt{3})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{1}{\left(\cos\theta + (\sqrt{3})\sin\theta\right)^2}\,d\theta = \frac{1}{\sqrt{3}}$. [4]

6



The diagram shows a sector AOB of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \pi$. The area of triangle AOB is half the area of the sector.

(i) Show that α satisfies the equation

$$x = 2\sin x.$$ [2]

(ii) Verify by calculation that α lies between $\frac{1}{2}\pi$ and $\frac{2}{3}\pi$. [2]

(iii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \tfrac{1}{3}\left(x_n + 4\sin x_n\right)$$

converges, then it converges to a root of the equation in part **(i)**. [2]

(iv) Use this iterative formula, with initial value $x_1 = 1.8$, to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 Let $I = \int_1^4 \frac{1}{x(4-\sqrt{x})}\,dx$.

(i) Use the substitution $u = \sqrt{x}$ to show that $I = \int_1^2 \frac{2}{u(4-u)}\,du$. [3]

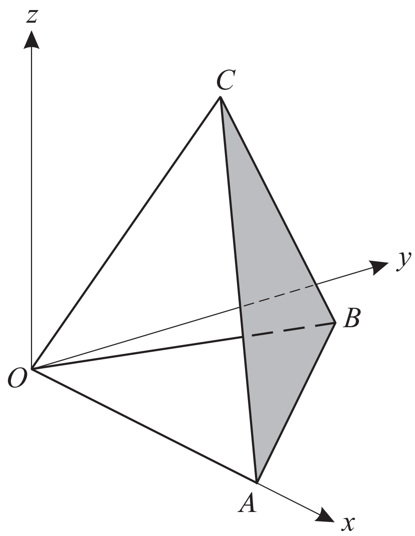
(ii) Hence show that $I = \frac{1}{2}\ln 3$. [6]

8 The complex number $\frac{2}{-1+\mathrm{i}}$ is denoted by u.

(i) Find the modulus and argument of u and u^2. [6]

(ii) Sketch an Argand diagram showing the points representing the complex numbers u and u^2. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z - u^2| < |z - u|$. [4]

9



The diagram shows a set of rectangular axes Ox, Oy and Oz, and three points A, B and C with position vectors $\overrightarrow{OA} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.

(i) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Calculate the acute angle between the planes ABC and OAB. [4]

10 A model for the height, h metres, of a certain type of tree at time t years after being planted assumes that, while the tree is growing, the rate of increase in height is proportional to $(9 - h)^{\frac{1}{3}}$. It is given that, when $t = 0$, $h = 1$ and $\frac{dh}{dt} = 0.2$.

(i) Show that h and t satisfy the differential equation

$$\frac{dh}{dt} = 0.1(9 - h)^{\frac{1}{3}}.$$ [2]

(ii) Solve this differential equation, and obtain an expression for h in terms of t. [7]

(iii) Find the maximum height of the tree and the time taken to reach this height after planting. [2]

(iv) Calculate the time taken to reach half the maximum height. [1]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2008**

1 hour 45 minutes

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Graph Paper
List of Formulae (MF9)

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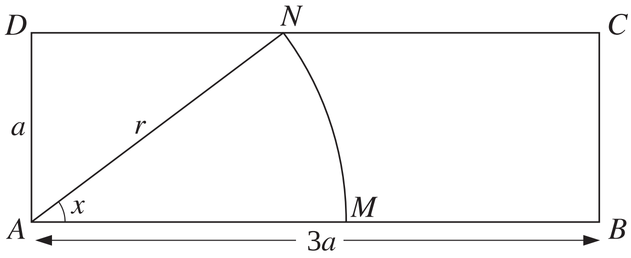
[Turn over

1 Solve the inequality $|x - 2| > 3|2x + 1|$. [4]

2 Solve, correct to 3 significant figures, the equation

$$e^x + e^{2x} = e^{3x}.$$ [5]

3



In the diagram, $ABCD$ is a rectangle with $AB = 3a$ and $AD = a$. A circular arc, with centre A and radius r, joins points M and N on AB and CD respectively. The angle MAN is x radians. The perimeter of the sector AMN is equal to half the perimeter of the rectangle.

(i) Show that x satisfies the equation

$$\sin x = \tfrac{1}{4}(2 + x).$$ [3]

(ii) This equation has only one root in the interval $0 < x < \frac{1}{2}\pi$. Use the iterative formula

$$x_{n+1} = \sin^{-1}\left(\frac{2 + x_n}{4}\right),$$

with initial value $x_1 = 0.8$, to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 **(i)** Show that the equation $\tan(30° + \theta) = 2\tan(60° - \theta)$ can be written in the form

$$\tan^2\theta + (6\sqrt{3})\tan\theta - 5 = 0.$$ [4]

(ii) Hence, or otherwise, solve the equation

$$\tan(30° + \theta) = 2\tan(60° - \theta),$$

for $0° \leqslant \theta \leqslant 180°$. [3]

5 The variable complex number z is given by

$$z = 2\cos\theta + \mathrm{i}(1 - 2\sin\theta),$$

where θ takes all values in the interval $-\pi < \theta \leqslant \pi$.

(i) Show that $|z - \mathrm{i}| = 2$, for all values of θ. Hence sketch, in an Argand diagram, the locus of the point representing z. [3]

(ii) Prove that the real part of $\dfrac{1}{z + 2 - \mathrm{i}}$ is constant for $-\pi < \theta < \pi$. [4]

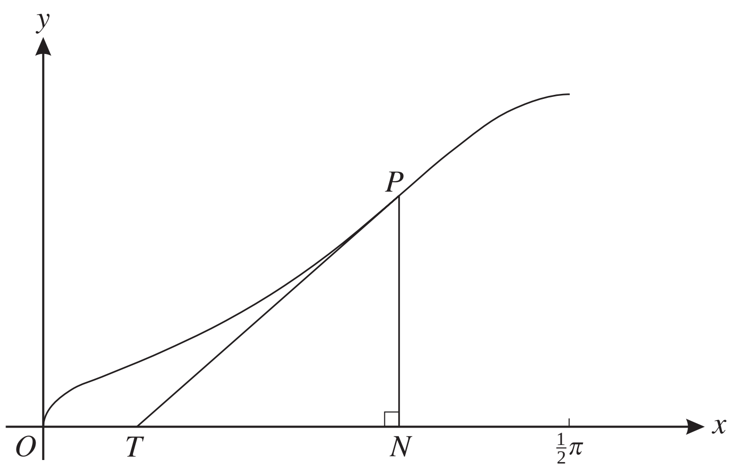
6 The equation of a curve is $xy(x + y) = 2a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [8]

7 Let $\mathrm{f}(x) \equiv \dfrac{x^2 + 3x + 3}{(x + 1)(x + 3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^3 \mathrm{f}(x)\,\mathrm{d}x = 3 - \tfrac{1}{2}\ln 2$. [4]

8



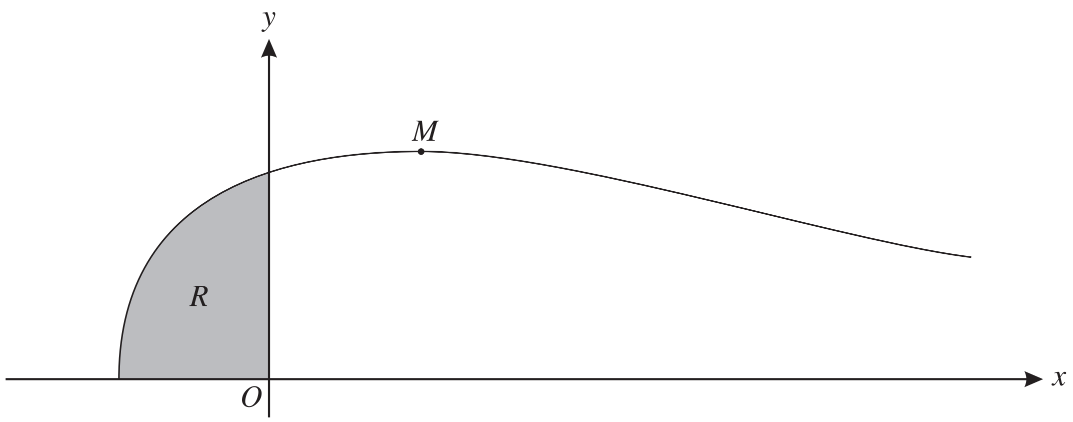
In the diagram the tangent to a curve at a general point P with coordinates (x, y) meets the x-axis at T. The point N on the x-axis is such that PN is perpendicular to the x-axis. The curve is such that, for all values of x in the interval $0 < x < \frac{1}{2}\pi$, the area of triangle PTN is equal to $\tan x$, where x is in radians.

(i) Using the fact that the gradient of the curve at P is $\dfrac{PN}{TN}$, show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{2}y^2 \cot x.$$ [3]

(ii) Given that $y = 2$ when $x = \frac{1}{6}\pi$, solve this differential equation to find the equation of the curve, expressing y in terms of x. [6]

9



The diagram shows the curve $y = e^{-\frac{1}{2}x}\sqrt{(1 + 2x)}$ and its maximum point M. The shaded region between the curve and the axes is denoted by R.

(i) Find the x-coordinate of M. [4]

(ii) Find by integration the volume of the solid obtained when R is rotated completely about the x-axis. Give your answer in terms of π and e. [6]

10 The points A and B have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} \qquad \text{and} \qquad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} + 3\mathbf{k}.$$

The line l has vector equation

$$\mathbf{r} = (1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}.$$

(i) Show that l does not intersect the line passing through A and B. [4]

(ii) The point P lies on l and is such that angle PAB is equal to $60°$. Given that the position vector of P is $(1 - 2t)\mathbf{i} + (5 + t)\mathbf{j} + (2 - t)\mathbf{k}$, show that $3t^2 + 7t + 2 = 0$. Hence find the only possible position vector of P. [6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
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MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2009**

1 hour 45 minutes

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4908186713

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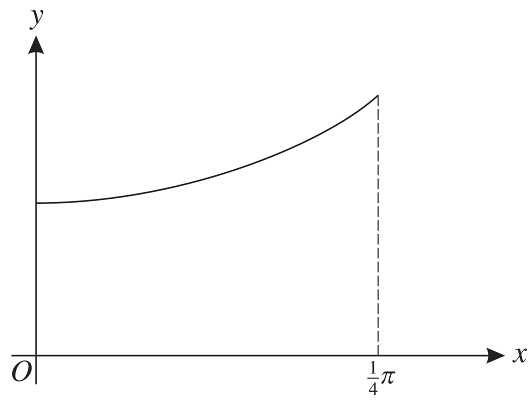
1 Solve the equation $\ln(2 + e^{-x}) = 2$, giving your answer correct to 2 decimal places. [4]

2



The diagram shows the curve $y = \sqrt{(1 + 2\tan^2 x)}$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$.

(i) Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{4}\pi} \sqrt{(1 + 2\tan^2 x)}\, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) The estimate found in part **(i)** is denoted by E. Explain, without further calculation, whether another estimate found using the trapezium rule with six intervals would be greater than E or less than E. [1]

3 **(i)** Prove the identity $\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot \theta$. [3]

(ii) Hence solve the equation $\operatorname{cosec} 2\theta + \cot 2\theta = 2$, for $0° \leqslant \theta \leqslant 360°$. [2]

4 The equation $x^3 - 2x - 2 = 0$ has one real root.

(i) Show by calculation that this root lies between $x = 1$ and $x = 2$. [2]

(ii) Prove that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 2}{3x_n^2 - 2}$$

converges, then it converges to this root. [2]

(iii) Use this iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 When $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, where a is a constant, is expanded in ascending powers of x, the coefficient of the term in x is zero.

(i) Find the value of a. [3]

(ii) When a has this value, find the term in x^3 in the expansion of $(1 + 2x)(1 + ax)^{\frac{2}{3}}$, simplifying the coefficient. [4]

6 The parametric equations of a curve are

$$x = a\cos^3 t, \quad y = a\sin^3 t,$$

where a is a positive constant and $0 < t < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x\sin t + y\cos t = a\sin t\cos t. \quad [3]$$

(iii) Hence show that, if this tangent meets the x-axis at X and the y-axis at Y, then the length of XY is always equal to a. [2]

7 **(i)** Solve the equation $z^2 + (2\sqrt{3})iz - 4 = 0$, giving your answers in the form $x + iy$, where x and y are real. [3]

(ii) Sketch an Argand diagram showing the points representing the roots. [1]

(iii) Find the modulus and argument of each root. [3]

(iv) Show that the origin and the points representing the roots are the vertices of an equilateral triangle. [1]

8 **(i)** Express $\frac{100}{x^2(10 - x)}$ in partial fractions. [4]

(ii) Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \frac{1}{100}x^2(10 - x),$$

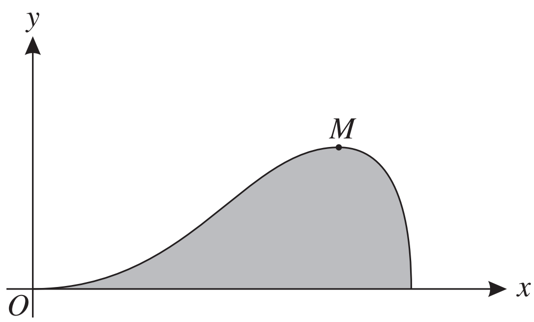
obtaining an expression for t in terms of x. [6]

9 The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k} + t(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$. It is given that l lies in the plane with equation $2x + by + cz = 1$, where b and c are constants.

(i) Find the values of b and c. [6]

(ii) The point P has position vector $2\mathbf{j} + 4\mathbf{k}$. Show that the perpendicular distance from P to l is $\sqrt{5}$. [5]

10



The diagram shows the curve $y = x^2\sqrt{(1 - x^2)}$ for $x \geqslant 0$ and its maximum point M.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Show, by means of the substitution $x = \sin\theta$, that the area A of the shaded region between the curve and the x-axis is given by

$$A = \frac{1}{4}\int_0^{\frac{1}{2}\pi} \sin^2 2\theta \, d\theta.$$ [3]

(iii) Hence obtain the exact value of A. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

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1 Solve the inequality $|x + 3a| > 2|x - 2a|$, where a is a positive constant. [4]

2 Solve the equation

$$\sin\theta = 2\cos 2\theta + 1,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 360°$. [6]

3 The variables x and y satisfy the equation $x^n y = C$, where n and C are constants. When $x = 1.10$, $y = 5.20$, and when $x = 3.20$, $y = 1.05$.

(i) Find the values of n and C. [5]

(ii) Explain why the graph of $\ln y$ against $\ln x$ is a straight line. [1]

4 **(i)** Using the expansions of $\cos(3x - x)$ and $\cos(3x + x)$, prove that

$$\tfrac{1}{2}(\cos 2x - \cos 4x) \equiv \sin 3x \sin x.$$ [3]

(ii) Hence show that

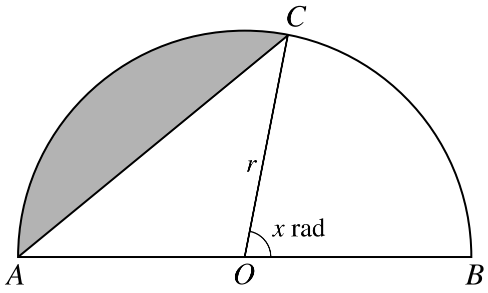
$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin 3x \sin x \,\mathrm{d}x = \tfrac{1}{8}\sqrt{3}.$$ [3]

5 Given that $y = 0$ when $x = 1$, solve the differential equation

$$xy\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 + 4,$$

obtaining an expression for y^2 in terms of x. [6]

6



The diagram shows a semicircle ACB with centre O and radius r. The angle BOC is x radians. The area of the shaded segment is a quarter of the area of the semicircle.

(i) Show that x satisfies the equation

$$x = \tfrac{3}{4}\pi - \sin x.$$ [3]

(ii) This equation has one root. Verify by calculation that the root lies between 1.3 and 1.5. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{3}{4}\pi - \sin x_n$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The complex number $2 + 2\mathrm{i}$ is denoted by u.

(i) Find the modulus and argument of u. [2]

(ii) Sketch an Argand diagram showing the points representing the complex numbers 1, i and u. Shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z - 1| \leqslant |z - \mathrm{i}|$ and $|z - u| \leqslant 1$. [4]

(iii) Using your diagram, calculate the value of $|z|$ for the point in this region for which $\arg z$ is least. [3]

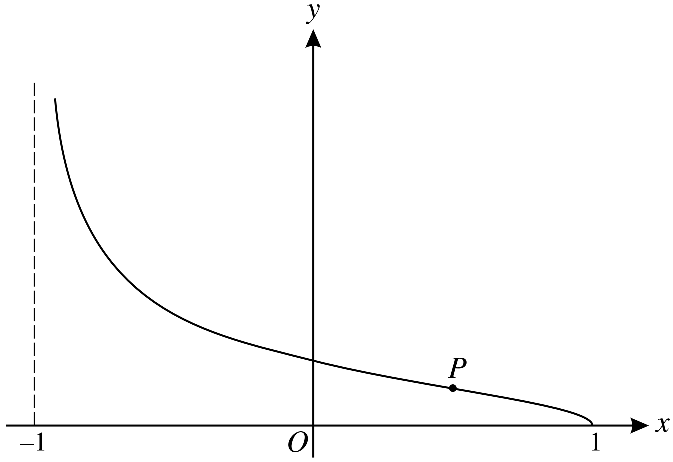
8 **(i)** Express $\dfrac{2}{(x+1)(x+3)}$ in partial fractions. [2]

(ii) Using your answer to part **(i)**, show that

$$\left(\frac{2}{(x+1)(x+3)}\right)^2 \equiv \frac{1}{(x+1)^2} - \frac{1}{x+1} + \frac{1}{x+3} + \frac{1}{(x+3)^2}.$$ [2]

(iii) Hence show that $\displaystyle\int_0^1 \frac{4}{(x+1)^2(x+3)^2}\,\mathrm{d}x = \tfrac{7}{12} - \ln\tfrac{3}{2}$. [5]

9



The diagram shows the curve $y = \sqrt{\left(\frac{1-x}{1+x}\right)}$.

(i) By first differentiating $\frac{1-x}{1+x}$, obtain an expression for $\frac{dy}{dx}$ in terms of x. Hence show that the gradient of the normal to the curve at the point (x, y) is $(1+x)\sqrt{(1-x^2)}$. [5]

(ii) The gradient of the normal to the curve has its maximum value at the point P shown in the diagram. Find, by differentiation, the x-coordinate of P. [4]

10 The lines l and m have vector equations

$$\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + s(\mathbf{i} - \mathbf{j} + 2\mathbf{k}) \quad \text{and} \quad \mathbf{r} = 4\mathbf{i} + 6\mathbf{j} + \mathbf{k} + t(2\mathbf{i} + 2\mathbf{j} + \mathbf{k})$$

respectively.

(i) Show that l and m intersect. [4]

(ii) Calculate the acute angle between the lines. [3]

(iii) Find the equation of the plane containing l and m, giving your answer in the form $ax + by + cz = d$. [5]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2010**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9385525965

READ THESE INSTRUCTIONS FIRST

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UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Solve the equation

$$\frac{2^x + 1}{2^x - 1} = 5,$$

giving your answer correct to 3 significant figures. [4]

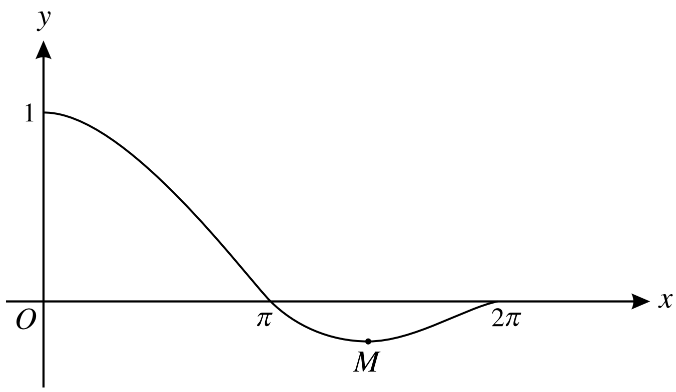
2 Show that $\int_0^{\pi} x^2 \sin x \, dx = \pi^2 - 4$. [5]

3 It is given that $\cos a = \frac{3}{5}$, where $0° < a < 90°$. Showing your working and without using a calculator to evaluate a,

(i) find the exact value of $\sin(a - 30°)$, [3]

(ii) find the exact value of $\tan 2a$, and hence find the exact value of $\tan 3a$. [4]

4



The diagram shows the curve $y = \dfrac{\sin x}{x}$ for $0 < x \leqslant 2\pi$, and its minimum point M.

(i) Show that the x-coordinate of M satisfies the equation

$$x = \tan x.$$ [4]

(ii) The iterative formula

$$x_{n+1} = \tan^{-1}(x_n) + \pi$$

can be used to determine the x-coordinate of M. Use this formula to determine the x-coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 The polynomial $2x^3 + 5x^2 + ax + b$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is 9.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

6 The equation of a curve is

$$x \ln y = 2x + 1.$$

(i) Show that $\dfrac{dy}{dx} = -\dfrac{y}{x^2}$. [4]

(ii) Find the equation of the tangent to the curve at the point where $y = 1$, giving your answer in the form $ax + by + c = 0$. [4]

7 The variables x and t are related by the differential equation

$$e^{2t}\frac{dx}{dt} = \cos^2 x,$$

where $t \geqslant 0$. When $t = 0$, $x = 0$.

(i) Solve the differential equation, obtaining an expression for x in terms of t. [6]

(ii) State what happens to the value of x when t becomes very large. [1]

(iii) Explain why x increases as t increases. [1]

8 The variable complex number z is given by

$$z = 1 + \cos 2\theta + i \sin 2\theta,$$

where θ takes all values in the interval $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(i) Show that the modulus of z is $2\cos\theta$ and the argument of z is θ. [6]

(ii) Prove that the real part of $\dfrac{1}{z}$ is constant. [3]

9 The plane p has equation $3x + 2y + 4z = 13$. A second plane q is perpendicular to p and has equation $ax + y + z = 4$, where a is a constant.

(i) Find the value of a. [3]

(ii) The line with equation $\mathbf{r} = \mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ meets the plane p at the point A and the plane q at the point B. Find the length of AB. [6]

10 **(i)** Find the values of the constants A, B, C and D such that

$$\frac{2x^3 - 1}{x^2(2x - 1)} \equiv A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{2x - 1}.$$ [5]

(ii) Hence show that

$$\int_1^2 \frac{2x^3 - 1}{x^2(2x - 1)}\,dx = \tfrac{3}{2} + \tfrac{1}{2}\ln\left(\tfrac{16}{27}\right).$$ [5]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2010**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1169365262

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UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 Solve the inequality $|x - 3| > 2|x + 1|$. [4]

2 The variables x and y satisfy the equation $y^3 = Ae^{2x}$, where A is a constant. The graph of $\ln y$ against x is a straight line.

(i) Find the gradient of this line. [2]

(ii) Given that the line intersects the axis of $\ln y$ at the point where $\ln y = 0.5$, find the value of A correct to 2 decimal places. [2]

3 Solve the equation

$$\tan(45^\circ - x) = 2\tan x,$$

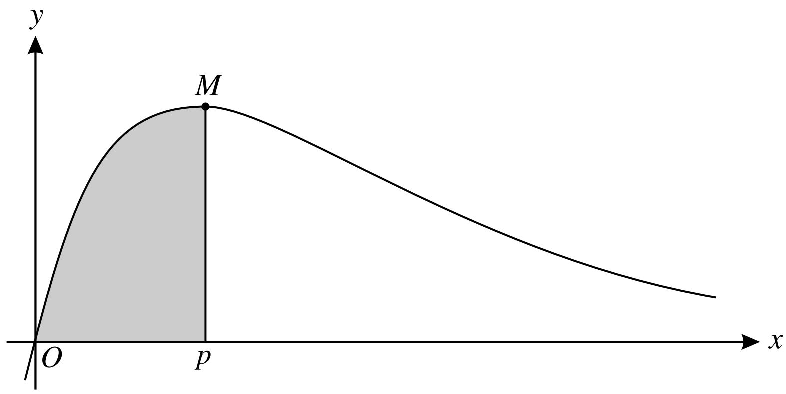
giving all solutions in the interval $0^\circ < x < 180^\circ$. [5]

4 Given that $x = 1$ when $t = 0$, solve the differential equation

$$\frac{dx}{dt} = \frac{1}{x} - \frac{x}{4},$$

obtaining an expression for x^2 in terms of t. [7]

5



The diagram shows the curve $y = e^{-x} - e^{-2x}$ and its maximum point M. The x-coordinate of M is denoted by p.

(i) Find the exact value of p. [4]

(ii) Show that the area of the shaded region bounded by the curve, the x-axis and the line $x = p$ is equal to $\frac{1}{8}$. [4]

6 The curve $y = \dfrac{\ln x}{x+1}$ has one stationary point.

(i) Show that the x-coordinate of this point satisfies the equation

$$x = \frac{x+1}{\ln x},$$

and that this x-coordinate lies between 3 and 4. [5]

(ii) Use the iterative formula

$$x_{n+1} = \frac{x_n + 1}{\ln x_n}$$

to determine the x-coordinate correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(i)** Prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [4]

(ii) Using this result, find the exact value of

$$\int_{\frac{1}{3}\pi}^{\frac{1}{2}\pi} \cos^3\theta \, d\theta.$$ [4]

8 **(a)** The equation $2x^3 - x^2 + 2x + 12 = 0$ has one real root and two complex roots. Showing your working, verify that $1 + i\sqrt{3}$ is one of the complex roots. State the other complex root. [4]

(b) On a sketch of an Argand diagram, show the point representing the complex number $1 + i\sqrt{3}$. On the same diagram, shade the region whose points represent the complex numbers z which satisfy both the inequalities $|z - 1 - i\sqrt{3}| \leqslant 1$ and $\arg z \leqslant \frac{1}{3}\pi$. [5]

9 **(i)** Express $\dfrac{4 + 5x - x^2}{(1-2x)(2+x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4 + 5x - x^2}{(1-2x)(2+x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

10 The straight line l has equation $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 4\mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$. The plane p has equation $3x - y + 2z = 9$. The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Find the acute angle between l and p. [4]

(iii) Find an equation for the plane which contains l and is perpendicular to p, giving your answer in the form $ax + by + cz = d$. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4001209312

READ THESE INSTRUCTIONS FIRST

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JC11 06_9709_31/FP

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International Examinations

1 Expand $\sqrt[3]{(1-6x)}$ in ascending powers of x up to and including the term in x^3, simplifying the coefficients. [4]

2 Find $\frac{dy}{dx}$ in each of the following cases:

(i) $y = \ln(1 + \sin 2x)$, [2]

(ii) $y = \frac{\tan x}{x}$. [2]

3 Points A and B have coordinates $(-1, 2, 5)$ and $(2, -2, 11)$ respectively. The plane p passes through B and is perpendicular to AB.

(i) Find an equation of p, giving your answer in the form $ax + by + cz = d$. [3]

(ii) Find the acute angle between p and the y-axis. [4]

4 The polynomial $f(x)$ is defined by

$$f(x) = 12x^3 + 25x^2 - 4x - 12.$$

(i) Show that $f(-2) = 0$ and factorise $f(x)$ completely. [4]

(ii) Given that

$$12 \times 27^y + 25 \times 9^y - 4 \times 3^y - 12 = 0,$$

state the value of 3^y and hence find y correct to 3 significant figures. [3]

5 The curve with equation

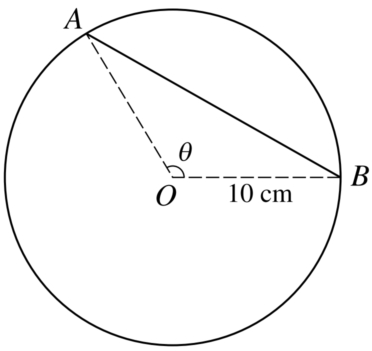
$$6e^{2x} + ke^y + e^{2y} = c,$$

where k and c are constants, passes through the point P with coordinates $(\ln 3, \ln 2)$.

(i) Show that $58 + 2k = c$. [2]

(ii) Given also that the gradient of the curve at P is -6, find the values of k and c. [5]

6



The diagram shows a circle with centre O and radius 10 cm. The chord AB divides the circle into two regions whose areas are in the ratio 1 : 4 and it is required to find the length of AB. The angle AOB is θ radians.

(i) Show that $\theta = \frac{2}{5}\pi + \sin\theta$. [3]

(ii) Showing all your working, use an iterative formula, based on the equation in part **(i)**, with an initial value of 2.1, to find θ correct to 2 decimal places. Hence find the length of AB in centimetres correct to 1 decimal place. [5]

7 The integral I is defined by $I = \int_0^2 4t^3 \ln(t^2 + 1)\,dt$.

(i) Use the substitution $x = t^2 + 1$ to show that $I = \int_1^5 (2x - 2)\ln x\,dx$. [3]

(ii) Hence find the exact value of I. [5]

8 The complex number u is defined by $u = \frac{6 - 3i}{1 + 2i}$.

(i) Showing all your working, find the modulus of u and show that the argument of u is $-\frac{1}{2}\pi$. [4]

(ii) For complex numbers z satisfying $\arg(z - u) = \frac{1}{4}\pi$, find the least possible value of $|z|$. [3]

(iii) For complex numbers z satisfying $|z - (1 + i)u| = 1$, find the greatest possible value of $|z|$. [3]

9 **(i)** Prove the identity $\cos 4\theta + 4\cos 2\theta \equiv 8\cos^4\theta - 3$. [4]

(ii) Hence

(a) solve the equation $\cos 4\theta + 4\cos 2\theta = 1$ for $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$, [3]

(b) find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^4\theta\,d\theta$. [3]

[Question 10 is printed on the next page.]

10 The number of birds of a certain species in a forested region is recorded over several years. At time t years, the number of birds is N, where N is treated as a continuous variable. The variation in the number of birds is modelled by

$$\frac{dN}{dt} = \frac{N(1800 - N)}{3600}.$$

It is given that $N = 300$ when $t = 0$.

(i) Find an expression for N in terms of t. [9]

(ii) According to the model, how many birds will there be after a long time? [1]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1540228500

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JC11 06_9709_32/RP

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Solve the inequality $|x| < |5 + 2x|$. [3]

2 **(i)** Show that the equation

$$\log_2(x + 5) = 5 - \log_2 x$$

can be written as a quadratic equation in x. [3]

(ii) Hence solve the equation

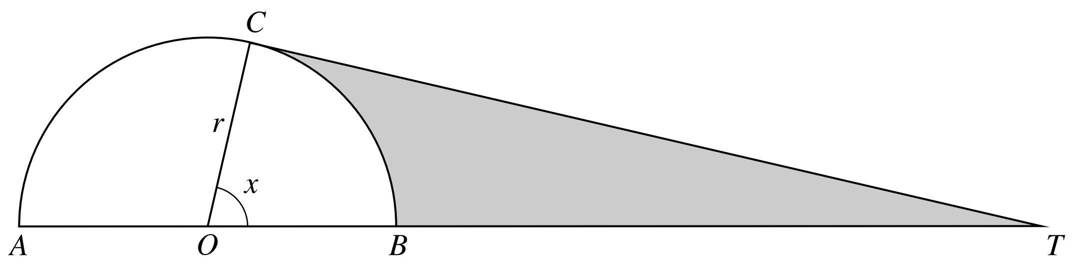
$$\log_2(x + 5) = 5 - \log_2 x.$$ [2]

3 Solve the equation

$$\cos\theta + 4\cos 2\theta = 3,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [5]

4



The diagram shows a semicircle ACB with centre O and radius r. The tangent at C meets AB produced at T. The angle BOC is x radians. The area of the shaded region is equal to the area of the semicircle.

(i) Show that x satisfies the equation

$$\tan x = x + \pi.$$ [3]

(ii) Use the iterative formula $x_{n+1} = \tan^{-1}(x_n + \pi)$ to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 The parametric equations of a curve are

$$x = \ln(\tan t), \quad y = \sin^2 t,$$

where $0 < t < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of t. [4]

(ii) Find the equation of the tangent to the curve at the point where $x = 0$. [3]

6 A certain curve is such that its gradient at a point (x, y) is proportional to xy. At the point $(1, 2)$ the gradient is 4.

(i) By setting up and solving a differential equation, show that the equation of the curve is $y = 2e^{x^2-1}$. [7]

(ii) State the gradient of the curve at the point $(-1, 2)$ and sketch the curve. [2]

7 **(a)** The complex number u is defined by $u = \dfrac{5}{a + 2i}$, where the constant a is real.

(i) Express u in the form $x + iy$, where x and y are real. [2]

(ii) Find the value of a for which $\arg(u^*) = \frac{3}{4}\pi$, where u^* denotes the complex conjugate of u. [3]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z| < 2$ and $|z| < |z - 2 - 2i|$. [4]

8 **(i)** Express $\dfrac{5x - x^2}{(1 + x)(2 + x^2)}$ in partial fractions. [5]

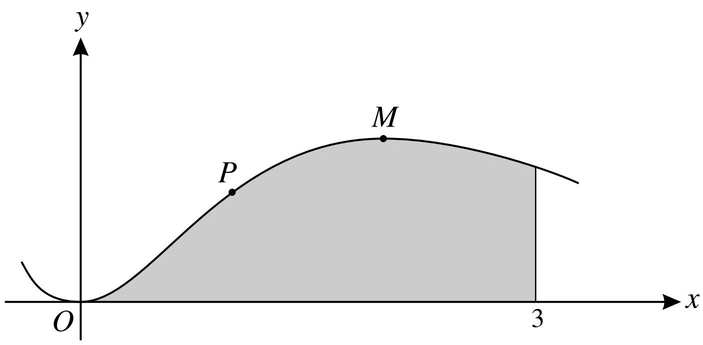
(ii) Hence obtain the expansion of $\dfrac{5x - x^2}{(1 + x)(2 + x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

9 Two planes have equations $x + 2y - 2z = 7$ and $2x + y + 3z = 5$.

(i) Calculate the acute angle between the planes. [4]

(ii) Find a vector equation for the line of intersection of the planes. [6]

10



The diagram shows the curve $y = x^2e^{-x}$.

(i) Show that the area of the shaded region bounded by the curve, the x-axis and the line $x = 3$ is equal to $2 - \dfrac{17}{e^3}$. [5]

(ii) Find the x-coordinate of the maximum point M on the curve. [4]

(iii) Find the x-coordinate of the point P at which the tangent to the curve passes through the origin. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

3311907393

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JC11 06_9709_33/RP

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1 Use logarithms to solve the equation $5^{2x-1} = 2(3^x)$, giving your answer correct to 3 significant figures. [4]

2 The curve $y = \frac{\ln x}{x^3}$ has one stationary point. Find the x-coordinate of this point. [4]

3 Show that $\int_0^1 (1-x)e^{-\frac{1}{2}x}\,dx = 4e^{-\frac{1}{2}} - 2$. [5]

4 **(i)** Show that the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = k$$

can be written in the form

$$(2\sqrt{3})(1 + \tan^2\theta) = k(1 - 3\tan^2\theta).$$ [4]

(ii) Hence solve the equation

$$\tan(60^\circ + \theta) + \tan(60^\circ - \theta) = 3\sqrt{3},$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 180^\circ$. [3]

5 The polynomial $ax^3 + bx^2 + 5x - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x - 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x - 2)$ the remainder is 12.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the quadratic factor of $p(x)$. [2]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\cot x = 1 + x^2,$$

where x is in radians, has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.5 and 0.8. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}\left(\frac{1}{1 + x_n^2}\right)$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(i)** Find the roots of the equation

$$z^2 + (2\sqrt{3})z + 4 = 0,$$

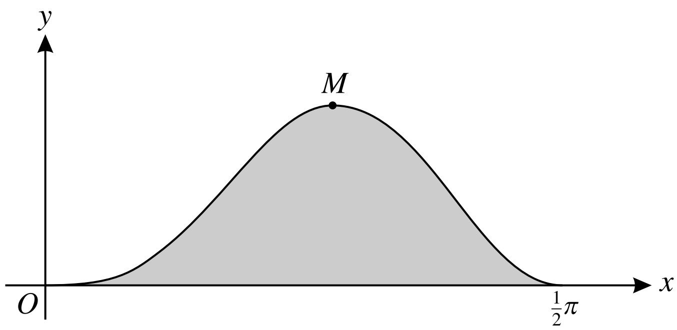
giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [2]

(ii) State the modulus and argument of each root. [3]

(iii) Showing all your working, verify that each root also satisfies the equation

$$z^6 = -64.$$ [3]

8



The diagram shows the curve $y = 5\sin^3 x \cos^2 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [5]

(ii) Using the substitution $u = \cos x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [5]

9 In a chemical reaction, a compound X is formed from two compounds Y and Z. The masses in grams of X, Y and Z present at time t seconds after the start of the reaction are x, $10 - x$ and $20 - x$ respectively. At any time the rate of formation of X is proportional to the product of the masses of Y and Z present at the time. When $t = 0$, $x = 0$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 2$.

(i) Show that x and t satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 0.01(10 - x)(20 - x).$$ [1]

(ii) Solve this differential equation and obtain an expression for x in terms of t. [9]

(iii) State what happens to the value of x when t becomes large. [1]

10 With respect to the origin O, the lines l and m have vector equations $\mathbf{r} = 2\mathbf{i} + \mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ and $\mathbf{r} = 2\mathbf{j} + 6\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ respectively.

(i) Prove that l and m do not intersect. [4]

(ii) Calculate the acute angle between the directions of l and m. [3]

(iii) Find the equation of the plane which is parallel to l and contains m, giving your answer in the form $ax + by + cz = d$. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9064786922

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JC12 06_9709_31/RP

UNIVERSITY *of* CAMBRIDGE International Examinations

1 Solve the equation $|4 - 2^x| = 10$, giving your answer correct to 3 significant figures. [3]

2 **(i)** Expand $\dfrac{1}{\sqrt{(1-4x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

(ii) Hence find the coefficient of x^2 in the expansion of $\dfrac{1+2x}{\sqrt{(4-16x)}}$. [2]

3 The polynomial $\text{p}(x)$ is defined by

$$\text{p}(x) = x^3 - 3ax + 4a,$$

where a is a constant.

(i) Given that $(x - 2)$ is a factor of $\text{p}(x)$, find the value of a. [2]

(ii) When a has this value,

(a) factorise $\text{p}(x)$ completely, [3]

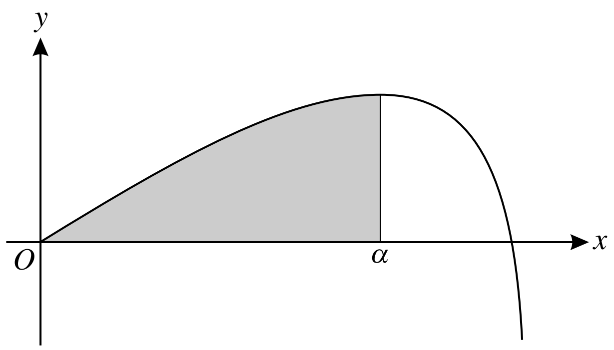
(b) find all the roots of the equation $\text{p}(x^2) = 0$. [2]

4 The complex number u is defined by $u = \dfrac{(1+2\text{i})^2}{2+\text{i}}$.

(i) Without using a calculator and showing your working, express u in the form $x + \text{i}y$, where x and y are real. [4]

(ii) Sketch an Argand diagram showing the locus of the complex number z such that $|z - u| = |u|$. [3]

5



The diagram shows the curve

$$y = 8\sin\tfrac{1}{2}x - \tan\tfrac{1}{2}x$$

for $0 \leqslant x < \pi$. The x-coordinate of the maximum point is α and the shaded region is enclosed by the curve and the lines $x = \alpha$ and $y = 0$.

(i) Show that $\alpha = \frac{2}{3}\pi$. [3]

(ii) Find the exact value of the area of the shaded region. [4]

6 The equation of a curve is $3x^2 - 4xy + y^2 = 45$.

(i) Find the gradient of the curve at the point $(2, -3)$. [4]

(ii) Show that there are no points on the curve at which the gradient is 1. [3]

7 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6xe^{3x}}{y^2}.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation and hence find the value of y when $x = 0.5$, giving your answer correct to 2 decimal places. [8]

8 The point P has coordinates $(-1, 4, 11)$ and the line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$.

(i) Find the perpendicular distance from P to l. [4]

(ii) Find the equation of the plane which contains P and l, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. [5]

9 By first expressing $\dfrac{4x^2 + 5x + 3}{2x^2 + 5x + 2}$ in partial fractions, show that

$$\int_0^4 \frac{4x^2 + 5x + 3}{2x^2 + 5x + 2}\, dx = 8 - \ln 9.$$ [10]

10 **(i)** It is given that $2\tan 2x + 5\tan^2 x = 0$. Denoting $\tan x$ by t, form an equation in t and hence show that either $t = 0$ or $t = \sqrt[3]{(t + 0.8)}$. [4]

(ii) It is given that there is exactly one real value of t satisfying the equation $t = \sqrt[3]{(t + 0.8)}$. Verify by calculation that this value lies between 1.2 and 1.3. [2]

(iii) Use the iterative formula $t_{n+1} = \sqrt[3]{(t_n + 0.8)}$ to find the value of t correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iv) Using the values of t found in previous parts of the question, solve the equation

$$2\tan 2x + 5\tan^2 x = 0$$

for $-\pi \leqslant x \leqslant \pi$. [3]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1202175411

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JC12 06_9709_32/RP

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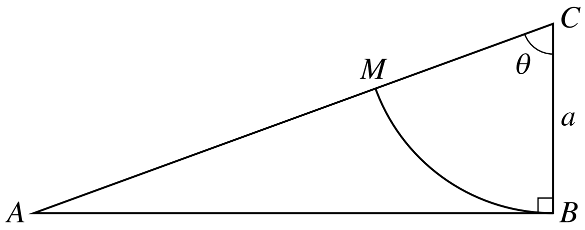
International Examinations

1 Solve the equation

$$\ln(3x + 4) = 2\ln(x + 1),$$

giving your answer correct to 3 significant figures. [4]

2



In the diagram, ABC is a triangle in which angle ABC is a right angle and $BC = a$. A circular arc, with centre C and radius a, joins B and the point M on AC. The angle ACB is θ radians. The area of the sector CMB is equal to one third of the area of the triangle ABC.

(i) Show that θ satisfies the equation

$$\tan\theta = 3\theta. \quad [2]$$

(ii) This equation has one root in the interval $0 < \theta < \frac{1}{2}\pi$. Use the iterative formula

$$\theta_{n+1} = \tan^{-1}(3\theta_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3 Expand $\sqrt{\left(\dfrac{1-x}{1+x}\right)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

4 Solve the equation

$$\operatorname{cosec} 2\theta = \sec\theta + \cot\theta,$$

giving all solutions in the interval $0° < \theta < 360°$. [6]

5 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{2x+y},$$

and $y = 0$ when $x = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [6]

6 The equation of a curve is $y = 3\sin x + 4\cos^3 x$.

(i) Find the x-coordinates of the stationary points of the curve in the interval $0 < x < \pi$. [6]

(ii) Determine the nature of the stationary point in this interval for which x is least. [2]

7 **Throughout this question the use of a calculator is not permitted.**

The complex number u is defined by

$$u = \frac{1+2\mathrm{i}}{1-3\mathrm{i}}.$$

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(ii) Show on a sketch of an Argand diagram the points A, B and C representing the complex numbers u, $1 + 2\mathrm{i}$ and $1 - 3\mathrm{i}$ respectively. [2]

(iii) By considering the arguments of $1 + 2\mathrm{i}$ and $1 - 3\mathrm{i}$, show that

$$\tan^{-1}2 + \tan^{-1}3 = \tfrac{3}{4}\pi.$$ [3]

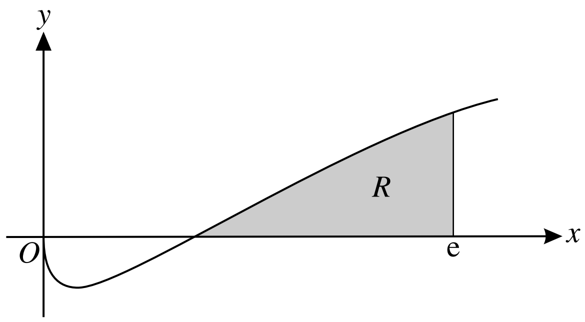
8 Let $I = \displaystyle\int_2^5 \frac{5}{x + \sqrt{(6-x)}}\,\mathrm{d}x.$

(i) Using the substitution $u = \sqrt{(6-x)}$, show that

$$I = \int_1^2 \frac{10u}{(3-u)(2+u)}\,\mathrm{d}u.$$ [4]

(ii) Hence show that $I = 2\ln\left(\frac{9}{2}\right)$. [6]

9



The diagram shows the curve $y = x^{\frac{1}{2}}\ln x$. The shaded region between the curve, the x-axis and the line $x = \mathrm{e}$ is denoted by R.

(i) Find the equation of the tangent to the curve at the point where $x = 1$, giving your answer in the form $y = mx + c$. [4]

(ii) Find by integration the volume of the solid obtained when the region R is rotated completely about the x-axis. Give your answer in terms of π and e. [7]

[Question 10 is printed on the next page.]

10 Two planes, m and n, have equations $x + 2y - 2z = 1$ and $2x - 2y + z = 7$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} - \mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l is parallel to m. [3]

(ii) Find the position vector of the point of intersection of l and n. [3]

(iii) A point P lying on l is such that its perpendicular distances from m and n are equal. Find the position vectors of the two possible positions for P and calculate the distance between them. [6]

[The perpendicular distance of a point with position vector $x_1\mathbf{i} + y_1\mathbf{j} + z_1\mathbf{k}$ from the plane $ax + by + cz = d$ is $\dfrac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{(a^2 + b^2 + c^2)}}$.]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC12 06_9709_33/FP

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Expand $\dfrac{1}{\sqrt{(4+3x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Solve the equation $\ln(2x+3) = 2\ln x + \ln 3$, giving your answer correct to 3 significant figures. [4]

3 The parametric equations of a curve are

$$x = \sin 2\theta - \theta, \quad y = \cos 2\theta + 2\sin\theta.$$

Show that $\dfrac{dy}{dx} = \dfrac{2\cos\theta}{1+2\sin\theta}$. [5]

4 The curve with equation $y = \dfrac{e^{2x}}{x^3}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

5 In a certain chemical process a substance A reacts with another substance B. The masses in grams of A and B present at time t seconds after the start of the process are x and y respectively. It is given that $\dfrac{dy}{dt} = -0.6xy$ and $x = 5e^{-3t}$. When $t = 0$, $y = 70$.

(i) Form a differential equation in y and t. Solve this differential equation and obtain an expression for y in terms of t. [6]

(ii) The percentage of the initial mass of B remaining at time t is denoted by p. Find the exact value approached by p as t becomes large. [2]

6 It is given that $\tan 3x = k\tan x$, where k is a constant and $\tan x \neq 0$.

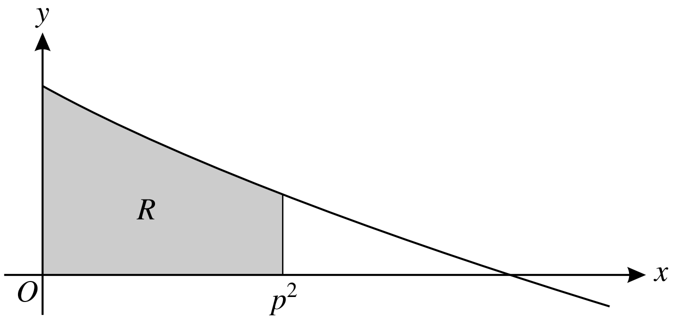
(i) By first expanding $\tan(2x + x)$, show that

$$(3k-1)\tan^2 x = k - 3.$$ [4]

(ii) Hence solve the equation $\tan 3x = k\tan x$ when $k = 4$, giving all solutions in the interval $0° < x < 180°$. [3]

(iii) Show that the equation $\tan 3x = k\tan x$ has no root in the interval $0° < x < 180°$ when $k = 2$. [1]

7



The diagram shows part of the curve $y = \cos(\sqrt{x})$ for $x \geqslant 0$, where x is in radians. The shaded region between the curve, the axes and the line $x = p^2$, where $p > 0$, is denoted by R. The area of R is equal to 1.

(i) Use the substitution $x = u^2$ to find $\int_0^{p^2} \cos(\sqrt{x})\,dx$. Hence show that $\sin p = \dfrac{3 - 2\cos p}{2p}$. [6]

(ii) Use the iterative formula $p_{n+1} = \sin^{-1}\left(\dfrac{3 - 2\cos p_n}{2p_n}\right)$, with initial value $p_1 = 1$, to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 Let $f(x) = \dfrac{4x^2 - 7x - 1}{(x+1)(2x-3)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\int_2^6 f(x)\,dx = 8 - \ln\left(\frac{49}{3}\right)$. [5]

9 The lines l and m have equations $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - \mathbf{k})$ respectively, where a and b are constants.

(i) Given that l and m intersect, show that

$$2a - b = 4.$$ [4]

(ii) Given also that l and m are perpendicular, find the values of a and b. [4]

(iii) When a and b have these values, find the position vector of the point of intersection of l and m. [2]

10 **(a)** The complex numbers u and w satisfy the equations

$$u - w = 4\text{i} \quad \text{and} \quad uw = 5.$$

Solve the equations for u and w, giving all answers in the form $x + \text{i}y$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 + 2\text{i}| \leqslant 2$, $\arg z \leqslant -\frac{1}{4}\pi$ and $\text{Re}\,z \geqslant 1$, where $\text{Re}\,z$ denotes the real part of z. [5]

(ii) Calculate the greatest possible value of $\text{Re}\,z$ for points lying in the shaded region. [1]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4064014490

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JC13 06_9709_31/FP

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1 Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

2 Expand $\dfrac{1 + 3x}{\sqrt{(1 + 2x)}}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [4]

3 Express $\dfrac{7x^2 - 3x + 2}{x(x^2 + 1)}$ in partial fractions. [5]

4 **(i)** Solve the equation $|4x - 1| = |x - 3|$. [3]

(ii) Hence solve the equation $\left|4^{y+1} - 1\right| = |4^y - 3|$ correct to 3 significant figures. [3]

5 For each of the following curves, find the gradient at the point where the curve crosses the y-axis:

(i) $y = \dfrac{1 + x^2}{1 + e^{2x}}$; [3]

(ii) $2x^3 + 5xy + y^3 = 8$. [4]

6 The points P and Q have position vectors, relative to the origin O, given by

$$\overrightarrow{OP} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \quad \text{and} \quad \overrightarrow{OQ} = -5\mathbf{i} + \mathbf{j} + \mathbf{k}.$$

The mid-point of PQ is the point A. The plane Π is perpendicular to the line PQ and passes through A.

(i) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

(ii) The straight line through P parallel to the x-axis meets Π at the point B. Find the distance AB, correct to 3 significant figures. [5]

7 **(a)** Without using a calculator, solve the equation

$$3w + 2\mathrm{i}w^* = 17 + 8\mathrm{i},$$

where w^* denotes the complex conjugate of w. Give your answer in the form $a + b\mathrm{i}$. [4]

(b) In an Argand diagram, the loci

$$\arg(z - 2\mathrm{i}) = \tfrac{1}{6}\pi \quad \text{and} \quad |z - 3| = |z - 3\mathrm{i}|$$

intersect at the point P. Express the complex number represented by P in the form $r\mathrm{e}^{\mathrm{i}\theta}$, giving the exact value of θ and the value of r correct to 3 significant figures. [5]

8 **(a)** Show that $\displaystyle\int_2^4 4x \ln x \, \mathrm{d}x = 56 \ln 2 - 12$. [5]

(b) Use the substitution $u = \sin 4x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{24}\pi} \cos^3 4x \, \mathrm{d}x$. [5]

9 **(i)** Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

(ii) Hence

(a) solve the equation $4\cos\theta + 3\sin\theta = 2$ for $0 < \theta < 2\pi$, [4]

(b) find $\displaystyle\int \frac{50}{(4\cos\theta + 3\sin\theta)^2}\,d\theta$. [3]

10 Liquid is flowing into a small tank which has a leak. Initially the tank is empty and, t minutes later, the volume of liquid in the tank is $V\,\text{cm}^3$. The liquid is flowing into the tank at a constant rate of $80\,\text{cm}^3$ per minute. Because of the leak, liquid is being lost from the tank at a rate which, at any instant, is equal to $kV\,\text{cm}^3$ per minute where k is a positive constant.

(i) Write down a differential equation describing this situation and solve it to show that

$$V = \frac{1}{k}(80 - 80e^{-kt}).$$ [7]

(ii) It is observed that $V = 500$ when $t = 15$, so that k satisfies the equation

$$k = \frac{4 - 4e^{-15k}}{25}.$$

Use an iterative formula, based on this equation, to find the value of k correct to 2 significant figures. Use an initial value of $k = 0.1$ and show the result of each iteration to 4 significant figures. [3]

(iii) Determine how much liquid there is in the tank 20 minutes after the liquid started flowing, and state what happens to the volume of liquid in the tank after a long time. [2]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

1 hour 45 minutes

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List of Formulae (MF9)

0157926880

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JC13 06_9709_32/FP

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 Solve the equation $|x - 2| = |\frac{1}{3}x|$. [3]

2 The sequence of values given by the iterative formula

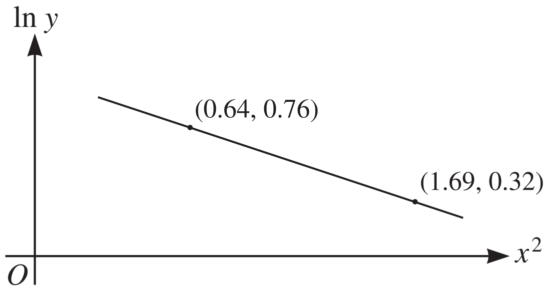
$$x_{n+1} = \frac{x_n(x_n^3 + 100)}{2(x_n^3 + 25)},$$

with initial value $x_1 = 3.5$, converges to α.

(i) Use this formula to calculate α correct to 4 decimal places, showing the result of each iteration to 6 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α. [2]

3



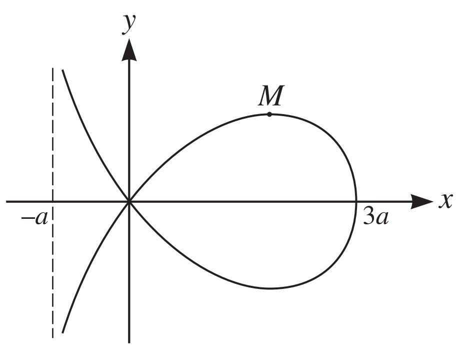
The variables x and y satisfy the equation $y = Ae^{-kx^2}$, where A and k are constants. The graph of $\ln y$ against x^2 is a straight line passing through the points (0.64, 0.76) and (1.69, 0.32), as shown in the diagram. Find the values of A and k correct to 2 decimal places. [5]

4 The polynomial $ax^3 - 20x^2 + x + 3$, where a is a constant, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$.

(i) Find the value of a. [3]

(ii) When a has this value, factorise $p(x)$ completely. [3]

5



The diagram shows the curve with equation

$$x^3 + xy^2 + ay^2 - 3ax^2 = 0,$$

where a is a positive constant. The maximum point on the curve is M. Find the x-coordinate of M in terms of a. [6]

6 **(i)** By differentiating $\dfrac{1}{\cos x}$, show that the derivative of $\sec x$ is $\sec x \tan x$. Hence show that if $y = \ln(\sec x + \tan x)$ then $\dfrac{dy}{dx} = \sec x$. [4]

(ii) Using the substitution $x = (\sqrt{3}) \tan \theta$, find the exact value of

$$\int_1^3 \frac{1}{\sqrt{(3 + x^2)}}\, dx,$$

expressing your answer as a single logarithm. [4]

7 **(i)** By first expanding $\cos(x + 45°)$, express $\cos(x + 45°) - (\sqrt{2}) \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$\cos(x + 45°) - (\sqrt{2}) \sin x = 2,$$

for $0° < x < 360°$. [4]

8 **(i)** Express $\dfrac{1}{x^2(2x + 1)}$ in the form $\dfrac{A}{x^2} + \dfrac{B}{x} + \dfrac{C}{2x + 1}$. [4]

(ii) The variables x and y satisfy the differential equation

$$y = x^2(2x + 1)\frac{dy}{dx},$$

and $y = 1$ when $x = 1$. Solve the differential equation and find the exact value of y when $x = 2$. Give your value of y in a form not involving logarithms. [7]

9 **(a)** The complex number w is such that $\operatorname{Re} w > 0$ and $w + 3w^* = iw^2$, where w^* denotes the complex conjugate of w. Find w, giving your answer in the form $x + iy$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z which satisfy both the inequalities $|z - 2i| \leqslant 2$ and $0 \leqslant \arg(z + 2) \leqslant \frac{1}{4}\pi$. Calculate the greatest value of $|z|$ for points in this region, giving your answer correct to 2 decimal places. [6]

10 The points A and B have position vectors $2\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ respectively. The plane p has equation $x + y = 5$.

(i) Find the position vector of the point of intersection of the line through A and B and the plane p. [4]

(ii) A second plane q has an equation of the form $x + by + cz = d$, where b, c and d are constants. The plane q contains the line AB, and the acute angle between the planes p and q is 60°. Find the equation of q. [7]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9836805872

READ THESE INSTRUCTIONS FIRST

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Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

JC13 06_9709_33/FP

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Solve the inequality $|4x + 3| > |x|$. [4]

2 It is given that $\ln(y + 1) - \ln y = 1 + 3\ln x$. Express y in terms of x, in a form not involving logarithms. [4]

3 Solve the equation $\tan 2x = 5\cot x$, for $0° < x < 180°$. [5]

4 **(i)** Express $(\sqrt{3})\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$, giving the exact values of R and α. [3]

(ii) Hence show that

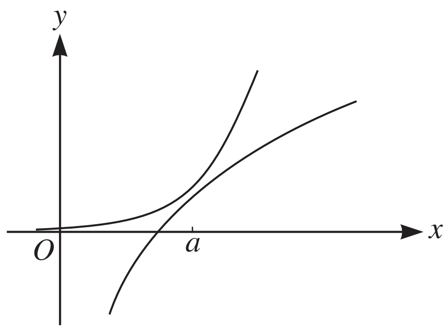
$$\int_{\frac{1}{6}\pi}^{\frac{1}{2}\pi} \frac{1}{\left((\sqrt{3})\cos x + \sin x\right)^2}\,dx = \frac{1}{4}\sqrt{3}.$$ [4]

5 The polynomial $8x^3 + ax^2 + bx + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x - 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $2x^2 - 1$. [3]

6



The diagram shows the curves $y = e^{2x-3}$ and $y = 2\ln x$. When $x = a$ the tangents to the curves are parallel.

(i) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [3]

(ii) Verify by calculation that this equation has a root between 1 and 2. [2]

(iii) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 The complex number z is defined by $z = a + \mathrm{i}b$, where a and b are real. The complex conjugate of z is denoted by z^*.

(i) Show that $|z|^2 = zz^*$ and that $(z - k\mathrm{i})^* = z^* + k\mathrm{i}$, where k is real. [2]

In an Argand diagram a set of points representing complex numbers z is defined by the equation $|z - 10\mathrm{i}| = 2|z - 4\mathrm{i}|$.

(ii) Show, by squaring both sides, that

$$zz^* - 2\mathrm{i}z^* + 2\mathrm{i}z - 12 = 0.$$

Hence show that $|z - 2\mathrm{i}| = 4$. [5]

(iii) Describe the set of points geometrically. [1]

8 The variables x and t satisfy the differential equation

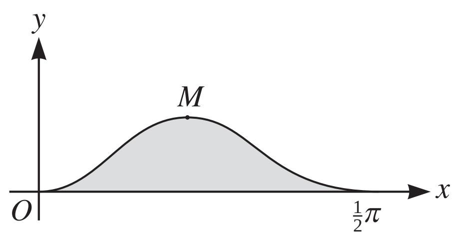
$$t\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{k - x^3}{2x^2},$$

for $t > 0$, where k is a constant. When $t = 1$, $x = 1$ and when $t = 4$, $x = 2$.

(i) Solve the differential equation, finding the value of k and obtaining an expression for x in terms of t. [9]

(ii) State what happens to the value of x as t becomes large. [1]

9



The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Find the x-coordinate of M. [6]

(ii) Using the substitution $u = \sin x$, find by integration the area of the shaded region bounded by the curve and the x-axis. [4]

10 The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} + \mathbf{k})$, where a is a constant. The plane p has equation $x + 2y + 2z = 6$. Find the value or values of a in each of the following cases.

(i) The line l is parallel to the plane p. [2]

(ii) The line l intersects the line passing through the points with position vectors $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $\mathbf{i} + \mathbf{j} - \mathbf{k}$. [4]

(iii) The acute angle between the line l and the plane p is $\tan^{-1} 2$. [5]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5374638582

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This document consists of **3** printed pages and **1** blank page.

JC14 06_9709_31/RP

CAMBRIDGE International Examinations

1 **(i)** Simplify $\sin 2\alpha \sec \alpha$. [2]

(ii) Given that $3\cos 2\beta + 7\cos \beta = 0$, find the exact value of $\cos \beta$. [3]

2 Use the substitution $u = 1 + 3\tan x$ to find the exact value of

$$\int_0^{\frac{1}{4}\pi} \frac{\sqrt{(1 + 3\tan x)}}{\cos^2 x}\, dx.$$ [5]

3 The parametric equations of a curve are

$$x = \ln(2t + 3), \quad y = \frac{3t + 2}{2t + 3}.$$

Find the gradient of the curve at the point where it crosses the y-axis. [6]

4 The variables x and y are related by the differential equation

$$\frac{dy}{dx} = \frac{6ye^{3x}}{2 + e^{3x}}.$$

Given that $y = 36$ when $x = 0$, find an expression for y in terms of x. [6]

5 The complex number z is defined by $z = \dfrac{9\sqrt{3} + 9i}{\sqrt{3} - i}$. Find, showing all your working,

(i) an expression for z in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, [5]

(ii) the two square roots of z, giving your answers in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. [3]

6 It is given that $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$.

(i) Show that $16x^3 - 24x^2 - 15x - 2 = 0$. [3]

(ii) By first using the factor theorem, factorise $16x^3 - 24x^2 - 15x - 2$ completely. [4]

(iii) Hence solve the equation $2\ln(4x - 5) + \ln(x + 1) = 3\ln 3$. [1]

7 The straight line l has equation $\mathbf{r} = 4\mathbf{i} - \mathbf{j} + 2\mathbf{k} + \lambda(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})$. The plane p passes through the point $(4, -1, 2)$ and is perpendicular to l.

(i) Find the equation of p, giving your answer in the form $ax + by + cz = d$. [2]

(ii) Find the perpendicular distance from the origin to p. [3]

(iii) A second plane q is parallel to p and the perpendicular distance between p and q is 14 units. Find the possible equations of q. [3]

8 **(i)** By sketching each of the graphs $y = \operatorname{cosec} x$ and $y = x(\pi - x)$ for $0 < x < \pi$, show that the equation

$$\operatorname{cosec} x = x(\pi - x)$$

has exactly two real roots in the interval $0 < x < \pi$. [3]

(ii) Show that the equation $\operatorname{cosec} x = x(\pi - x)$ can be written in the form $x = \dfrac{1 + x^2 \sin x}{\pi \sin x}$. [2]

(iii) The two real roots of the equation $\operatorname{cosec} x = x(\pi - x)$ in the interval $0 < x < \pi$ are denoted by α and β, where $\alpha < \beta$.

(a) Use the iterative formula

$$x_{n+1} = \frac{1 + x_n^2 \sin x_n}{\pi \sin x_n}$$

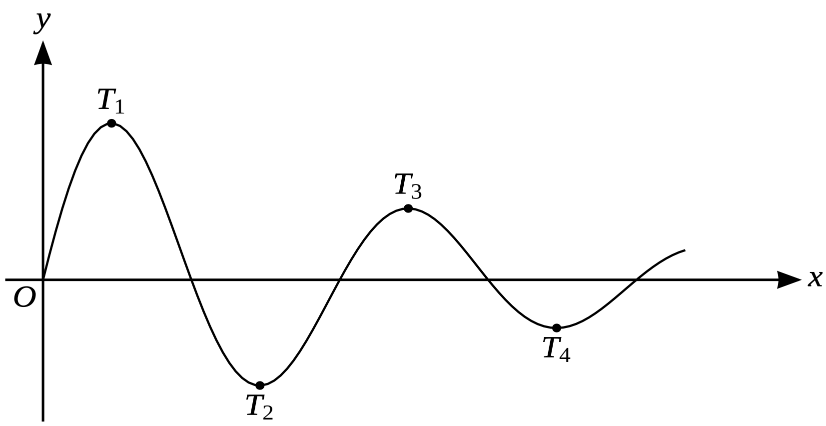
to find α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(b) Deduce the value of β correct to 2 decimal places. [1]

9 **(i)** Express $\dfrac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{4 + 12x + x^2}{(3 - x)(1 + 2x)^2}$ in ascending powers of x, up to and including the term in x^2. [5]

10



The diagram shows the curve $y = 10e^{-\frac{1}{2}x} \sin 4x$ for $x \geqslant 0$. The stationary points are labelled T_1, T_2, T_3, … as shown.

(i) Find the x-coordinates of T_1 and T_2, giving each x-coordinate correct to 3 decimal places. [6]

(ii) It is given that the x-coordinate of T_n is greater than 25. Find the least possible value of n. [4]

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

1929888667

READ THESE INSTRUCTIONS FIRST

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This document consists of **4** printed pages.

JC14 06_9709_32/RP

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International Examinations

[Turn over

1 Find the set of values of x satisfying the inequality

$$|x + 2a| > 3|x - a|,$$

where a is a positive constant. [4]

2 Solve the equation

$$2\ln(5 - e^{-2x}) = 1,$$

giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$\cos(x + 30°) = 2\cos x,$$

giving all solutions in the interval $-180° < x < 180°$. [5]

4 The parametric equations of a curve are

$$x = t - \tan t, \quad y = \ln(\cos t),$$

for $-\frac{1}{2}\pi < t < \frac{1}{2}\pi$.

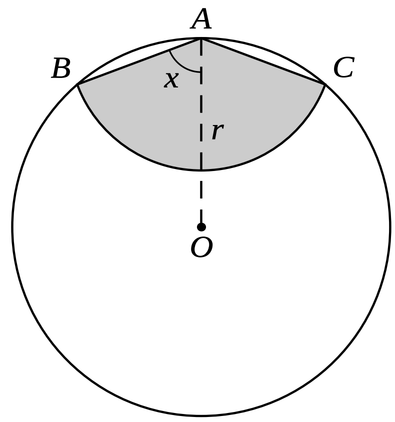
(i) Show that $\frac{dy}{dx} = \cot t$. [5]

(ii) Hence find the x-coordinate of the point on the curve at which the gradient is equal to 2. Give your answer correct to 3 significant figures. [2]

5 **(i)** The polynomial $f(x)$ is of the form $(x - 2)^2 g(x)$, where $g(x)$ is another polynomial. Show that $(x - 2)$ is a factor of $f'(x)$. [2]

(ii) The polynomial $x^5 + ax^4 + 3x^3 + bx^2 + a$, where a and b are constants, has a factor $(x - 2)^2$. Using the factor theorem and the result of part **(i)**, or otherwise, find the values of a and b. [5]

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r. A circular arc with centre A meets the circumference at B and C. The angle OAB is equal to x radians. The shaded region is bounded by AB, AC and the circular arc with centre A joining B and C. The perimeter of the shaded region is equal to half the circumference of the circle.

(i) Show that $x = \cos^{-1}\left(\dfrac{\pi}{4+4x}\right)$. [3]

(ii) Verify by calculation that x lies between 1 and 1.5. [2]

(iii) Use the iterative formula

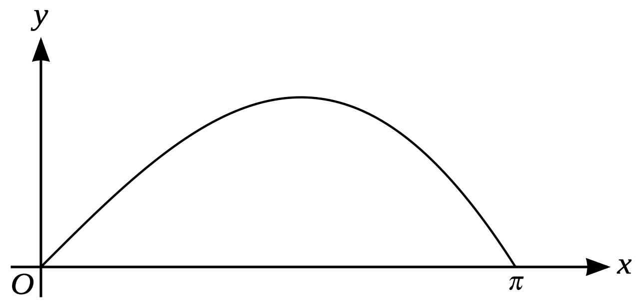
$$x_{n+1} = \cos^{-1}\left(\frac{\pi}{4+4x_n}\right)$$

to determine the value of x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 **(a)** It is given that $-1 + (\sqrt{5})\mathrm{i}$ is a root of the equation $z^3 + 2z + a = 0$, where a is real. Showing your working, find the value of a, and write down the other complex root of this equation. [4]

(b) The complex number w has modulus 1 and argument 2θ radians. Show that $\dfrac{w-1}{w+1} = \mathrm{i}\tan\theta$. [4]

8



The diagram shows the curve $y = x\cos\frac{1}{2}x$ for $0 \leqslant x \leqslant \pi$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and show that $4\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} + y + 4\sin\frac{1}{2}x = 0$. [5]

(ii) Find the exact value of the area of the region enclosed by this part of the curve and the x-axis. [5]

9 The population of a country at time t years is N millions. At any time, N is assumed to increase at a rate proportional to the product of N and $(1 - 0.01N)$. When $t = 0$, $N = 20$ and $\frac{dN}{dt} = 0.32$.

(i) Treating N and t as continuous variables, show that they satisfy the differential equation

$$\frac{dN}{dt} = 0.02N(1 - 0.01N).$$ [1]

(ii) Solve the differential equation, obtaining an expression for t in terms of N. [8]

(iii) Find the time at which the population will be double its value at $t = 0$. [1]

10 Referred to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}.$$

(i) Find the exact value of the cosine of angle BAC. [4]

(ii) Hence find the exact value of the area of triangle ABC. [3]

(iii) Find the equation of the plane which is parallel to the y-axis and contains the line through B and C. Give your answer in the form $ax + by + cz = d$. [5]

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

7019745292

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[Turn over

1 Solve the equation $\log_{10}(x+9) = 2 + \log_{10} x$. [3]

2 Expand $(1 + 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 **(i)** Show that the equation

$$\tan(x - 60°) + \cot x = \sqrt{3}$$

can be written in the form

$$2\tan^2 x + (\sqrt{3})\tan x - 1 = 0.$$ [3]

(ii) Hence solve the equation

$$\tan(x - 60°) + \cot x = \sqrt{3},$$

for $0° < x < 180°$. [3]

4 The equation $x = \dfrac{10}{e^{2x} - 1}$ has one positive real root, denoted by α.

(i) Show that α lies between $x = 1$ and $x = 2$. [2]

(ii) Show that if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(1 + \frac{10}{x_n}\right)$$

converges, then it converges to α. [2]

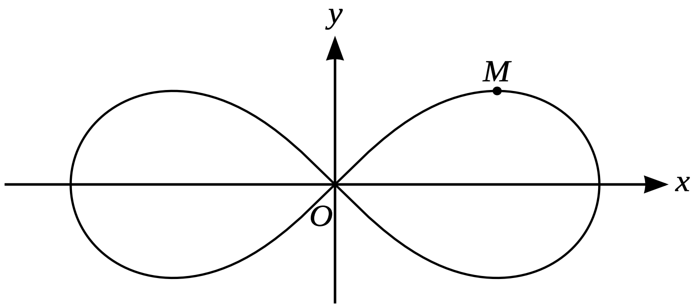
(iii) Use this iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 The variables x and θ satisfy the differential equation

$$2\cos^2\theta \frac{dx}{d\theta} = \sqrt{(2x + 1)},$$

and $x = 0$ when $\theta = \frac{1}{4}\pi$. Solve the differential equation and obtain an expression for x in terms of θ. [7]

6



The diagram shows the curve $(x^2 + y^2)^2 = 2(x^2 - y^2)$ and one of its maximum points M. Find the coordinates of M. [7]

7 **(a)** The complex number $\dfrac{3 - 5\mathrm{i}}{1 + 4\mathrm{i}}$ is denoted by u. Showing your working, express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 - \mathrm{i}| \leqslant 1$ and $|z - \mathrm{i}| \leqslant |z - 2|$. [4]

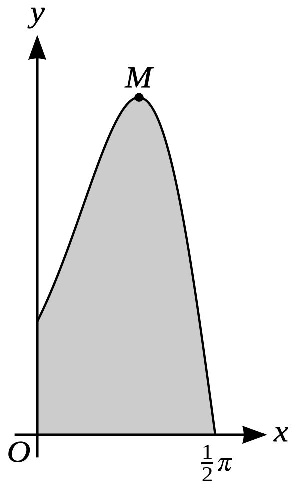
(ii) Calculate the maximum value of $\arg z$ for points lying in the shaded region. [2]

8 Let $\mathrm{f}(x) = \dfrac{6 + 6x}{(2 - x)(2 + x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{2 + x^2}$. [4]

(ii) Show that $\displaystyle\int_{-1}^{1} \mathrm{f}(x)\,\mathrm{d}x = 3\ln 3$. [5]

9



The diagram shows the curve $y = \mathrm{e}^{2\sin x}\cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \sin x$, find the exact value of the area of the shaded region bounded by the curve and the axes. [5]

(ii) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$ and the plane p has equation $2x + 3y - 5z = 18$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Find the acute angle between l and p. [4]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

* 8 2 5 4 1 4 9 8 6 5 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 Use logarithms to solve the equation $2^{5x} = 3^{2x+1}$, giving the answer correct to 3 significant figures. [4]

2 Use the trapezium rule with three intervals to find an approximation to

$$\int_0^3 |3^x - 10| \, dx. \quad [4]$$

3 Show that, for small values of x^2,

$$(1 - 2x^2)^{-2} - (1 + 6x^2)^{\frac{2}{3}} \approx kx^4,$$

where the value of the constant k is to be determined. [6]

4 The equation of a curve is

$$y = 3\cos 2x + 7\sin x + 2.$$

Find the x-coordinates of the stationary points in the interval $0 \leqslant x \leqslant \pi$. Give each answer correct to 3 significant figures. [7]

5 **(a)** Find $\int (4 + \tan^2 2x)\, dx$. [3]

(b) Find the exact value of $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \frac{\sin(x + \frac{1}{6}\pi)}{\sin x}\, dx$. [5]

6 The straight line l_1 passes through the points (0, 1, 5) and (2, −2, 1). The straight line l_2 has equation $\mathbf{r} = 7\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 5\mathbf{k})$.

(i) Show that the lines l_1 and l_2 are skew. [6]

(ii) Find the acute angle between the direction of the line l_2 and the direction of the x-axis. [3]

7 Given that $y = 1$ when $x = 0$, solve the differential equation

$$\frac{dy}{dx} = 4x(3y^2 + 10y + 3),$$

obtaining an expression for y in terms of x. [9]

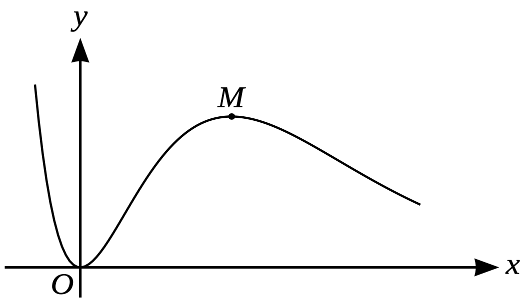
8 The complex number w is defined by $w = \frac{22 + 4i}{(2 - i)^2}$.

(i) Without using a calculator, show that $w = 2 + 4i$. [3]

(ii) It is given that p is a real number such that $\frac{1}{4}\pi \leqslant \arg(w + p) \leqslant \frac{3}{4}\pi$. Find the set of possible values of p. [3]

(iii) The complex conjugate of w is denoted by w^*. The complex numbers w and w^* are represented in an Argand diagram by the points S and T respectively. Find, in the form $|z - a| = k$, the equation of the circle passing through S, T and the origin. [3]

9

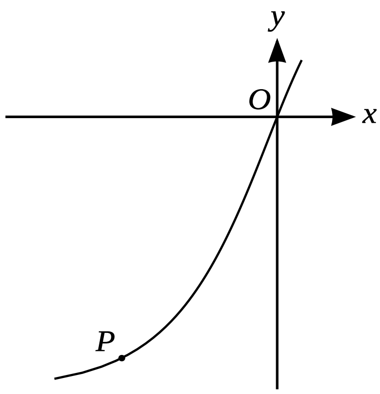


The diagram shows the curve $y = x^2e^{2-x}$ and its maximum point M.

(i) Show that the x-coordinate of M is 2. [3]

(ii) Find the exact value of $\int_0^2 x^2e^{2-x}\,dx$. [6]

10



The diagram shows part of the curve with parametric equations

$$x = 2\ln(t+2), \qquad y = t^3 + 2t + 3.$$

(i) Find the gradient of the curve at the origin. [5]

(ii) At the point P on the curve, the value of the parameter is p. It is given that the gradient of the curve at P is $\frac{1}{2}$.

(a) Show that $p = \dfrac{1}{3p^2+2} - 2$. [1]

(b) By first using an iterative formula based on the equation in part **(a)**, determine the coordinates of the point P. Give the result of each iteration to 5 decimal places and each coordinate of P correct to 2 decimal places. [4]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

2399507033

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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At the end of the examination, fasten all your work securely together.
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The total number of marks for this paper is 75.
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This document consists of **3** printed pages and **1** blank page.

JC15 06_9709_32/RP

1 Use the trapezium rule with three intervals to estimate the value of

$$\int_0^{\frac{1}{2}\pi} \ln(1 + \sin x)\,dx,$$

giving your answer correct to 2 decimal places. [3]

2 Using the substitution $u = 4^x$, solve the equation $4^x + 4^2 = 4^{x+2}$, giving your answer correct to 3 significant figures. [4]

3 A curve has equation $y = \cos x \cos 2x$. Find the x-coordinate of the stationary point on the curve in the interval $0 < x < \frac{1}{2}\pi$, giving your answer correct to 3 significant figures. [6]

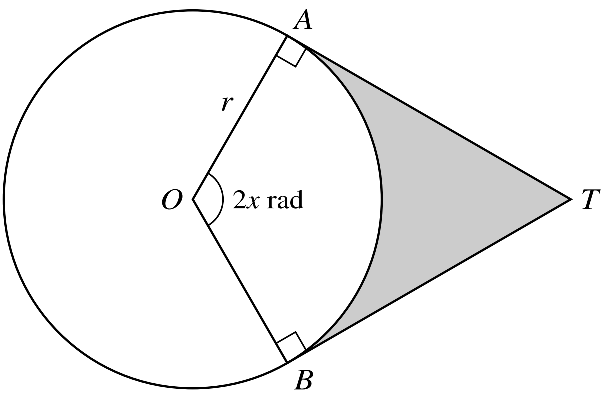
4 **(i)** Express $3\sin\theta + 2\cos\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$3\sin\theta + 2\cos\theta = 1,$$

for $0° < \theta < 180°$. [3]

5



The diagram shows a circle with centre O and radius r. The tangents to the circle at the points A and B meet at T, and the angle AOB is $2x$ radians. The shaded region is bounded by the tangents AT and BT, and by the minor arc AB. The perimeter of the shaded region is equal to the circumference of the circle.

(i) Show that x satisfies the equation

$$\tan x = \pi - x.$$ [3]

(ii) This equation has one root in the interval $0 < x < \frac{1}{2}\pi$. Verify by calculation that this root lies between 1 and 1.3. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tan^{-1}(\pi - x_n)$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 Let $I = \int_0^1 \frac{\sqrt{x}}{2 - \sqrt{x}}\, dx$.

(i) Using the substitution $u = 2 - \sqrt{x}$, show that $I = \int_1^2 \frac{2(2-u)^2}{u}\, du$. [4]

(ii) Hence show that $I = 8\ln 2 - 5$. [4]

7 The complex number u is given by $u = -1 + (4\sqrt{3})i$.

(i) Without using a calculator and showing all your working, find the two square roots of u. Give your answers in the form $a + ib$, where the real numbers a and b are exact. [5]

(ii) On an Argand diagram, sketch the locus of points representing complex numbers z satisfying the relation $|z - u| = 1$. Determine the greatest value of $\arg z$ for points on this locus. [4]

8 Let $f(x) = \frac{5x^2 + x + 6}{(3 - 2x)(x^2 + 4)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 The number of organisms in a population at time t is denoted by x. Treating x as a continuous variable, the differential equation satisfied by x and t is

$$\frac{dx}{dt} = \frac{xe^{-t}}{k + e^{-t}},$$

where k is a positive constant.

(i) Given that $x = 10$ when $t = 0$, solve the differential equation, obtaining a relation between x, k and t. [6]

(ii) Given also that $x = 20$ when $t = 1$, show that $k = 1 - \frac{2}{e}$. [2]

(iii) Show that the number of organisms never reaches 48, however large t becomes. [2]

10 The points A and B have position vectors given by $\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OB} = \mathbf{i} + \mathbf{j} + 5\mathbf{k}$. The line l has equation $\mathbf{r} = \mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(3\mathbf{i} + \mathbf{j} - \mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

(ii) Find the equation of the plane containing the line l and the point A. Give your answer in the form $ax + by + cz = d$. [6]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2015**

1 hour 45 minutes

9684619653

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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JC15 06_9709_33/RP

CAMBRIDGE International Examinations

1 Solve the equation $\ln(x+4) = 2\ln x + \ln 4$, giving your answer correct to 3 significant figures. [4]

2 Solve the inequality $|x-2| > 2x-3$. [4]

3 Solve the equation $\cot 2x + \cot x = 3$ for $0° < x < 180°$. [6]

4 The curve with equation $y = \dfrac{e^{2x}}{4+e^{3x}}$ has one stationary point. Find the exact values of the coordinates of this point. [6]

5 The parametric equations of a curve are

$$x = a\cos^4 t, \quad y = a\sin^4 t,$$

where a is a positive constant.

(i) Express $\dfrac{dy}{dx}$ in terms of t. [3]

(ii) Show that the equation of the tangent to the curve at the point with parameter t is

$$x\sin^2 t + y\cos^2 t = a\sin^2 t\cos^2 t.$$ [3]

(iii) Hence show that if the tangent meets the x-axis at P and the y-axis at Q, then

$$OP + OQ = a,$$

where O is the origin. [2]

6 It is given that $\int_0^a x\cos x\,dx = 0.5$, where $0 < a < \frac{1}{2}\pi$.

(i) Show that a satisfies the equation $\sin a = \dfrac{1.5 - \cos a}{a}$. [4]

(ii) Verify by calculation that a is greater than 1. [2]

(iii) Use the iterative formula

$$a_{n+1} = \sin^{-1}\left(\frac{1.5 - \cos a_n}{a_n}\right)$$

to determine the value of a correct to 4 decimal places, giving the result of each iteration to 6 decimal places. [3]

7 The number of micro-organisms in a population at time t is denoted by M. At any time the variation in M is assumed to satisfy the differential equation

$$\frac{dM}{dt} = k(\sqrt{M})\cos(0.02t),$$

where k is a constant and M is taken to be a continuous variable. It is given that when $t = 0$, $M = 100$.

(i) Solve the differential equation, obtaining a relation between M, k and t. [5]

(ii) Given also that $M = 196$ when $t = 50$, find the value of k. [2]

(iii) Obtain an expression for M in terms of t and find the least possible number of micro-organisms. [2]

8 The complex number $1 - \mathrm{i}$ is denoted by u.

(i) Showing your working and without using a calculator, express

$$\frac{\mathrm{i}}{u}$$

in the form $x + \mathrm{i}y$, where x and y are real. [2]

(ii) On an Argand diagram, sketch the loci representing complex numbers z satisfying the equations $|z - u| = |z|$ and $|z - \mathrm{i}| = 2$. [4]

(iii) Find the argument of each of the complex numbers represented by the points of intersection of the two loci in part **(ii)**. [3]

9 Two planes have equations $x + 3y - 2z = 4$ and $2x + y + 3z = 5$. The planes intersect in the straight line l.

(i) Calculate the acute angle between the two planes. [4]

(ii) Find a vector equation for the line l. [6]

10 Let $\mathrm{f}(x) = \dfrac{11x + 7}{(2x - 1)(x + 2)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_1^2 \mathrm{f}(x)\,dx = \tfrac{1}{4} + \ln\left(\tfrac{9}{4}\right)$. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

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JC16 06_9709_31/RP

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International Examinations

1 **(i)** Solve the equation $2|x-1| = 3|x|$. [3]

(ii) Hence solve the equation $2|5^x - 1| = 3|5^x|$, giving your answer correct to 3 significant figures. [2]

2 Find the exact value of $\int_0^{\frac{1}{2}} x\mathrm{e}^{-2x}\,\mathrm{d}x$. [5]

3 By expressing the equation $\operatorname{cosec}\theta = 3\sin\theta + \cot\theta$ in terms of $\cos\theta$ only, solve the equation for $0° < \theta < 180°$. [5]

4 The variables x and y satisfy the differential equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} = y(1-2x^2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x in a form not involving logarithms. [6]

5 The curve with equation $y = \sin x \cos 2x$ has one stationary point in the interval $0 < x < \frac{1}{2}\pi$. Find the x-coordinate of this point, giving your answer correct to 3 significant figures. [6]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$5\mathrm{e}^{-x} = \sqrt{x}$$

has one root. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2}\ln\left(\frac{25}{x_n}\right)$$

converges, then it converges to the root of the equation in part **(i)**. [2]

(iii) Use this iterative formula, with initial value $x_1 = 1$, to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The equation of a curve is $x^3 - 3x^2y + y^3 = 3$.

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x^2 - 2xy}{x^2 - y^2}$. [4]

(ii) Find the coordinates of the points on the curve where the tangent is parallel to the x-axis. [5]

8 Let $\mathrm{f}(x) = \dfrac{4x^2 + 12}{(x+1)(x-3)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 With respect to the origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k}, \quad \overrightarrow{OB} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + \mathbf{k}, \quad \overrightarrow{OD} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k}.$$

(i) Find the equation of the plane containing A, B and C, giving your answer in the form $ax + by + cz = d$. [6]

(ii) The line through D parallel to OA meets the plane with equation $x + 2y - z = 7$ at the point P. Find the position vector of P and show that the length of DP is $2\sqrt{(14)}$. [5]

10 **(a)** Showing all your working and without the use of a calculator, find the square roots of the complex number $7 - (6\sqrt{2})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [5]

(b) **(i)** On an Argand diagram, sketch the loci of points representing complex numbers w and z such that $|w - 1 - 2\mathrm{i}| = 1$ and $\arg(z - 1) = \frac{3}{4}\pi$. [4]

(ii) Calculate the least value of $|w - z|$ for points on these loci. [2]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

* 5 4 0 2 6 4 8 4 3 9 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC16 06_9709_32/2R

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International Examinations

1 Use logarithms to solve the equation $4^{3x-1} = 3(5^x)$, giving your answer correct to 3 decimal places. [4]

2 Expand $\dfrac{1}{\sqrt{(1-2x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 Find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} x^2 \sin 2x \, dx$. [5]

4 The curve with equation $y = \dfrac{(\ln x)^2}{x}$ has two stationary points. Find the exact values of the coordinates of these points. [6]

5 **(i)** Prove the identity $\cos 4\theta - 4\cos 2\theta \equiv 8\sin^4\theta - 3$. [4]

(ii) Hence solve the equation

$$\cos 4\theta = 4\cos 2\theta + 3,$$

for $0° \leqslant \theta \leqslant 360°$. [4]

6 The variables x and θ satisfy the differential equation

$$(3 + \cos 2\theta)\frac{dx}{d\theta} = x \sin 2\theta,$$

and it is given that $x = 3$ when $\theta = \frac{1}{4}\pi$.

(i) Solve the differential equation and obtain an expression for x in terms of θ. [7]

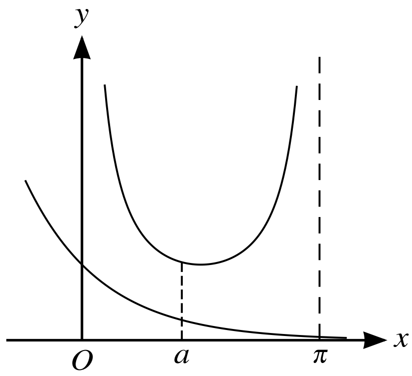
(ii) State the least value taken by x. [1]

7 Let $f(x) = \dfrac{4x^2 + 7x + 4}{(2x+1)(x+2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Show that $\displaystyle\int_0^4 f(x)\, dx = 8 - \ln 3$. [5]

8



The diagram shows the curve $y = \operatorname{cosec} x$ for $0 < x < \pi$ and part of the curve $y = e^{-x}$. When $x = a$, the tangents to the curves are parallel.

(i) By differentiating $\dfrac{1}{\sin x}$, show that if $y = \operatorname{cosec} x$ then $\dfrac{dy}{dx} = -\operatorname{cosec} x \cot x$. [3]

(ii) By equating the gradients of the curves at $x = a$, show that

$$a = \tan^{-1}\left(\frac{e^a}{\sin a}\right).$$ [2]

(iii) Verify by calculation that a lies between 1 and 1.5. [2]

(iv) Use an iterative formula based on the equation in part **(ii)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

9 The points A, B and C have position vectors, relative to the origin O, given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\overrightarrow{OB} = 4\mathbf{j} + \mathbf{k}$ and $\overrightarrow{OC} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$. A fourth point D is such that the quadrilateral $ABCD$ is a parallelogram.

(i) Find the position vector of D and verify that the parallelogram is a rhombus. [5]

(ii) The plane p is parallel to OA and the line BC lies in p. Find the equation of p, giving your answer in the form $ax + by + cz = d$. [5]

10 **(a)** Showing all necessary working, solve the equation $iz^2 + 2z - 3i = 0$, giving your answers in the form $x + iy$, where x and y are real and exact. [5]

(b) **(i)** On a sketch of an Argand diagram, show the locus representing complex numbers satisfying the equation $|z| = |z - 4 - 3i|$. [2]

(ii) Find the complex number represented by the point on the locus where $|z|$ is least. Find the modulus and argument of this complex number, giving the argument correct to 2 decimal places. [3]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2016**

1 hour 45 minutes

* 6 7 6 8 9 3 9 7 6 1 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC16 06_9709_33/RP

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1 Solve the inequality $2|x-2| > |3x+1|$. [4]

2 The variables x and y satisfy the relation $3^y = 4^{2-x}$.

(i) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

(ii) Calculate the exact x-coordinate of the point of intersection of this line with the line with equation $y = 2x$, simplifying your answer. [2]

3 **(i)** Express $(\sqrt{5})\cos x + 2\sin x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$(\sqrt{5})\cos\tfrac{1}{2}x + 2\sin\tfrac{1}{2}x = 1.2,$$

for $0° < x < 360°$. [3]

4 The parametric equations of a curve are

$$x = t + \cos t, \qquad y = \ln(1+\sin t),$$

where $-\tfrac{1}{2}\pi < t < \tfrac{1}{2}\pi$.

(i) Show that $\dfrac{dy}{dx} = \sec t$. [5]

(ii) Hence find the x-coordinates of the points on the curve at which the gradient is equal to 3. Give your answers correct to 3 significant figures. [3]

5 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = e^{-2y}\tan^2 x,$$

for $0 \leqslant x < \tfrac{1}{2}\pi$, and it is given that $y = 0$ when $x = 0$. Solve the differential equation and calculate the value of y when $x = \tfrac{1}{4}\pi$. [8]

6 The curve with equation $y = x^2\cos\tfrac{1}{2}x$ has a stationary point at $x = p$ in the interval $0 < x < \pi$.

(i) Show that p satisfies the equation $\tan\tfrac{1}{2}p = \dfrac{4}{p}$. [3]

(ii) Verify by calculation that p lies between 2 and 2.5. [2]

(iii) Use the iterative formula $p_{n+1} = 2\tan^{-1}\left(\dfrac{4}{p_n}\right)$ to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 Let $I = \int_0^1 \frac{x^5}{(1+x^2)^3}\,dx$.

(i) Using the substitution $u = 1 + x^2$, show that $I = \int_1^2 \frac{(u-1)^2}{2u^3}\,du$. [3]

(ii) Hence find the exact value of I. [5]

8 The points A and B have position vectors, relative to the origin O, given by $\overrightarrow{OA} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\overrightarrow{OB} = 2\mathbf{i} + 3\mathbf{k}$. The line l has vector equation $\mathbf{r} = 2\mathbf{i} - 2\mathbf{j} - \mathbf{k} + \mu(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$.

(i) Show that the line passing through A and B does not intersect l. [4]

(ii) Show that the length of the perpendicular from A to l is $\frac{1}{\sqrt{2}}$. [5]

9 **Throughout this question the use of a calculator is not permitted.**

The complex numbers $-1 + 3i$ and $2 - i$ are denoted by u and v respectively. In an Argand diagram with origin O, the points A, B and C represent the numbers u, v and $u + v$ respectively.

(i) Sketch this diagram and state fully the geometrical relationship between OB and AC. [4]

(ii) Find, in the form $x + iy$, where x and y are real, the complex number $\frac{u}{v}$. [3]

(iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

10 Let $f(x) = \frac{10x - 2x^2}{(x+3)(x-1)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

4382153981

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.

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CAMBRIDGE International Examinations

[Turn over

1 Solve the inequality $|2x + 1| < 3|x - 2|$. [4]

2 Expand $\frac{1}{\sqrt[3]{(1+6x)}}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

3 It is given that $x = \ln(1 - y) - \ln y$, where $0 < y < 1$.

(i) Show that $y = \dfrac{e^{-x}}{1 + e^{-x}}$. [2]

(ii) Hence show that $\int_0^1 y \, dx = \ln\left(\frac{2e}{e + 1}\right)$. [4]

4 The parametric equations of a curve are

$$x = \ln \cos \theta, \quad y = 3\theta - \tan \theta,$$

where $0 \leqslant \theta < \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of $\tan \theta$. [5]

(ii) Find the exact y-coordinate of the point on the curve at which the gradient of the normal is equal to 1. [3]

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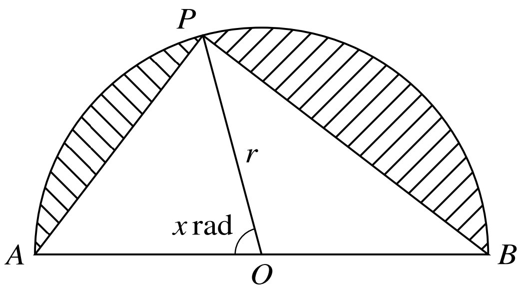

The diagram shows a semicircle with centre O, radius r and diameter AB. The point P on its circumference is such that the area of the minor segment on AP is equal to half the area of the minor segment on BP. The angle AOP is x radians.

(i) Show that x satisfies the equation $x = \frac{1}{3}(\pi + \sin x)$. [3]

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(ii) Verify by calculation that x lies between 1 and 1.5. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

6 The plane with equation $2x + 2y - z = 5$ is denoted by m. Relative to the origin O, the points A and B have coordinates $(3, 4, 0)$ and $(-1, 0, 2)$ respectively.

(i) Show that the plane m bisects AB at right angles. [5]

A second plane p is parallel to m and nearer to O. The perpendicular distance between the planes is 1.

(ii) Find the equation of p, giving your answer in the form $ax + by + cz = d$. [3]

7 Throughout this question the use of a calculator is not permitted.

The complex numbers u and w are defined by $u = -1 + 7\mathrm{i}$ and $w = 3 + 4\mathrm{i}$.

(i) Showing all your working, find in the form $x + \mathrm{i}y$, where x and y are real, the complex numbers $u - 2w$ and $\dfrac{u}{w}$. [4]

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In an Argand diagram with origin O, the points A, B and C represent the complex numbers u, w and $u - 2w$ respectively.

(ii) Prove that angle $AOB = \frac{1}{4}\pi$. [2]

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(iii) State fully the geometrical relation between the line segments OB and CA. [2]

8 **(i)** By first expanding $2\sin(x - 30°)$, express $2\sin(x - 30°) - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

(ii) Hence solve the equation

$$2\sin(x - 30°) - \cos x = 1,$$

for $0° < x < 180°$. [3]

9 **(i)** Express $\dfrac{1}{x(2x+3)}$ in partial fractions. [2]

(ii) The variables x and y satisfy the differential equation

$$x(2x+3)\frac{dy}{dx} = y,$$

and it is given that $y = 1$ when $x = 1$. Solve the differential equation and calculate the value of y when $x = 9$, giving your answer correct to 3 significant figures. [7]

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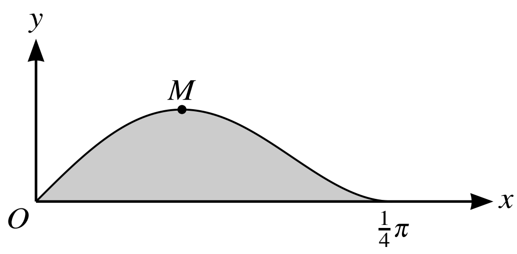

The diagram shows the curve $y = \sin x \cos^2 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$ and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

(ii) Find the x-coordinate of M. Give your answer correct to 2 decimal places. [6]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

9035132693

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
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DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

JC17 06_9709_32/2R

CAMBRIDGE International Examinations

[Turn over

1 Solve the equation $\ln(x^2 + 1) = 1 + 2\ln x$, giving your answer correct to 3 significant figures. [3]

2 Solve the inequality $|x - 3| < 3x - 4$. [4]

3 **(i)** Express the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ in the form $a\cos^4\theta + b\cos^2\theta + c = 0$, where a, b and c are constants to be determined. [3]

(ii) Hence solve the equation $\cot\theta - 2\tan\theta = \sin 2\theta$ for $90° < \theta < 180°$. [2]

4 The parametric equations of a curve are

$$x = t^2 + 1, \quad y = 4t + \ln(2t - 1).$$

(i) Express $\dfrac{dy}{dx}$ in terms of t. [3]

(ii) Find the equation of the normal to the curve at the point where $t = 1$. Give your answer in the form $ax + by + c = 0$. [3]

5 In a certain chemical process a substance A reacts with and reduces a substance B. The masses of A and B at time t after the start of the process are x and y respectively. It is given that $\frac{dy}{dt} = -0.2xy$ and $x = \frac{10}{(1+t)^2}$. At the beginning of the process $y = 100$.

(i) Form a differential equation in y and t, and solve this differential equation. [6]

(ii) Find the exact value approached by the mass of B as t becomes large. State what happens to the mass of A as t becomes large. [2]

6 **Throughout this question the use of a calculator is not permitted.**

The complex number $2 - \mathrm{i}$ is denoted by u.

(i) It is given that u is a root of the equation $x^3 + ax^2 - 3x + b = 0$, where the constants a and b are real. Find the values of a and b. [4]

(ii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| < 1$ and $|z| < |z + \mathrm{i}|$. [4]

7 **(i)** Prove that if $y = \dfrac{1}{\cos\theta}$ then $\dfrac{dy}{d\theta} = \sec\theta\tan\theta$. [2]

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(ii) Prove the identity $\dfrac{1+\sin\theta}{1-\sin\theta} \equiv 2\sec^2\theta + 2\sec\theta\tan\theta - 1$. [3]

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(iii) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} \frac{1 + \sin\theta}{1 - \sin\theta}\, d\theta$. [4]

8 Let $f(x) = \dfrac{5x^2 - 7x + 4}{(3x+2)(x^2+5)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 Relative to the origin O, the point A has position vector given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$. The line l has equation $\mathbf{r} = 9\mathbf{i} - \mathbf{j} + 8\mathbf{k} + \mu(3\mathbf{i} - \mathbf{j} + 2\mathbf{k})$.

(i) Find the position vector of the foot of the perpendicular from A to l. Hence find the position vector of the reflection of A in l. [5]

(ii) Find the equation of the plane through the origin which contains l. Give your answer in the form $ax + by + cz = d$. [3]

(iii) Find the exact value of the perpendicular distance of A from this plane. [3]

[Turn over

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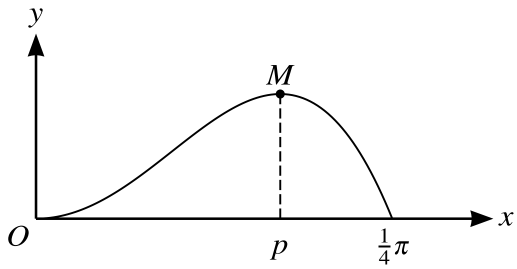

The diagram shows the curve $y = x^2 \cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The curve has a maximum point at M where $x = p$.

(i) Show that p satisfies the equation $p = \frac{1}{2}\tan^{-1}\left(\frac{1}{p}\right)$. [3]

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(ii) Use the iterative formula $p_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{p_n}\right)$ to determine the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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(iii) Find, showing all necessary working, the exact area of the region bounded by the curve and the x-axis. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

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JC17 06_9709_33/RP

CAMBRIDGE International Examinations

1 Prove the identity $\frac{\cot x - \tan x}{\cot x + \tan x} \equiv \cos 2x$. [3]

2 Expand $(3 + 2x)^{-3}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [4]

3 Using the substitution $u = e^x$, solve the equation $4e^{-x} = 3e^x + 4$. Give your answer correct to 3 significant figures. [4]

4 Find the exact value of $\int_0^{\frac{1}{2}\pi} \theta \sin \frac{1}{2}\theta \, d\theta$. [4]

5 A curve has equation $y = \frac{2}{3}\ln(1 + 3\cos^2 x)$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Express $\frac{dy}{dx}$ in terms of $\tan x$. [4]

(ii) Hence find the x-coordinate of the point on the curve where the gradient is -1. Give your answer correct to 3 significant figures. [2]

6 The equation $\cot x = 1 - x$ has one root in the interval $0 < x < \pi$, denoted by α.

(i) Show by calculation that α is greater than 2.5. [2]

(ii) Show that, if a sequence of values in the interval $0 < x < \pi$ given by the iterative formula $x_{n+1} = \pi + \tan^{-1}\left(\dfrac{1}{1 - x_n}\right)$ converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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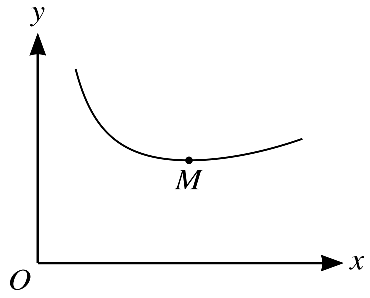

The diagram shows a sketch of the curve $y = \dfrac{e^{\frac{1}{2}x}}{x}$ for $x > 0$, and its minimum point M.

(i) Find the x-coordinate of M. [4]

(ii) Use the trapezium rule with two intervals to estimate the value of

$$\int_1^3 \frac{e^{\frac{1}{2}x}}{x}\,dx,$$

giving your answer correct to 2 decimal places. [3]

(iii) The estimate found in part **(ii)** is denoted by E. Explain, without further calculation, whether another estimate found using the trapezium rule with four intervals would be greater than E or less than E. [1]

8 In a certain chemical reaction, a compound A is formed from a compound B. The masses of A and B at time t after the start of the reaction are x and y respectively and the sum of the masses is equal to 50 throughout the reaction. At any time the rate of increase of the mass of A is proportional to the mass of B at that time.

(i) Explain why $\dfrac{dx}{dt} = k(50 - x)$, where k is a constant. [1]

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It is given that $x = 0$ when $t = 0$, and $x = 25$ when $t = 10$.

(ii) Solve the differential equation in part **(i)** and express x in terms of t. [8]

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9 Let $f(x) = \dfrac{3x^2 - 4}{x^2(3x + 2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_1^2 f(x)\,dx = \ln\left(\frac{25}{8}\right) - 1.$ [5]

[Turn over

10 The points A and B have position vectors given by $\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + \mathbf{j} + \mathbf{k}$. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + m\mathbf{k} + \mu(\mathbf{i} - 2\mathbf{j} - 4\mathbf{k})$, where m is a constant.

(i) Given that the line l intersects the line passing through A and B, find the value of m. [5]

(ii) Find the equation of the plane which is parallel to $\mathbf{i} - 2\mathbf{j} - 4\mathbf{k}$ and contains the points A and B. Give your answer in the form $ax + by + cz = d$. [5]

11 Throughout this question the use of a calculator is not permitted.

(a) The complex numbers z and w satisfy the equations

$$z + (1 + \mathrm{i})w = \mathrm{i} \qquad \text{and} \qquad (1 - \mathrm{i})z + \mathrm{i}w = 1.$$

Solve the equations for z and w, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [6]

(b) The complex numbers u and v are given by $u = 1 + (2\sqrt{3})\text{i}$ and $v = 3 + 2\text{i}$. In an Argand diagram, u and v are represented by the points A and B. A third point C lies in the first quadrant and is such that $BC = 2AB$ and angle $ABC = 90°$. Find the complex number z represented by C, giving your answer in the form $x + \text{i}y$, where x and y are real and exact. [4]

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Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2262006173

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC18 06_9709_11/2R

CAMBRIDGE International Examinations

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1 **(i)** Find the first three terms in the expansion, in ascending powers of x, of $(1 - 2x)^5$. [2]

(ii) Given that the coefficient of x^2 in the expansion of $(1 + ax + 2x^2)(1 - 2x)^5$ is 12, find the value of the constant a. [3]

2 A point is moving along the curve $y = 2x + \frac{5}{x}$ in such a way that the x-coordinate is increasing at a constant rate of 0.02 units per second. Find the rate of change of the y-coordinate when $x = 1$. [4]

3 A curve is such that $\frac{dy}{dx} = \frac{12}{(2x+1)^2}$. The point (1, 1) lies on the curve. Find the coordinates of the point at which the curve intersects the x-axis. [6]

4 **(i)** Prove the identity $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) \equiv \sin^3\theta + \cos^3\theta$. [3]

(ii) Hence solve the equation $(\sin\theta + \cos\theta)(1 - \sin\theta\cos\theta) = 3\cos^3\theta$ for $0° \leqslant \theta \leqslant 360°$. [3]

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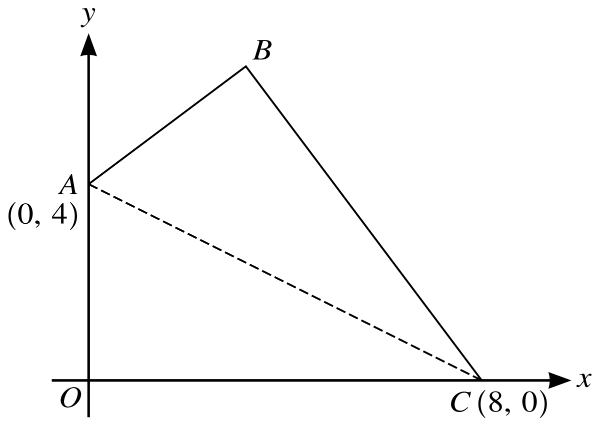

The diagram shows a kite *OABC* in which *AC* is the line of symmetry. The coordinates of *A* and *C* are (0, 4) and (8, 0) respectively and *O* is the origin.

(i) Find the equations of *AC* and *OB*. [4]

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(ii) Find, by calculation, the coordinates of B. [3]

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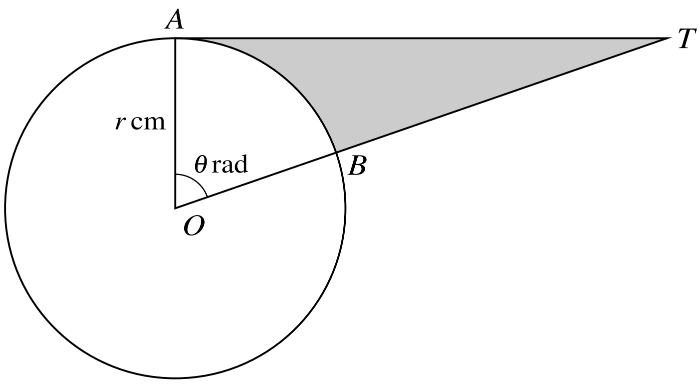

The diagram shows a circle with centre O and radius r cm. The points A and B lie on the circle and AT is a tangent to the circle. Angle $AOB = \theta$ radians and OBT is a straight line.

(i) Express the area of the shaded region in terms of r and θ. [3]

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(ii) In the case where $r = 3$ and $\theta = 1.2$, find the perimeter of the shaded region. [4]

7 Relative to an origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} -1 \\ 3 \\ 5 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}.$$

(i) Find $\overrightarrow{AC}$. [1]

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(ii) The point M is the mid-point of AC. Find the unit vector in the direction of $\overrightarrow{OM}$. [3]

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(iii) Evaluate $\overrightarrow{AB}.\overrightarrow{AC}$ and hence find angle BAC. [4]

8 **(a)** A geometric progression has a second term of 12 and a sum to infinity of 54. Find the possible values of the first term of the progression. [4]

(b) The nth term of a progression is $p + qn$, where p and q are constants, and S_n is the sum of the first n terms.

(i) Find an expression, in terms of p, q and n, for S_n. [3]

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(ii) Given that $S_4 = 40$ and $S_6 = 72$, find the values of p and q. [2]

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9 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto \tfrac{1}{2}x - 2,$$
$$g : x \mapsto 4 + x - \tfrac{1}{2}x^2.$$

(i) Find the points of intersection of the graphs of $y = f(x)$ and $y = g(x)$. [3]

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(ii) Find the set of values of x for which $f(x) > g(x)$. [2]

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(iii) Find an expression for $\mathrm{fg}(x)$ and deduce the range of fg. [4]

The function h is defined by $\mathrm{h} : x \mapsto 4 + x - \frac{1}{2}x^2$ for $x \geqslant k$.

(iv) Find the smallest value of k for which h has an inverse. [2]

10 The curve with equation $y = x^3 - 2x^2 + 5x$ passes through the origin.

(i) Show that the curve has no stationary points. [3]

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(ii) Denoting the gradient of the curve by m, find the stationary value of m and determine its nature. [5]

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(iii) Showing all necessary working, find the area of the region enclosed by the curve, the x-axis and the line $x = 6$. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 6 6 5 3 0 0 0 6 7 4 *

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **20** printed pages.

JC18 06_9709_12/2RP **R**

[Turn over

1 The coefficient of x^2 in the expansion of $\left(2 + \frac{x}{2}\right)^6 + (a + x)^5$ is 330. Find the value of the constant a. [5]

2 The equation of a curve is $y = x^2 - 6x + k$, where k is a constant.

(i) Find the set of values of k for which the whole of the curve lies above the x-axis. [2]

(ii) Find the value of k for which the line $y + 2x = 7$ is a tangent to the curve. [3]

3 A company producing salt from sea water changed to a new process. The amount of salt obtained each week increased by 2% of the amount obtained in the preceding week. It is given that in the first week after the change the company obtained 8000 kg of salt.

(i) Find the amount of salt obtained in the 12th week after the change. [3]

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(ii) Find the total amount of salt obtained in the first 12 weeks after the change. [2]

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4 The function f is such that $f(x) = a + b\cos x$ for $0 \leqslant x \leqslant 2\pi$. It is given that $f\left(\frac{1}{3}\pi\right) = 5$ and $f(\pi) = 11$.

(i) Find the values of the constants a and b. [3]

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(ii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [3]

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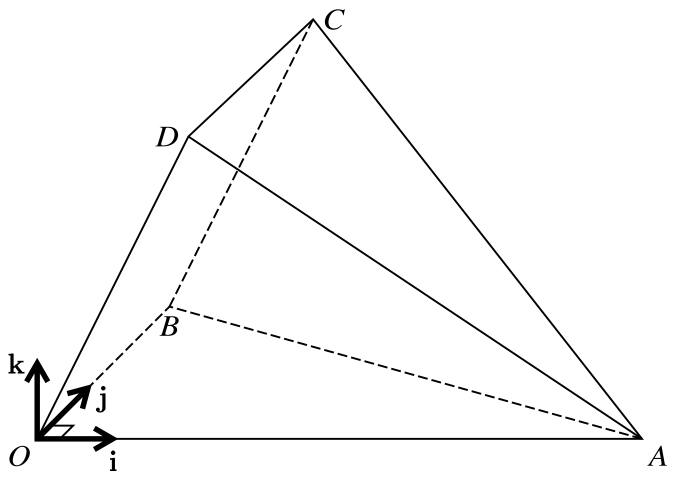

The diagram shows a three-dimensional shape. The base OAB is a horizontal triangle in which angle AOB is 90°. The side $OBCD$ is a rectangle and the side OAD lies in a vertical plane. Unit vectors $\mathbf{i}$ and $\mathbf{j}$ are parallel to OA and OB respectively and the unit vector $\mathbf{k}$ is vertical. The position vectors of A, B and D are given by $\overrightarrow{OA} = 8\mathbf{i}$, $\overrightarrow{OB} = 5\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 4\mathbf{k}$.

(i) Express each of the vectors $\overrightarrow{DA}$ and $\overrightarrow{CA}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

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(ii) Use a scalar product to find angle *CAD*. [4]

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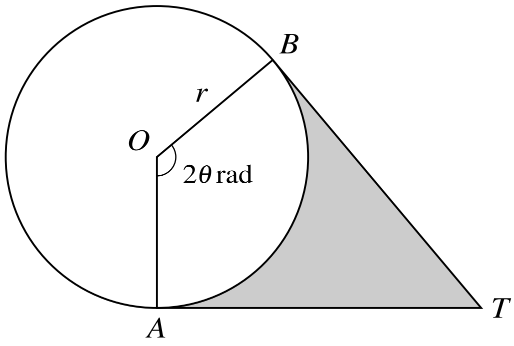

The diagram shows points A and B on a circle with centre O and radius r. The tangents to the circle at A and B meet at T. The shaded region is bounded by the minor arc AB and the lines AT and BT. Angle AOB is 2θ radians.

(i) In the case where the area of the sector AOB is the same as the area of the shaded region, show that $\tan\theta = 2\theta$. [3]

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(ii) In the case where $r = 8$ cm and the length of the minor arc AB is 19.2 cm, find the area of the shaded region. [3]

7 The function f is defined by $\mathrm{f} : x \mapsto 7 - 2x^2 - 12x$ for $x \in \mathbb{R}$.

(i) Express $7 - 2x^2 - 12x$ in the form $a - 2(x + b)^2$, where a and b are constants. [2]

(ii) State the coordinates of the stationary point on the curve $y = \mathrm{f}(x)$. [1]

The function g is defined by $g : x \mapsto 7 - 2x^2 - 12x$ for $x \geqslant k$.

(iii) State the smallest value of k for which g has an inverse. [1]

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(iv) For this value of k, find $g^{-1}(x)$. [3]

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8 Points A and B have coordinates (h, h) and $(4h + 6, 5h)$ respectively. The equation of the perpendicular bisector of AB is $3x + 2y = k$. Find the values of the constants h and k. [7]

9 A curve is such that $\frac{dy}{dx} = \sqrt{(4x + 1)}$ and $(2, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) A point P moves along the curve in such a way that the y-coordinate is increasing at a constant rate of 0.06 units per second. Find the rate of change of the x-coordinate when P passes through (2, 5). [2]

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(iii) Show that $\frac{d^2y}{dx^2} \times \frac{dy}{dx}$ is constant. [2]

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10 **(i)** Solve the equation $2\cos x + 3\sin x = 0$, for $0° \leqslant x \leqslant 360°$. [3]

(ii) Sketch, on the same diagram, the graphs of $y = 2\cos x$ and $y = -3\sin x$ for $0° \leqslant x \leqslant 360°$. [3]

(iii) Use your answers to parts **(i)** and **(ii)** to find the set of values of x for $0° \leqslant x \leqslant 360°$ for which $2\cos x + 3\sin x > 0$. [2]

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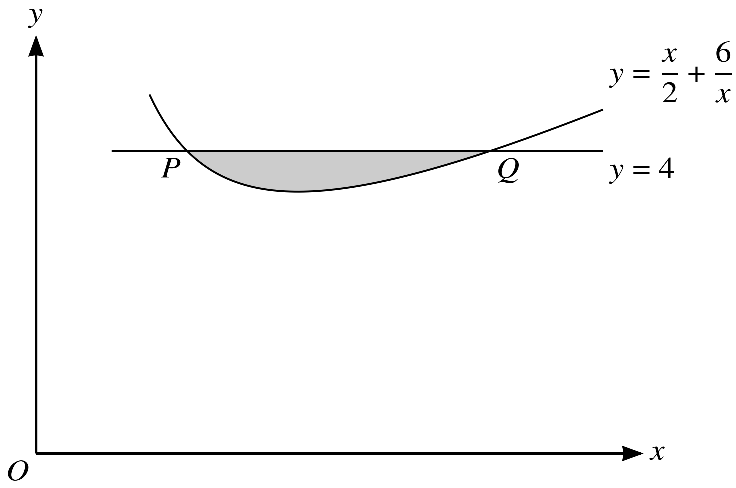

The diagram shows part of the curve $y = \frac{x}{2} + \frac{6}{x}$. The line $y = 4$ intersects the curve at the points P and Q.

(i) Show that the tangents to the curve at P and Q meet at a point on the line $y = x$. [6]

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(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. Give your answer in terms of π. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

8124433764

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

JC18 06_9709_13/RP

CAMBRIDGE International Examinations

1 Express $3x^2 - 12x + 7$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

2 Find the coefficient of $\frac{1}{x}$ in the expansion of $\left(x - \frac{2}{x}\right)^5$. [3]

3 The common ratio of a geometric progression is 0.99. Express the sum of the first 100 terms as a percentage of the sum to infinity, giving your answer correct to 2 significant figures. [5]

4 A curve with equation $y = f(x)$ passes through the point $A\ (3,\ 1)$ and crosses the y-axis at B. It is given that $f'(x) = (3x - 1)^{-\frac{1}{3}}$. Find the y-coordinate of B. [6]

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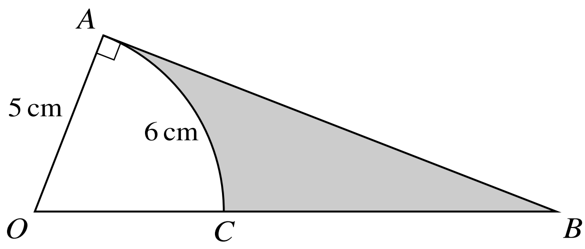

The diagram shows a triangle OAB in which angle $OAB = 90°$ and $OA = 5$ cm. The arc AC is part of a circle with centre O. The arc has length 6 cm and it meets OB at C. Find the area of the shaded region. [5]

6 The coordinates of points A and B are $(-3k-1,\ k+3)$ and $(k+3,\ 3k+5)$ respectively, where k is a constant $(k \neq -1)$.

(i) Find and simplify the gradient of AB, showing that it is independent of k. [2]

(ii) Find and simplify the equation of the perpendicular bisector of AB. [5]

7 **(a)** **(i)** Express $\dfrac{\tan^2\theta - 1}{\tan^2\theta + 1}$ in the form $a\sin^2\theta + b$, where a and b are constants to be found. [3]

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(ii) Hence, or otherwise, and showing all necessary working, solve the equation

$$\frac{\tan^2\theta - 1}{\tan^2\theta + 1} = \frac{1}{4}$$

for $-90° \leqslant \theta \leqslant 0°$. [2]

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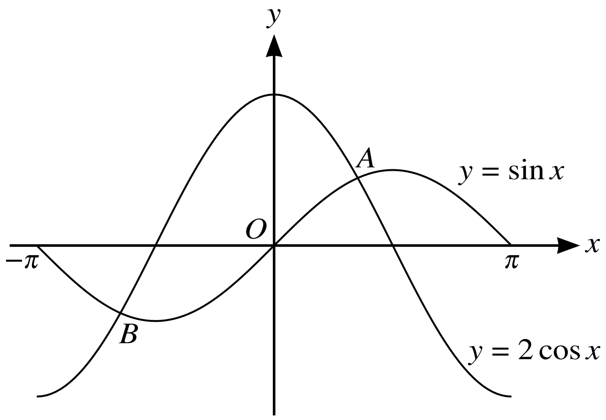

The diagram shows the graphs of $y = \sin x$ and $y = 2\cos x$ for $-\pi \leqslant x \leqslant \pi$. The graphs intersect at the points A and B.

(i) Find the x-coordinate of A. [2]

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(ii) Find the y-coordinate of B. [2]

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8 **(i)** The tangent to the curve $y = x^3 - 9x^2 + 24x - 12$ at a point A is parallel to the line $y = 2 - 3x$. Find the equation of the tangent at A. [6]

(ii) The function f is defined by $f(x) = x^3 - 9x^2 + 24x - 12$ for $x > k$, where k is a constant. Find the smallest value of k for f to be an increasing function. [2]

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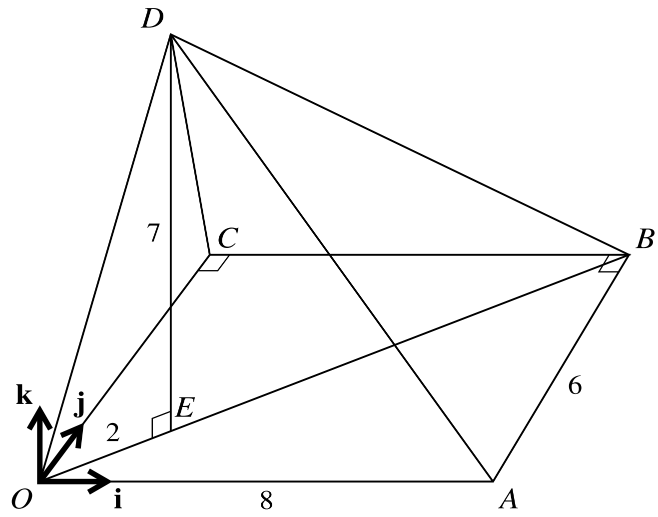

The diagram shows a pyramid *OABCD* with a horizontal rectangular base *OABC*. The sides *OA* and *AB* have lengths of 8 units and 6 units respectively. The point *E* on *OB* is such that *OE* = 2 units. The point *D* of the pyramid is 7 units vertically above *E*. Unit vectors **i**, **j** and **k** are parallel to *OA*, *OC* and *ED* respectively.

(i) Show that $\overrightarrow{OE} = 1.6\mathbf{i} + 1.2\mathbf{j}$. [2]

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(ii) Use a scalar product to find angle *BDO*. [7]

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10 The one-one function f is defined by $f(x) = (x-2)^2 + 2$ for $x \geqslant c$, where c is a constant.

(i) State the smallest possible value of c. [1]

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In parts **(ii)** and **(iii)** the value of c is 4.

(ii) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [3]

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(iii) Solve the equation $ff(x) = 51$, giving your answer in the form $a + \sqrt{b}$. [5]

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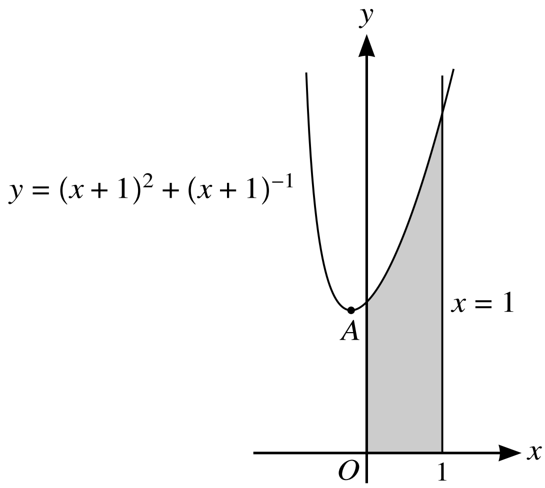

The diagram shows part of the curve $y = (x + 1)^2 + (x + 1)^{-1}$ and the line $x = 1$. The point A is the minimum point on the curve.

(i) Show that the x-coordinate of A satisfies the equation $2(x + 1)^3 = 1$ and find the exact value of $\frac{d^2y}{dx^2}$ at A. [5]

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(ii) Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x-axis. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

2969009541

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

Cambridge Assessment International Education

1 Use the trapezium rule with 3 intervals to estimate the value of

$$\int_0^3 |2^x - 4| \, dx.$$ [3]

2 Showing all necessary working, solve the equation $\ln(2x - 3) = 2\ln x - \ln(x - 1)$. Give your answer correct to 2 decimal places. [4]

3 Find the gradient of the curve $x^3 + 3xy^2 - y^3 = 1$ at the point with coordinates $(1, 3)$. [4]

4 By first expressing the equation $\cot\theta - \cot(\theta + 45°) = 3$ as a quadratic equation in $\tan\theta$, solve the equation for $0° < \theta < 180°$. [6]

5 **(i)** Differentiate $\dfrac{1}{\sin^2\theta}$ with respect to θ. [2]

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(ii) The variables x and θ satisfy the differential equation

$$x\tan\theta\frac{dx}{d\theta} + \operatorname{cosec}^2\theta = 0,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 4$ when $\theta = \frac{1}{6}\pi$. Solve the differential equation, obtaining an expression for x in terms of θ. [6]

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6 **(i)** By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3\sin x - 4\sin^3 x$. [4]

(ii) Hence, showing all necessary working, find the exact value of $\int_0^{\frac{1}{3}\pi} \sin^3 x \, dx$. [4]

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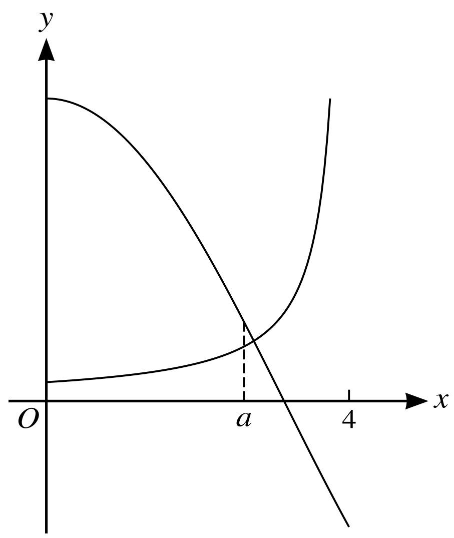

The diagram shows the curves $y = 4\cos\frac{1}{2}x$ and $y = \dfrac{1}{4-x}$, for $0 \leqslant x < 4$. When $x = a$, the tangents to the curves are perpendicular.

(i) Show that $a = 4 - \sqrt{(2\sin\frac{1}{2}a)}$. [4]

(ii) Verify by calculation that a lies between 2 and 3. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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8 Let $f(x) = \dfrac{16 - 17x}{(2 + x)(3 - x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

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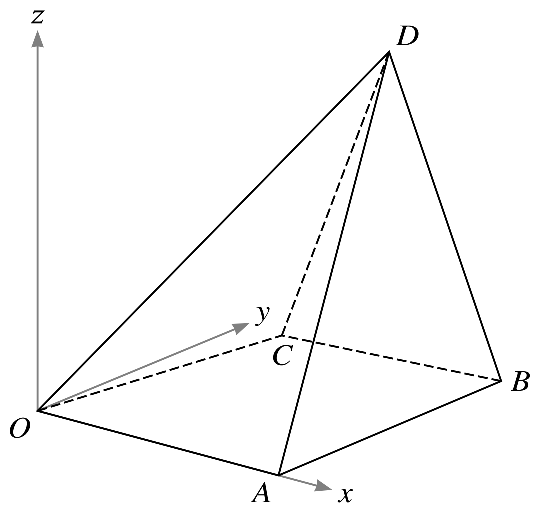

The diagram shows a set of rectangular axes Ox, Oy and Oz, and four points A, B, C and D with position vectors $\overrightarrow{OA} = 3\mathbf{i}$, $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = \mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$.

(i) Find the equation of the plane BCD, giving your answer in the form $ax + by + cz = d$. [6]

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(ii) Calculate the acute angle between the planes BCD and $OABC$. [4]

10 Throughout this question the use of a calculator is not permitted.

The complex number $(\sqrt{3}) + \mathrm{i}$ is denoted by u.

(i) Express u in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. Hence or otherwise state the exact values of the modulus and argument of u^4. [5]

(ii) Verify that u is a root of the equation $z^3 - 8z + 8\sqrt{3} = 0$ and state the other complex root of this equation. [3]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leqslant 2$ and $\operatorname{Im} z \geqslant 2$, where $\operatorname{Im} z$ denotes the imaginary part of z. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6271108665

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

JC19 06_9709_32/FP

Cambridge Assessment International Education

1 Find the coefficient of x^3 in the expansion of $(3 - x)(1 + 3x)^{\frac{1}{3}}$ in ascending powers of x. [4]

2 Showing all necessary working, solve the equation $9^x = 3^x + 12$. Give your answer correct to 2 decimal places. [4]

3 Showing all necessary working, solve the equation $\cot 2\theta = 2\tan\theta$ for $0° < \theta < 180°$. [5]

4 Find the exact coordinates of the point on the curve $y = \dfrac{x}{1 + \ln x}$ at which the gradient of the tangent is equal to $\frac{1}{4}$. [7]

5 **Throughout this question the use of a calculator is not permitted.**

It is given that the complex number $-1 + (\sqrt{3})\mathrm{i}$ is a root of the equation

$$kx^3 + 5x^2 + 10x + 4 = 0,$$

where k is a real constant.

(i) Write down another root of the equation. [1]

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(ii) Find the value of k and the third root of the equation. [6]

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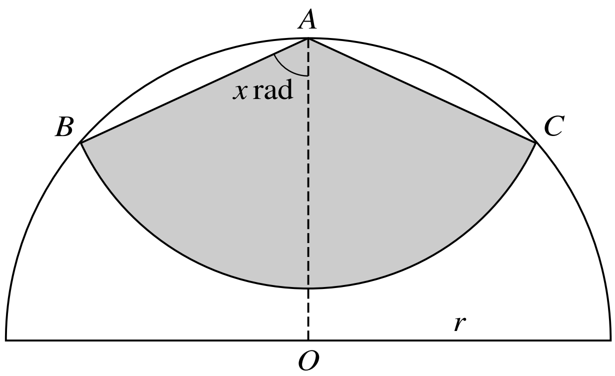

In the diagram, A is the mid-point of the semicircle with centre O and radius r. A circular arc with centre A meets the semicircle at B and C. The angle OAB is equal to x radians. The area of the shaded region bounded by AB, AC and the arc with centre A is equal to half the area of the semicircle.

(i) Use triangle OAB to show that $AB = 2r\cos x$. [1]

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(ii) Hence show that $x = \cos^{-1}\sqrt{\left(\dfrac{\pi}{16x}\right)}$. [2]

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(iii) Verify by calculation that x lies between 1 and 1.5. [2]

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(iv) Use an iterative formula based on the equation in part **(ii)** to determine x correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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7 The variables x and y satisfy the differential equation $\frac{dy}{dx} = xe^{x+y}$. It is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation, obtaining y in terms of x. [7]

(ii) Explain why x can only take values that are less than 1. [1]

8 Let $f(x) = \dfrac{10x + 9}{(2x + 1)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence show that $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \frac{1}{2}\ln\frac{9}{5} + \frac{1}{5}$. [5]

9 The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l does not intersect the line passing through A and B. [5]

(ii) The plane m is perpendicular to AB and passes through the mid-point of AB. The plane m intersects the line l at the point P. Find the equation of m and the position vector of P. [5]

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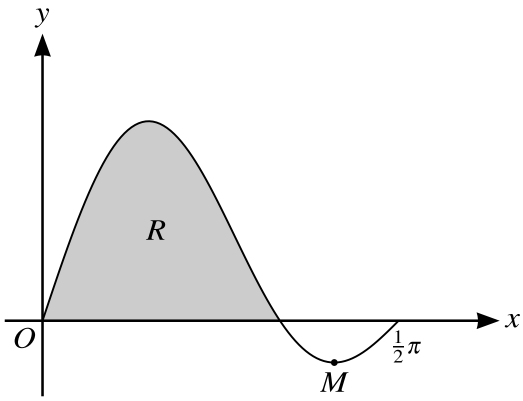

The diagram shows the curve $y = \sin 3x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$ and its minimum point M. The shaded region R is bounded by the curve and the x-axis.

(i) By expanding $\sin(3x + x)$ and $\sin(3x - x)$ show that

$$\sin 3x \cos x = \tfrac{1}{2}(\sin 4x + \sin 2x).$$ [3]

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(ii) Using the result of part **(i)** and showing all necessary working, find the exact area of the region R. [4]

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(iii) Using the result of part **(i)**, express $\frac{dy}{dx}$ in terms of $\cos 2x$ and hence find the x-coordinate of M, giving your answer correct to 2 decimal places. [5]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

3275076176

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **20** printed pages.

1 Use logarithms to solve the equation $5^{3-2x} = 4(7^x)$, giving your answer correct to 3 decimal places. [4]

2 Show that $\int_0^{\frac{1}{4}\pi} x^2 \cos 2x \, dx = \frac{1}{32}(\pi^2 - 8)$. [5]

3 Let $f(\theta) = \dfrac{1 - \cos 2\theta + \sin 2\theta}{1 + \cos 2\theta + \sin 2\theta}$.

(i) Show that $f(\theta) = \tan \theta$. [3]

(ii) Hence show that $\int_{\frac{1}{6}\pi}^{\frac{1}{4}\pi} \mathrm{f}(\theta)\,\mathrm{d}\theta = \frac{1}{2}\ln\frac{3}{2}$. [4]

4 The equation of a curve is $y = \dfrac{1 + e^{-x}}{1 - e^{-x}}$, for $x > 0$.

(i) Show that $\dfrac{dy}{dx}$ is always negative. [3]

(ii) The gradient of the curve is equal to -1 when $x = a$. Show that a satisfies the equation $e^{2a} - 4e^{a} + 1 = 0$. Hence find the exact value of a. [4]

5 The variables x and y satisfy the differential equation

$$(x+1)y\frac{\mathrm{d}y}{\mathrm{d}x} = y^2 + 5.$$

It is given that $y = 2$ when $x = 0$. Solve the differential equation obtaining an expression for y^2 in terms of x. [7]

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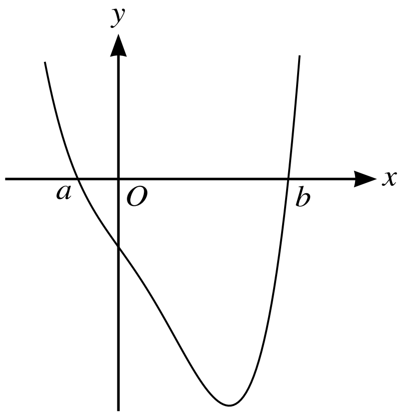

The diagram shows the curve $y = x^4 - 2x^3 - 7x - 6$. The curve intersects the x-axis at the points $(a, 0)$ and $(b, 0)$, where $a < b$. It is given that b is an integer.

(i) Find the value of b. [1]

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(ii) Hence show that a satisfies the equation $a = -\frac{1}{3}(2 + a^2 + a^3)$. [4]

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(iii) Use an iterative formula based on the equation in part **(ii)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

7 The curve $y = \sin\left(x + \frac{1}{3}\pi\right)\cos x$ has two stationary points in the interval $0 \leqslant x \leqslant \pi$.

(i) Find $\frac{dy}{dx}$. [2]

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(ii) By considering the formula for $\cos(A + B)$, show that, at the stationary points on the curve, $\cos\left(2x + \frac{1}{3}\pi\right) = 0$. [2]

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(iii) Hence find the exact x-coordinates of the stationary points. [3]

8 **Throughout this question the use of a calculator is not permitted.**

The complex number u is defined by

$$u = \frac{4\mathrm{i}}{1 - (\sqrt{3})\mathrm{i}}.$$

(i) Express u in the form $x + \mathrm{i}y$, where x and y are real and exact. [3]

(ii) Find the exact modulus and argument of u. [2]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| < 2$ and $|z - u| < |z|$. [4]

9 Let $f(x) = \dfrac{2x(5-x)}{(3+x)(1-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x up to and including the term in x^3. [5]

10 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} - 2\mathbf{k})$.

(i) The point P has position vector $4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$. Find the length of the perpendicular from P to l. [5]

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(ii) It is given that l lies in the plane with equation $ax + by + 2z = 13$, where a and b are constants. Find the values of a and b. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2016**

1 hour 45 minutes

0572757842

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC16 03_9709_32/RP

CAMBRIDGE International Examinations

[Turn over

1 Solve the equation $\ln(x^2 + 4) = 2\ln x + \ln 4$, giving your answer in an exact form. [3]

2 Express the equation $\tan(\theta + 45°) - 2\tan(\theta - 45°) = 4$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0° \leqslant \theta \leqslant 180°$. [6]

3 The equation $x^5 - 3x^3 + x^2 - 4 = 0$ has one positive root.

(i) Verify by calculation that this root lies between 1 and 2. [2]

(ii) Show that the equation can be rearranged in the form

$$x = \sqrt[3]{\left(3x + \frac{4}{x^2} - 1\right)}.$$ [1]

(iii) Use an iterative formula based on this rearrangement to determine the positive root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 The polynomial $4x^3 + ax + 2$, where a is a constant, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a. [2]

(ii) When a has this value,

(a) factorise $\mathrm{p}(x)$, [2]

(b) solve the inequality $\mathrm{p}(x) > 0$, justifying your answer. [3]

5 Let $I = \displaystyle\int_0^1 \frac{9}{(3 + x^2)^2}\,\mathrm{d}x$.

(i) Using the substitution $x = (\sqrt{3})\tan\theta$, show that $I = \sqrt{3}\displaystyle\int_0^{\frac{1}{6}\pi} \cos^2\theta\,\mathrm{d}\theta$. [3]

(ii) Hence find the exact value of I. [4]

6 A curve has equation

$$\sin y \ln x = x - 2\sin y,$$

for $-\frac{1}{2}\pi \leqslant y \leqslant \frac{1}{2}\pi$.

(i) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [5]

(ii) Hence find the exact x-coordinate of the point on the curve at which the tangent is parallel to the x-axis. [3]

7 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = xe^{x+y},$$

and it is given that $y = 0$ when $x = 0$.

(i) Solve the differential equation and obtain an expression for y in terms of x. [7]

(ii) Explain briefly why x can only take values less than 1. [1]

8 The line l has equation $\mathbf{r} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}$. The plane p has equation $\mathbf{r}.\begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix} = 6$.

(i) Show that l is parallel to p. [3]

(ii) A line m lies in the plane p and is perpendicular to l. The line m passes through the point with coordinates $(5, 3, 1)$. Find a vector equation for m. [6]

9 Let $f(x) = \dfrac{3x^3 + 6x - 8}{x(x^2 + 2)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x} + \dfrac{Cx + D}{x^2 + 2}$. [5]

(ii) Show that $\int_1^2 f(x)\,dx = 3 - \ln 4$. [5]

10 **(a)** Find the complex number z satisfying the equation $z^* + 1 = 2iz$, where z^* denotes the complex conjugate of z. Give your answer in the form $x + iy$, where x and y are real. [5]

(b) **(i)** On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z + 1 - 3i| \leqslant 1$ and $\operatorname{Im} z \geqslant 3$, where $\operatorname{Im} z$ denotes the imaginary part of z. [4]

(ii) Determine the difference between the greatest and least values of $\arg z$ for points lying in this region. [2]

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Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 8 4 5 0 9 4 7 7 1 7 *

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC17 03_9709_32/RP

1 Solve the equation $\ln(1 + 2^x) = 2$, giving your answer correct to 3 decimal places. [3]

2 Solve the inequality $|x - 4| < 2|3x + 1|$. [4]

3 **(i)** By sketching suitable graphs, show that the equation $e^{-\frac{1}{2}x} = 4 - x^2$ has one positive root and one negative root. [2]

(ii) Verify by calculation that the negative root lies between −1 and −1.5. [2]

(iii) Use the iterative formula $x_{n+1} = -\sqrt{\left(4 - e^{-\frac{1}{2}x_n}\right)}$ to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

4 **(i)** Express $8\cos\theta - 15\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the exact value of R and giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$8\cos 2x - 15\sin 2x = 4,$$

for $0° < x < 180°$. [4]

5 The curve with equation $y = e^{-ax} \tan x$, where a is a positive constant, has only one point in the interval $0 < x < \frac{1}{2}\pi$ at which the tangent is parallel to the x-axis. Find the value of a and state the exact value of the x-coordinate of this point. [7]

6 The line l has equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$. The plane p has equation $3x + y - 5z = 20$.

(i) Show that the line l lies in the plane p. [3]

(ii) A second plane is parallel to l, perpendicular to p and contains the point with position vector $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [5]

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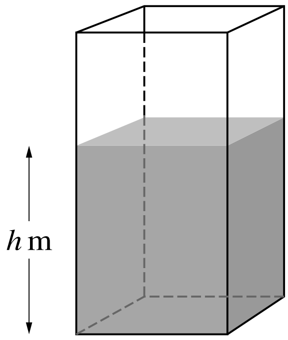

A water tank has vertical sides and a horizontal rectangular base, as shown in the diagram. The area of the base is $2\,\text{m}^2$. At time $t = 0$ the tank is empty and water begins to flow into it at a rate of $1\,\text{m}^3$ per hour. At the same time water begins to flow out from the base at a rate of $0.2\sqrt{h}\,\text{m}^3$ per hour, where h m is the depth of water in the tank at time t hours.

(i) Form a differential equation satisfied by h and t, and show that the time T hours taken for the depth of water to reach 4 m is given by

$$T = \int_0^4 \frac{10}{5 - \sqrt{h}}\,dh.$$ [3]

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(ii) Using the substitution $u = 5 - \sqrt{h}$, find the value of T. [6]

8 **Throughout this question the use of a calculator is not permitted.**

The polynomial $z^4 + 3z^2 + 6z + 10$ is denoted by $p(z)$. The complex number $-1 + i$ is denoted by u.

(i) Showing all your working, verify that u is a root of the equation $p(z) = 0$. [3]

(ii) Find the other three roots of the equation $p(z) = 0$. [7]

9 Let $f(x) = \dfrac{x(6-x)}{(2+x)(4+x^2)}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

[Turn over

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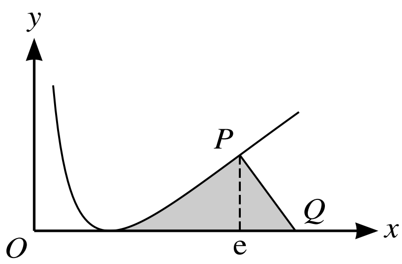

The diagram shows the curve $y = (\ln x)^2$. The x-coordinate of the point P is equal to e, and the normal to the curve at P meets the x-axis at Q.

(i) Find the x-coordinate of Q. [4]

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(ii) Show that $\int \ln x \, dx = x \ln x - x + c$, where c is a constant. [1]

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(iii) Using integration by parts, or otherwise, find the exact value of the area of the shaded region between the curve, the x-axis and the normal PQ. [5]

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Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

4755885761

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **February/March 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

JC19 03_9709_32/FP

Cambridge Assessment International Education

[Turn over

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1 **(i)** Show that the equation $\log_{10}(x-4) = 2 - \log_{10} x$ can be written as a quadratic equation in x. [3]

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(ii) Hence solve the equation $\log_{10}(x-4) = 2 - \log_{10} x$, giving your answer correct to 3 significant figures. [2]

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2 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{2x_n^6 + 12x_n}{3x_n^5 + 8},$$

with initial value $x_1 = 2$, converges to α.

(i) Use the formula to calculate α correct to 4 decimal places. Give the result of each iteration to 6 decimal places. [3]

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(ii) State an equation satisfied by α and hence find the exact value of α. [2]

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3 **(i)** Given that $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$, find the exact value of $\tan\theta$ in a form involving surds. You need not simplify your answer. [4]

(ii) Hence solve the equation $\sin(\theta + 45°) + 2\cos(\theta + 60°) = 3\cos\theta$ for $0° < \theta < 360°$. [2]

4 Show that $\int_1^4 x^{-\frac{3}{2}} \ln x \, dx = 2 - \ln 4.$ [5]

5 The variables x and y satisfy the relation $\sin y = \tan x$, where $-\frac{1}{2}\pi < y < \frac{1}{2}\pi$. Show that

$$\frac{dy}{dx} = \frac{1}{\cos x \sqrt{(\cos 2x)}}.$$ [5]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = ky^3e^{-x},$$

where k is a constant. It is given that $y = 1$ when $x = 0$, and that $y = \sqrt{e}$ when $x = 1$. Solve the differential equation, obtaining an expression for y in terms of x. [7]

7 (a) Showing all working and without using a calculator, solve the equation

$$(1 + i)z^2 - (4 + 3i)z + 5 + i = 0.$$

Give your answers in the form $x + iy$, where x and y are real. [6]

(b) The complex number u is given by

$$u = -1 - \mathrm{i}.$$

On a sketch of an Argand diagram show the point representing u. Shade the region whose points represent complex numbers satisfying the inequalities $|z| < |z - 2\mathrm{i}|$ and $\frac{1}{4}\pi < \arg(z - u) < \frac{1}{2}\pi$. [4]

8 Let $\mathrm{f}(x) = \dfrac{12 + 12x - 4x^2}{(2 + x)(3 - 2x)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 Two planes have equations $2x + 3y - z = 1$ and $x - 2y + z = 3$.

(i) Find the acute angle between the planes. [4]

(ii) Find a vector equation for the line of intersection of the planes. [6]

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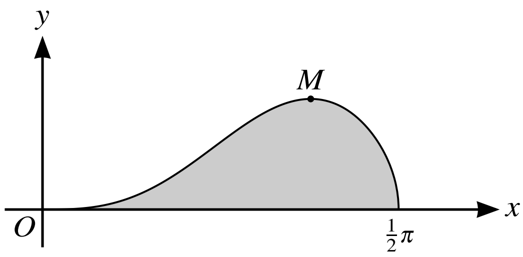

The diagram shows the curve $y = \sin^3 x \sqrt{(\cos x)}$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(i) Using the substitution $u = \cos x$, find by integration the exact area of the shaded region bounded by the curve and the x-axis. [6]

(ii) Showing all your working, find the x-coordinate of M, giving your answer correct to 3 decimal places. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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CAMBRIDGE INTERNATIONAL EXAMINATIONS

General Certificate of Education Advanced Subsidiary Level
General Certificate of Education Advanced Level

HIGHER MATHEMATICS **8719/3**

MATHEMATICS **9709/3**

PAPER 3 Pure Mathematics 3 **(P3)**

OCTOBER/NOVEMBER SESSION 2002

1 hour 45 minutes

Additional materials:
- Answer paper
- Graph paper
- List of Formulae (MF9)

TIME 1 hour 45 minutes

INSTRUCTIONS TO CANDIDATES

Write your name, Centre number and candidate number in the spaces provided on the answer paper/answer booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

This question paper consists of 4 printed pages.

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

1 Solve the inequality $|9 - 2x| < 1$. [3]

2 Find the exact value of $\int_1^2 x \ln x \, dx$. [4]

3 (i) Show that the equation

$$\log_{10}(x+5) = 2 - \log_{10} x$$

may be written as a quadratic equation in x. [3]

(ii) Hence find the value of x satisfying the equation

$$\log_{10}(x+5) = 2 - \log_{10} x.$$ [2]

4 The curve $y = e^x + 4e^{-2x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether the stationary point is a maximum or a minimum point. [2]

5 (i) Express $4 \sin \theta - 3 \cos \theta$ in the form $R \sin(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, stating the value of α correct to 2 decimal places. [3]

Hence

(ii) solve the equation

$$4 \sin \theta - 3 \cos \theta = 2,$$

giving all values of θ such that $0° < \theta < 360°$, [4]

(iii) write down the greatest value of $\dfrac{1}{4 \sin \theta - 3 \cos \theta + 6}$. [1]

6 Let $f(x) = \dfrac{6 + 7x}{(2 - x)(1 + x^2)}$.

(i) Express $f(x)$ in partial fractions. [4]

(ii) Show that, when x is sufficiently small for x^4 and higher powers to be neglected,

$$f(x) = 3 + 5x - \tfrac{1}{2}x^2 - \tfrac{15}{4}x^3.$$ [5]

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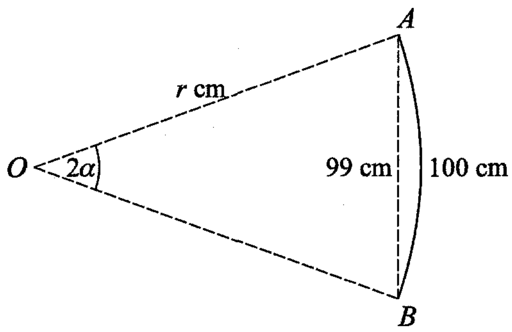

The diagram shows a curved rod AB of length 100 cm which forms an arc of a circle. The end points A and B of the rod are 99 cm apart. The circle has radius r cm and the arc AB subtends an angle of 2α radians at O, the centre of the circle.

(i) Show that α satisfies the equation $\frac{99}{100}x = \sin x$. [3]

(ii) Given that this equation has exactly one root in the interval $0 < x < \frac{1}{2}\pi$, verify by calculation that this root lies between 0.1 and 0.5. [2]

(iii) Show that if the sequence of values given by the iterative formula

$$x_{n+1} = 50\sin x_n - 48.5x_n$$

converges, then it converges to a root of the equation in part **(i)**. [2]

(iv) Use this iterative formula, with initial value $x_1 = 0.25$, to find α correct to 3 decimal places, showing the result of each iteration. [2]

8 **(a)** Find the two square roots of the complex number $-3 + 4\mathrm{i}$, giving your answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) The complex number z is given by

$$z = \frac{-1 + 3\mathrm{i}}{2 + \mathrm{i}}.$$

(i) Express z in the form $x + \mathrm{i}y$, where x and y are real. [2]

(ii) Show on a sketch of an Argand diagram, with origin O, the points A, B and C representing the complex numbers $-1 + 3\mathrm{i}$, $2 + \mathrm{i}$ and z respectively. [1]

(iii) State an equation relating the lengths OA, OB and OC. [1]

9 In an experiment to study the spread of a soil disease, an area of $10\,\text{m}^2$ of soil was exposed to infection. In a simple model, it is assumed that the infected area grows at a rate which is proportional to the product of the infected area and the uninfected area. Initially, $5\,\text{m}^2$ was infected and the rate of growth of the infected area was $0.1\,\text{m}^2$ per day. At time t days after the start of the experiment, an area $a\,\text{m}^2$ is infected and an area $(10 - a)\,\text{m}^2$ is uninfected.

(i) Show that $\dfrac{\mathrm{d}a}{\mathrm{d}t} = 0.004a(10 - a)$. [2]

(ii) By first expressing $\dfrac{1}{a(10 - a)}$ in partial fractions, solve this differential equation, obtaining an expression for t in terms of a. [6]

(iii) Find the time taken for 90% of the soil area to become infected, according to this model. [2]

10 With respect to the origin O, the points A, B, C, D have position vectors given by

$$\overrightarrow{OA} = 4\mathbf{i} + \mathbf{k}, \quad \overrightarrow{OB} = 5\mathbf{i} - 2\mathbf{j} - 2\mathbf{k}, \quad \overrightarrow{OC} = \mathbf{i} + \mathbf{j}, \quad \overrightarrow{OD} = -\mathbf{i} - 4\mathbf{k}.$$

(i) Calculate the acute angle between the lines AB and CD. [4]

(ii) Prove that the lines AB and CD intersect. [4]

(iii) The point P has position vector $\mathbf{i} + 5\mathbf{j} + 6\mathbf{k}$. Show that the perpendicular distance from P to the line AB is equal to $\sqrt{3}$. [4]

CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2003

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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The total number of marks for this paper is 75.
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The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
Local Examinations Syndicate

1 Solve the inequality $|2^x - 8| < 5$. [4]

2 Expand $(2 + x^2)^{-2}$ in ascending powers of x, up to and including the term in x^4, simplifying the coefficients. [4]

3 Solve the equation

$$\cos\theta + 3\cos 2\theta = 2,$$

giving all solutions in the interval $0° \leqslant \theta \leqslant 180°$. [5]

4 The equation of a curve is

$$\sqrt{x} + \sqrt{y} = \sqrt{a},$$

where a is a positive constant.

(i) Express $\frac{dy}{dx}$ in terms of x and y. [3]

(ii) The straight line with equation $y = x$ intersects the curve at the point P. Find the equation of the tangent to the curve at P. [3]

5 **(i)** By sketching suitable graphs, show that the equation

$$\sec x = 3 - x^2$$

has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

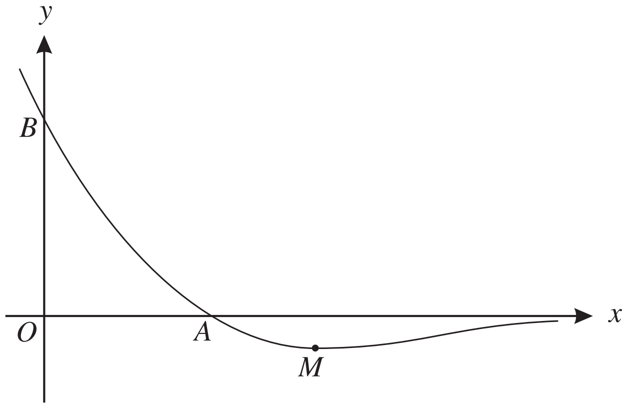
(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \cos^{-1}\left(\frac{1}{3 - x_n^2}\right)$$

converges, then it converges to a root of the equation given in part **(i)**. [2]

(iii) Use this iterative formula, with initial value $x_1 = 1$, to determine the root in the interval $0 < x < \frac{1}{2}\pi$ correct to 2 decimal places, showing the result of each iteration. [3]

6



The diagram shows the curve $y = (3 - x)e^{-2x}$ and its minimum point M. The curve intersects the x-axis at A and the y-axis at B.

(i) Calculate the x-coordinate of M. [4]

(ii) Find the area of the region bounded by OA, OB and the curve, giving your answer in terms of e. [5]

7 The complex number u is given by $u = \frac{7 + 4i}{3 - 2i}$.

(i) Express u in the form $x + iy$, where x and y are real. [3]

(ii) Sketch an Argand diagram showing the point representing the complex number u. Show on the same diagram the locus of the complex number z such that $|z - u| = 2$. [3]

(iii) Find the greatest value of $\arg z$ for points on this locus. [3]

8 Let $f(x) = \frac{x^3 - x - 2}{(x - 1)(x^2 + 1)}$.

(i) Express $f(x)$ in the form

$$A + \frac{B}{x - 1} + \frac{Cx + D}{x^2 + 1},$$

where A, B, C and D are constants. [5]

(ii) Hence show that $\int_2^3 f(x)\,dx = 1$. [4]

9 Compressed air is escaping from a container. The pressure of the air in the container at time t is P, and the constant atmospheric pressure of the air outside the container is A. The rate of decrease of P is proportional to the square root of the pressure difference $(P - A)$. Thus the differential equation connecting P and t is

$$\frac{dP}{dt} = -k\sqrt{(P - A)},$$

where k is a positive constant.

(i) Find, in any form, the general solution of this differential equation. [3]

(ii) Given that $P = 5A$ when $t = 0$, and that $P = 2A$ when $t = 2$, show that $k = \sqrt{A}$. [4]

(iii) Find the value of t when $P = A$. [2]

(iv) Obtain an expression for P in terms of A and t. [2]

10 The lines l and m have vector equations

$$\mathbf{r} = \mathbf{i} - 2\mathbf{k} + s(2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = 6\mathbf{i} - 5\mathbf{j} + 4\mathbf{k} + t(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$$

respectively.

(i) Show that l and m intersect, and find the position vector of their point of intersection. [5]

(ii) Find the equation of the plane containing l and m, giving your answer in the form $ax + by + cz = d$. [6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2004

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
At the end of the examination, fasten all your work securely together.

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You are reminded of the need for clear presentation in your answers.

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UNIVERSITY *of* CAMBRIDGE International Examinations

1 Expand $\frac{1}{(2+x)^3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Solve the equation

$$\ln(1+x) = 1 + \ln x,$$

giving your answer correct to 2 significant figures. [4]

3 The polynomial $2x^3 + ax^2 - 4$ is denoted by $p(x)$. It is given that $(x-2)$ is a factor of $p(x)$.

(i) Find the value of a. [2]

When a has this value,

(ii) factorise $p(x)$, [2]

(iii) solve the inequality $p(x) > 0$, justifying your answer. [2]

4 **(i)** Show that the equation

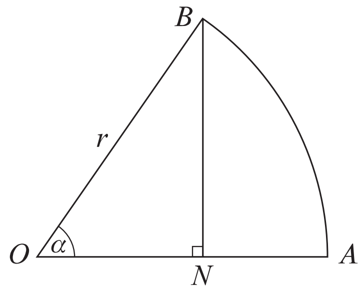
$$\tan(45^\circ + x) = 2\tan(45^\circ - x)$$

can be written in the form

$$\tan^2 x - 6\tan x + 1 = 0.$$ [4]

(ii) Hence solve the equation $\tan(45^\circ + x) = 2\tan(45^\circ - x)$, for $0^\circ < x < 90^\circ$. [3]

5



The diagram shows a sector OAB of a circle with centre O and radius r. The angle AOB is α radians, where $0 < \alpha < \frac{1}{2}\pi$. The point N on OA is such that BN is perpendicular to OA. The area of the triangle ONB is half the area of the sector OAB.

(i) Show that α satisfies the equation $\sin 2x = x$. [3]

(ii) By sketching a suitable pair of graphs, show that this equation has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sin(2x_n),$$

with initial value $x_1 = 1$, to find α correct to 2 decimal places, showing the result of each iteration. [3]

6 The complex numbers $1 + 3\mathrm{i}$ and $4 + 2\mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, where x and y are real, the complex numbers $u - v$ and $\dfrac{u}{v}$. [3]

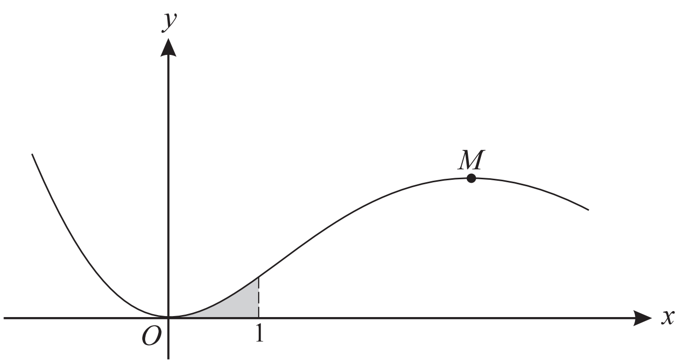
(ii) State the argument of $\dfrac{u}{v}$. [1]

In an Argand diagram, with origin O, the points A, B and C represent the numbers u, v and $u - v$ respectively.

(iii) State fully the geometrical relationship between OC and BA. [2]

(iv) Prove that angle $AOB = \frac{1}{4}\pi$ radians. [2]

7



The diagram shows the curve $y = x^2 e^{-\frac{1}{2}x}$.

(i) Find the x-coordinate of M, the maximum point of the curve. [4]

(ii) Find the area of the shaded region enclosed by the curve, the x-axis and the line $x = 1$, giving your answer in terms of e. [5]

8 An appropriate form for expressing $\dfrac{3x}{(x+1)(x-2)}$ in partial fractions is

$$\frac{A}{x+1} + \frac{B}{x-2},$$

where A and B are constants.

(a) Without evaluating any constants, state appropriate forms for expressing the following in partial fractions:

(i) $\dfrac{4x}{(x+4)(x^2+3)}$, [1]

(ii) $\dfrac{2x+1}{(x-2)(x+2)^2}$. [2]

(b) Show that $\displaystyle\int_3^4 \frac{3x}{(x+1)(x-2)}\,dx = \ln 5$. [6]

9 The lines l and m have vector equations

$$\mathbf{r} = 2\mathbf{i} - \mathbf{j} + 4\mathbf{k} + s(\mathbf{i} + \mathbf{j} - \mathbf{k}) \qquad \text{and} \qquad \mathbf{r} = -2\mathbf{i} + 2\mathbf{j} + \mathbf{k} + t(-2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

respectively.

(i) Show that l and m do not intersect. [4]

The point P lies on l and the point Q has position vector $2\mathbf{i} - \mathbf{k}$.

(ii) Given that the line PQ is perpendicular to l, find the position vector of P. [4]

(iii) Verify that Q lies on m and that PQ is perpendicular to m. [2]

10 A rectangular reservoir has a horizontal base of area $1000\,\text{m}^2$. At time $t = 0$, it is empty and water begins to flow into it at a constant rate of $30\,\text{m}^3\,\text{s}^{-1}$. At the same time, water begins to flow out at a rate proportional to $\sqrt{h}$, where h m is the depth of the water at time t s. When $h = 1$, $\frac{dh}{dt} = 0.02$.

(i) Show that h satisfies the differential equation

$$\frac{dh}{dt} = 0.01(3 - \sqrt{h}).$$ [3]

It is given that, after making the substitution $x = 3 - \sqrt{h}$, the equation in part **(i)** becomes

$$(x - 3)\frac{dx}{dt} = 0.005x.$$

(ii) Using the fact that $x = 3$ when $t = 0$, solve this differential equation, obtaining an expression for t in terms of x. [5]

(iii) Find the time at which the depth of water reaches 4 m. [2]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

HIGHER MATHEMATICS **8719/03**

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2005

1 hour 45 minutes

Additional materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

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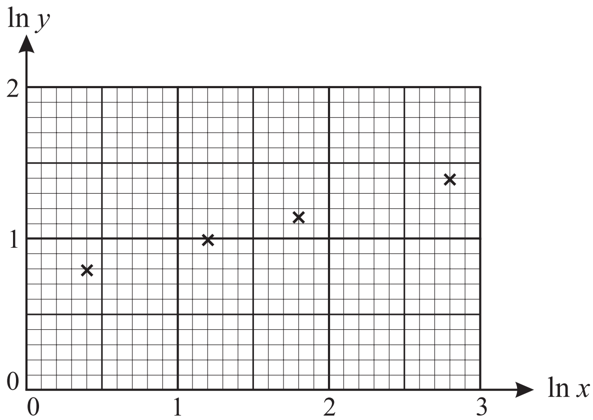
UNIVERSITY *of* CAMBRIDGE International Examinations

1 Given that a is a positive constant, solve the inequality

$$|x - 3a| > |x - a|.$$ [4]

2



Two variable quantities x and y are related by the equation $y = Ax^n$, where A and n are constants. The diagram shows the result of plotting $\ln y$ against $\ln x$ for four pairs of values of x and y. Use the diagram to estimate the values of A and n. [5]

3 The equation of a curve is $y = x + \cos 2x$. Find the x-coordinates of the stationary points of the curve for which $0 \leqslant x \leqslant \pi$, and determine the nature of each of these stationary points. [7]

4 The equation $x^3 - x - 3 = 0$ has one real root, α.

(i) Show that α lies between 1 and 2. [2]

Two iterative formulae derived from this equation are as follows:

$$x_{n+1} = x_n^3 - 3, \qquad (A)$$

$$x_{n+1} = (x_n + 3)^{\frac{1}{3}}. \qquad (B)$$

Each formula is used with initial value $x_1 = 1.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [5]

5 By expressing $8 \sin \theta - 6 \cos \theta$ in the form $R \sin(\theta - \alpha)$, solve the equation

$$8 \sin \theta - 6 \cos \theta = 7,$$

for $0° \leqslant \theta \leqslant 360°$. [7]

6 **(i)** Use the substitution $x = \sin^2\theta$ to show that

$$\int \sqrt{\left(\frac{x}{1-x}\right)}\,\mathrm{d}x = \int 2\sin^2\theta\,\mathrm{d}\theta. \qquad [4]$$

(ii) Hence find the exact value of

$$\int_0^{\frac{1}{4}} \sqrt{\left(\frac{x}{1-x}\right)}\,\mathrm{d}x. \qquad [4]$$

7 The equation $2x^3 + x^2 + 25 = 0$ has one real root and two complex roots.

(i) Verify that $1 + 2\mathrm{i}$ is one of the complex roots. [3]

(ii) Write down the other complex root of the equation. [1]

(iii) Sketch an Argand diagram showing the point representing the complex number $1 + 2\mathrm{i}$. Show on the same diagram the set of points representing the complex numbers z which satisfy

$$|z| = |z - 1 - 2\mathrm{i}|. \qquad [4]$$

8 In a certain chemical reaction the amount, x grams, of a substance present is decreasing. The rate of decrease of x is proportional to the product of x and the time, t seconds, since the start of the reaction. Thus x and t satisfy the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -kxt,$$

where k is a positive constant. At the start of the reaction, when $t = 0$, $x = 100$.

(i) Solve this differential equation, obtaining a relation between x, k and t. [5]

(ii) 20 seconds after the start of the reaction the amount of substance present is 90 grams. Find the time after the start of the reaction at which the amount of substance present is 50 grams. [3]

9 **(i)** Express $\dfrac{3x^2 + x}{(x+2)(x^2+1)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{3x^2 + x}{(x+2)(x^2+1)}$ in ascending powers of x, up to and including the term in x^3. [5]

10 The straight line l passes through the points A and B with position vectors

$$2\mathbf{i} + 2\mathbf{j} + \mathbf{k} \qquad \text{and} \qquad \mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$$

respectively. This line intersects the plane p with equation $x - 2y + 2z = 6$ at the point C.

(i) Find the position vector of C. [4]

(ii) Find the acute angle between l and p. [4]

(iii) Show that the perpendicular distance from A to p is equal to 2. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)**

October/November 2006

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

1 Find the set of values of x satisfying the inequality $|3^x - 8| < 0.5$, giving 3 significant figures in your answer. [4]

2 Solve the equation

$$\tan x \tan 2x = 1,$$

giving all solutions in the interval $0° < x < 180°$. [4]

3 The curve with equation $y = 6e^x - e^{3x}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

4 Given that $y = 2$ when $x = 0$, solve the differential equation

$$y\frac{dy}{dx} = 1 + y^2,$$

obtaining an expression for y^2 in terms of x. [6]

5 **(i)** Simplify $\left(\sqrt{(1+x)} + \sqrt{(1-x)}\right)\left(\sqrt{(1+x)} - \sqrt{(1-x)}\right)$, showing your working, and deduce that

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}} = \frac{\sqrt{(1+x)} - \sqrt{(1-x)}}{2x}.$$ [2]

(ii) Using this result, or otherwise, obtain the expansion of

$$\frac{1}{\sqrt{(1+x)} + \sqrt{(1-x)}}$$

in ascending powers of x, up to and including the term in x^2. [4]

6 The equation of a curve is $x^3 + 2y^3 = 3xy$.

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{2y^2 - x}$. [4]

(ii) Find the coordinates of the point, other than the origin, where the curve has a tangent which is parallel to the x-axis. [5]

7 The line l has equation $\mathbf{r} = \mathbf{j} + \mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation $x + 2y + 3z = 5$.

(i) Show that the line l lies in the plane p. [3]

(ii) A second plane is perpendicular to the plane p, parallel to the line l and contains the point with position vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$. Find the equation of this plane, giving your answer in the form $ax + by + cz = d$. [6]

8 Let $\mathrm{f}(x) = \dfrac{7x + 4}{(2x + 1)(x + 1)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence show that $\displaystyle\int_0^2 \mathrm{f}(x)\,\mathrm{d}x = 2 + \ln \tfrac{5}{3}$. [5]

9 The complex number u is given by

$$u = \frac{3 + \mathrm{i}}{2 - \mathrm{i}}.$$

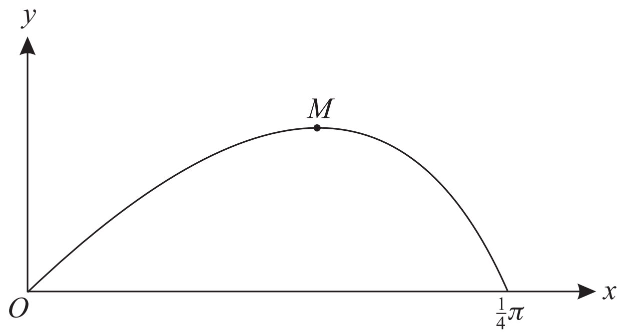
(i) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

(ii) Find the modulus and argument of u. [2]

(iii) Sketch an Argand diagram showing the point representing the complex number u. Show on the same diagram the locus of the point representing the complex number z such that $|z - u| = 1$. [3]

(iv) Using your diagram, calculate the least value of $|z|$ for points on this locus. [2]

10



The diagram shows the curve $y = x\cos 2x$ for $0 \leqslant x \leqslant \frac{1}{4}\pi$. The point M is a maximum point.

(i) Show that the x-coordinate of M satisfies the equation $1 = 2x\tan 2x$. [3]

(ii) The equation in part **(i)** can be rearranged in the form $x = \dfrac{1}{2}\tan^{-1}\left(\dfrac{1}{2x}\right)$. Use the iterative formula

$$x_{n+1} = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x_n}\right),$$

with initial value $x_1 = 0.4$, to calculate the x-coordinate of M correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

(iii) Use integration by parts to find the exact area of the region enclosed between the curve and the x-axis from 0 to $\frac{1}{4}\pi$. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2007**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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1 Find the exact value of the constant k for which $\displaystyle\int_1^k \frac{1}{2x-1}\,dx = 1$. [4]

2 The polynomial $x^4 + 3x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $x^2 + x + 2$ is a factor of $p(x)$. Find the value of a and the other quadratic factor of $p(x)$. [4]

3 Use integration by parts to show that

$$\int_2^4 \ln x\,dx = 6\ln 2 - 2. \quad [4]$$

4 The curve with equation $y = e^{-x}\sin x$ has one stationary point for which $0 \leqslant x \leqslant \pi$.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

5 **(i)** Show that the equation

$$\tan(45^\circ + x) - \tan x = 2$$

can be written in the form

$$\tan^2 x + 2\tan x - 1 = 0. \quad [3]$$

(ii) Hence solve the equation

$$\tan(45^\circ + x) - \tan x = 2,$$

giving all solutions in the interval $0^\circ \leqslant x \leqslant 180^\circ$. [4]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$2 - x = \ln x$$

has only one root. [2]

(ii) Verify by calculation that this root lies between 1.4 and 1.7. [2]

(iii) Show that this root also satisfies the equation

$$x = \tfrac{1}{3}(4 + x - 2\ln x). \quad [1]$$

(iv) Use the iterative formula

$$x_{n+1} = \tfrac{1}{3}(4 + x_n - 2\ln x_n),$$

with initial value $x_1 = 1.5$, to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The number of insects in a population t days after the start of observations is denoted by N. The variation in the number of insects is modelled by a differential equation of the form

$$\frac{dN}{dt} = kN\cos(0.02t),$$

where k is a constant and N is taken to be a continuous variable. It is given that $N = 125$ when $t = 0$.

(i) Solve the differential equation, obtaining a relation between N, k and t. [5]

(ii) Given also that $N = 166$ when $t = 30$, find the value of k. [2]

(iii) Obtain an expression for N in terms of t, and find the least value of N predicted by this model. [3]

8 **(a)** The complex number z is given by $z = \frac{4 - 3i}{1 - 2i}$.

(i) Express z in the form $x + iy$, where x and y are real. [2]

(ii) Find the modulus and argument of z. [2]

(b) Find the two square roots of the complex number $5 - 12i$, giving your answers in the form $x + iy$, where x and y are real. [6]

9 **(i)** Express $\frac{2 - x + 8x^2}{(1-x)(1+2x)(2+x)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{2 - x + 8x^2}{(1-x)(1+2x)(2+x)}$ in ascending powers of x, up to and including the term in x^2. [5]

10 The straight line l has equation $\mathbf{r} = \mathbf{i} + 6\mathbf{j} - 3\mathbf{k} + s(\mathbf{i} - 2\mathbf{j} + 2\mathbf{k})$. The plane p has equation $(\mathbf{r} - 3\mathbf{i}).(2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) = 0$. The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Find the acute angle between l and p. [4]

(iii) Find a vector equation for the line which lies in p, passes through A and is perpendicular to l. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2008**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

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This document consists of **4** printed pages.

UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

1 Solve the equation

$$\ln(x + 2) = 2 + \ln x,$$

giving your answer correct to 3 decimal places. [3]

2 Expand $(1 + x)\sqrt{(1 - 2x)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

3 The curve $y = \dfrac{e^x}{\cos x}$, for $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$, has one stationary point. Find the x-coordinate of this point. [5]

4 The parametric equations of a curve are

$$x = a(2\theta - \sin 2\theta), \qquad y = a(1 - \cos 2\theta).$$

Show that $\dfrac{dy}{dx} = \cot\theta$. [5]

5 The polynomial $4x^3 - 4x^2 + 3x + a$, where a is a constant, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $2x^2 - 3x + 3$.

(i) Find the value of a. [3]

(ii) When a has this value, solve the inequality $p(x) < 0$, justifying your answer. [3]

6 **(i)** Express $5\sin x + 12\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation

$$5\sin 2\theta + 12\cos 2\theta = 11,$$

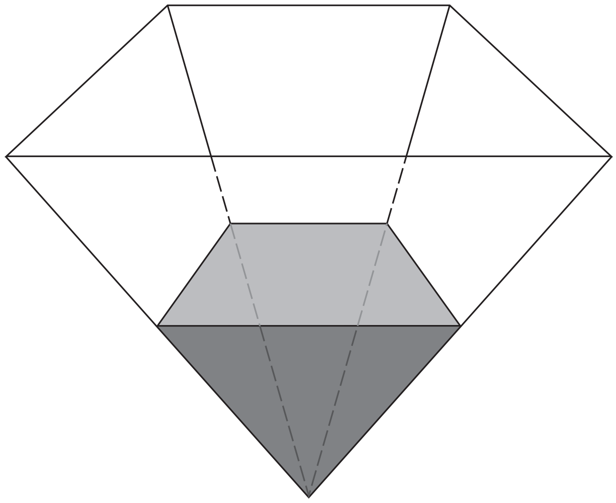
giving all solutions in the interval $0° < \theta < 180°$. [5]

7 Two planes have equations $2x - y - 3z = 7$ and $x + 2y + 2z = 0$.

(i) Find the acute angle between the planes. [4]

(ii) Find a vector equation for their line of intersection. [6]

8



An underground storage tank is being filled with liquid as shown in the diagram. Initially the tank is empty. At time t hours after filling begins, the volume of liquid is V m^3 and the depth of liquid is h m. It is given that $V = \frac{4}{3}h^3$.

The liquid is poured in at a rate of 20 m^3 per hour, but owing to leakage, liquid is lost at a rate proportional to h^2. When $h = 1$, $\frac{dh}{dt} = 4.95$.

(i) Show that h satisfies the differential equation

$$\frac{dh}{dt} = \frac{5}{h^2} - \frac{1}{20}.$$ [4]

(ii) Verify that $\frac{20h^2}{100 - h^2} \equiv -20 + \frac{2000}{(10-h)(10+h)}$. [1]

(iii) Hence solve the differential equation in part **(i)**, obtaining an expression for t in terms of h. [5]

9 The constant a is such that $\int_0^a x e^{\frac{1}{2}x}\, dx = 6$.

(i) Show that a satisfies the equation

$$x = 2 + e^{-\frac{1}{2}x}.$$ [5]

(ii) By sketching a suitable pair of graphs, show that this equation has only one root. [2]

(iii) Verify by calculation that this root lies between 2 and 2.5. [2]

(iv) Use an iterative formula based on the equation in part **(i)** to calculate the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

10 The complex number w is given by $w = -\frac{1}{2} + \mathrm{i}\frac{\sqrt{3}}{2}$.

(i) Find the modulus and argument of w. [2]

(ii) The complex number z has modulus R and argument θ, where $-\frac{1}{3}\pi < \theta < \frac{1}{3}\pi$. State the modulus and argument of wz and the modulus and argument of $\frac{z}{w}$. [4]

(iii) Hence explain why, in an Argand diagram, the points representing z, wz and $\frac{z}{w}$ are the vertices of an equilateral triangle. [2]

(iv) In an Argand diagram, the vertices of an equilateral triangle lie on a circle with centre at the origin. One of the vertices represents the complex number $4 + 2\mathrm{i}$. Find the complex numbers represented by the other two vertices. Give your answers in the form $x + \mathrm{i}y$, where x and y are real and exact. [4]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2009**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5226711904

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1 Solve the inequality $2 - 3x < |x - 3|$. [4]

2 Solve the equation $3^{x+2} = 3^x + 3^2$, giving your answer correct to 3 significant figures. [4]

3 The sequence of values given by the iterative formula

$$x_{n+1} = \frac{3x_n}{4} + \frac{15}{x_n^3},$$

with initial value $x_1 = 3$, converges to α.

(i) Use this iterative formula to find α correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

(ii) State an equation satisfied by α and hence find the exact value of α. [2]

4 A curve has equation $y = e^{-3x} \tan x$. Find the x-coordinates of the stationary points on the curve in the interval $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$. Give your answers correct to 3 decimal places. [6]

5 **(i)** Prove the identity $\cos 4\theta - 4\cos 2\theta + 3 \equiv 8\sin^4\theta$. [4]

(ii) Using this result find, in simplified form, the exact value of

$$\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \sin^4\theta \, d\theta.$$ [4]

6 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \mathbf{i} - \mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}.$$

The mid-point of AB is M. The point N lies on AC between A and C and is such that $AN = 2NC$.

(i) Find a vector equation of the line MN. [4]

(ii) It is given that MN intersects BC at the point P. Find the position vector of P. [4]

7 The complex number $-2 + i$ is denoted by u.

(i) Given that u is a root of the equation $x^3 - 11x - k = 0$, where k is real, find the value of k. [3]

(ii) Write down the other complex root of this equation. [1]

(iii) Find the modulus and argument of u. [2]

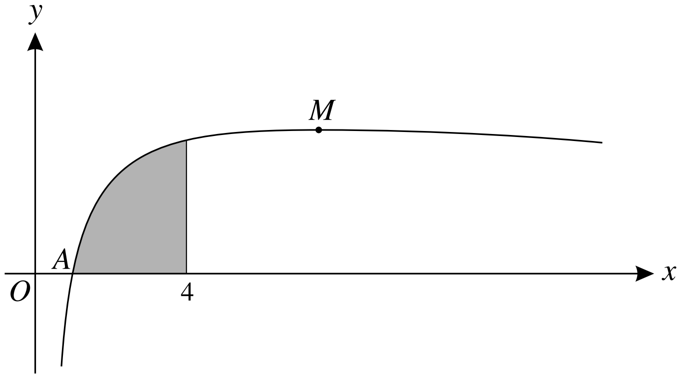
(iv) Sketch an Argand diagram showing the point representing u. Shade the region whose points represent the complex numbers z satisfying both the inequalities

$$|z| < |z - 2| \quad \text{and} \quad 0 < \arg(z - u) < \tfrac{1}{4}\pi.$$ [4]

8 **(i)** Express $\dfrac{5x+3}{(x+1)^2(3x+2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{5x+3}{(x+1)^2(3x+2)}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [5]

9



The diagram shows the curve $y = \dfrac{\ln x}{\sqrt{x}}$ and its maximum point M. The curve cuts the x-axis at the point A.

(i) State the coordinates of A. [1]

(ii) Find the exact value of the x-coordinate of M. [4]

(iii) Using integration by parts, show that the area of the shaded region bounded by the curve, the x-axis and the line $x = 4$ is equal to $8\ln 2 - 4$. [5]

10 In a model of the expansion of a sphere of radius r cm, it is assumed that, at time t seconds after the start, the rate of increase of the surface area of the sphere is proportional to its volume. When $t = 0$, $r = 5$ and $\dfrac{dr}{dt} = 2$.

(i) Show that r satisfies the differential equation

$$\frac{dr}{dt} = 0.08r^2.$$ [4]

[The surface area A and volume V of a sphere of radius r are given by the formulae $A = 4\pi r^2$, $V = \frac{4}{3}\pi r^3$.]

(ii) Solve this differential equation, obtaining an expression for r in terms of t. [5]

(iii) Deduce from your answer to part **(ii)** the set of values that t can take, according to this model. [1]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2009**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 Solve the equation

$$\ln(5 - x) = \ln 5 - \ln x,$$

giving your answers correct to 3 significant figures. [4]

2 The equation $x^3 - 8x - 13 = 0$ has one real root.

(i) Find the two consecutive integers between which this root lies. [2]

(ii) Use the iterative formula

$$x_{n+1} = (8x_n + 13)^{\frac{1}{3}}$$

to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

3 The equation of a curve is $x^3 - x^2y - y^3 = 3$.

(i) Find $\frac{dy}{dx}$ in terms of x and y. [4]

(ii) Find the equation of the tangent to the curve at the point (2, 1), giving your answer in the form $ax + by + c = 0$. [2]

4 The angles α and β lie in the interval $0° < x < 180°$, and are such that

$$\tan \alpha = 2 \tan \beta \quad \text{and} \quad \tan(\alpha + \beta) = 3.$$

Find the possible values of α and β. [6]

5 The polynomial $2x^3 + ax^2 + bx - 4$, where a and b are constants, is denoted by $\mathrm{p}(x)$. The result of differentiating $\mathrm{p}(x)$ with respect to x is denoted by $\mathrm{p}'(x)$. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$ and of $\mathrm{p}'(x)$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $\mathrm{p}(x)$ completely. [3]

6 **(i)** Use the substitution $x = 2 \tan \theta$ to show that

$$\int_0^2 \frac{8}{(4 + x^2)^2}\, dx = \int_0^{\frac{1}{4}\pi} \cos^2 \theta \, d\theta. \quad [4]$$

(ii) Hence find the exact value of

$$\int_0^2 \frac{8}{(4 + x^2)^2}\, dx. \quad [4]$$

7 The complex numbers $-2 + \mathrm{i}$ and $3 + \mathrm{i}$ are denoted by u and v respectively.

(i) Find, in the form $x + \mathrm{i}y$, the complex numbers

(a) $u + v$, [1]

(b) $\frac{u}{v}$, showing all your working. [3]

(ii) State the argument of $\frac{u}{v}$. [1]

In an Argand diagram with origin O, the points A, B and C represent the complex numbers u, v and $u + v$ respectively.

(iii) Prove that angle $AOB = \frac{3}{4}\pi$. [2]

(iv) State fully the geometrical relationship between the line segments OA and BC. [2]

8 **(i)** Express $\frac{1 + x}{(1 - x)(2 + x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\frac{1 + x}{(1 - x)(2 + x^2)}$ in ascending powers of x, up to and including the term in x^2. [5]

9 The temperature of a quantity of liquid at time t is θ. The liquid is cooling in an atmosphere whose temperature is constant and equal to A. The rate of decrease of θ is proportional to the temperature difference $(\theta - A)$. Thus θ and t satisfy the differential equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}t} = -k(\theta - A),$$

where k is a positive constant.

(i) Find, in any form, the solution of this differential equation, given that $\theta = 4A$ when $t = 0$. [5]

(ii) Given also that $\theta = 3A$ when $t = 1$, show that $k = \ln \frac{3}{2}$. [1]

(iii) Find θ in terms of A when $t = 2$, expressing your answer in its simplest form. [3]

10 The plane p has equation $2x - 3y + 6z = 16$. The plane q is parallel to p and contains the point with position vector $\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.

(i) Find the equation of q, giving your answer in the form $ax + by + cz = d$. [2]

(ii) Calculate the perpendicular distance between p and q. [3]

(iii) The line l is parallel to the plane p and also parallel to the plane with equation $x - 2y + 2z = 5$. Given that l passes through the origin, find a vector equation for l. [5]

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General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2010**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

3454847368

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1 Solve the inequality $2|x-3| > |3x+1|$. [4]

2 Solve the equation

$$\ln(1+x^2) = 1 + 2\ln x,$$

giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$\cos(\theta + 60^\circ) = 2\sin\theta,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$. [5]

4 **(i)** By sketching suitable graphs, show that the equation

$$4x^2 - 1 = \cot x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.6 and 1. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{2}\sqrt{(1 + \cot x_n)}$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 Let $I = \displaystyle\int_0^1 \frac{x^2}{\sqrt{(4-x^2)}}\,dx$.

(i) Using the substitution $x = 2\sin\theta$, show that

$$I = \int_0^{\frac{1}{6}\pi} 4\sin^2\theta \, d\theta.$$ [3]

(ii) Hence find the exact value of I. [4]

6 The complex number z is given by

$$z = (\sqrt{3}) + \mathrm{i}.$$

(i) Find the modulus and argument of z. [2]

(ii) The complex conjugate of z is denoted by z^*. Showing your working, express in the form $x + \mathrm{i}y$, where x and y are real,

(a) $2z + z^*$,

(b) $\dfrac{\mathrm{i}z^*}{z}$.

[4]

(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z and $\mathrm{i}z^*$ respectively. Prove that angle $AOB = \frac{1}{6}\pi$. [3]

7 With respect to the origin O, the points A and B have position vectors given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line AB and OP is perpendicular to AB.

(i) Find a vector equation for the line AB. [1]

(ii) Find the position vector of P. [4]

(iii) Find the equation of the plane which contains AB and which is perpendicular to the plane OAB, giving your answer in the form $ax + by + cz = d$. [4]

8 Let $\mathrm{f}(x) = \dfrac{3x}{(1+x)(1+2x^2)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^3. [5]

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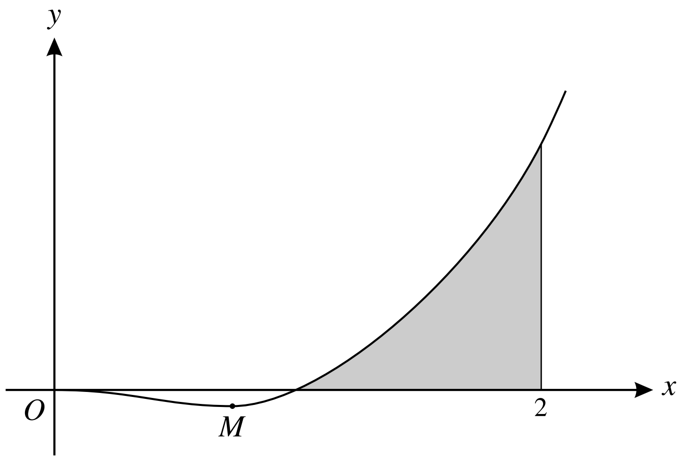
9



The diagram shows the curve $y = x^3 \ln x$ and its minimum point M.

(i) Find the exact coordinates of M. [5]

(ii) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 2$. [5]

10 A certain substance is formed in a chemical reaction. The mass of substance formed t seconds after the start of the reaction is x grams. At any time the rate of formation of the substance is proportional to $(20 - x)$. When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 1$.

(i) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.05(20 - x).$$ [2]

(ii) Find, in any form, the solution of this differential equation. [5]

(iii) Find x when $t = 10$, giving your answer correct to 1 decimal place. [2]

(iv) State what happens to the value of x as t becomes very large. [1]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2010**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4772556275

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2 Solve the equation

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giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$\cos(\theta + 60^\circ) = 2\sin\theta,$$

giving all solutions in the interval $0^\circ \leqslant \theta \leqslant 360^\circ$. [5]

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$$4x^2 - 1 = \cot x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 0.6 and 1. [2]

(iii) Use the iterative formula

$$x_{n+1} = \tfrac{1}{2}\sqrt{(1 + \cot x_n)}$$

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 Let $I = \displaystyle\int_0^1 \frac{x^2}{\sqrt{(4 - x^2)}}\,dx$.

(i) Using the substitution $x = 2\sin\theta$, show that

$$I = \int_0^{\frac{1}{6}\pi} 4\sin^2\theta\,d\theta.$$ [3]

(ii) Hence find the exact value of I. [4]

6 The complex number z is given by

$$z = (\sqrt{3}) + \mathrm{i}.$$

(i) Find the modulus and argument of z. [2]

(ii) The complex conjugate of z is denoted by z^*. Showing your working, express in the form $x + \mathrm{i}y$, where x and y are real,

(a) $2z + z^*$,

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[4]

(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z and $\mathrm{i}z^*$ respectively. Prove that angle $AOB = \frac{1}{6}\pi$. [3]

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(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

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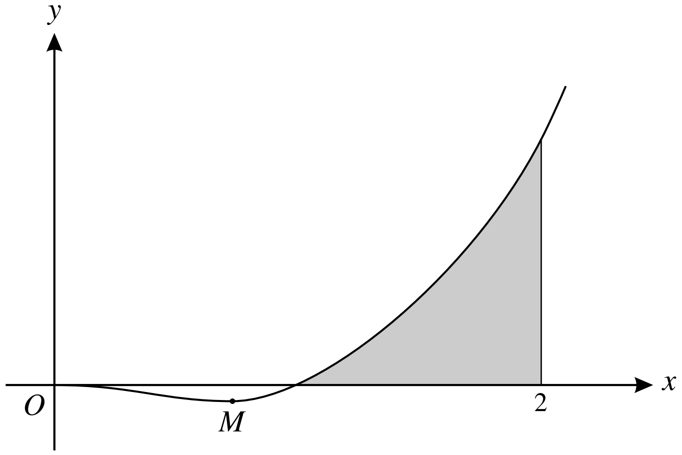

The diagram shows the curve $y = x^3 \ln x$ and its minimum point M.

(i) Find the exact coordinates of M. [5]

(ii) Find the exact area of the shaded region bounded by the curve, the x-axis and the line $x = 2$. [5]

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$$\frac{dx}{dt} = 0.05(20 - x).$$ [2]

(ii) Find, in any form, the solution of this differential equation. [5]

(iii) Find x when $t = 10$, giving your answer correct to 1 decimal place. [2]

(iv) State what happens to the value of x as t becomes very large. [1]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2010**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 Expand $(1 + 2x)^{-3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

2 The parametric equations of a curve are

$$x = \frac{t}{2t + 3}, \qquad y = e^{-2t}.$$

Find the gradient of the curve at the point for which $t = 0$. [5]

3 The complex number w is defined by $w = 2 + i$.

(i) Showing your working, express w^2 in the form $x + iy$, where x and y are real. Find the modulus of w^2. [3]

(ii) Shade on an Argand diagram the region whose points represent the complex numbers z which satisfy

$$|z - w^2| \leqslant |w^2|.$$ [3]

4 It is given that $f(x) = 4\cos^2 3x$.

(i) Find the exact value of $f'\left(\frac{1}{9}\pi\right)$. [3]

(ii) Find $\int f(x)\,dx$. [3]

5 Show that $\displaystyle\int_0^7 \frac{2x + 7}{(2x + 1)(x + 2)}\,dx = \ln 50$. [7]

6 The straight line l passes through the points with coordinates $(-5, 3, 6)$ and $(5, 8, 1)$. The plane p has equation $2x - y + 4z = 9$.

(i) Find the coordinates of the point of intersection of l and p. [4]

(ii) Find the acute angle between l and p. [4]

7 **(i)** Given that $\displaystyle\int_1^a \frac{\ln x}{x^2}\,dx = \frac{2}{5}$, show that $a = \frac{5}{3}(1 + \ln a)$. [5]

(ii) Use an iteration formula based on the equation $a = \frac{5}{3}(1 + \ln a)$ to find the value of a correct to 2 decimal places. Use an initial value of 4 and give the result of each iteration to 4 decimal places. [3]

8 **(i)** Express $(\sqrt{6})\cos\theta + (\sqrt{10})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence, in each of the following cases, find the smallest positive angle θ which satisfies the equation

(a) $(\sqrt{6})\cos\theta + (\sqrt{10})\sin\theta = -4$, [2]

(b) $(\sqrt{6})\cos\frac{1}{2}\theta + (\sqrt{10})\sin\frac{1}{2}\theta = 3$. [4]

9 A biologist is investigating the spread of a weed in a particular region. At time t weeks after the start of the investigation, the area covered by the weed is $A\,\text{m}^2$. The biologist claims that the rate of increase of A is proportional to $\sqrt{(2A - 5)}$.

(i) Write down a differential equation representing the biologist's claim. [1]

(ii) At the start of the investigation, the area covered by the weed was $7\,\text{m}^2$ and, 10 weeks later, the area covered was $27\,\text{m}^2$. Assuming that the biologist's claim is correct, find the area covered 20 weeks after the start of the investigation. [9]

10 The polynomial $\text{p}(z)$ is defined by

$$\text{p}(z) = z^3 + mz^2 + 24z + 32,$$

where m is a constant. It is given that $(z + 2)$ is a factor of $\text{p}(z)$.

(i) Find the value of m. [2]

(ii) Hence, showing all your working, find

(a) the three roots of the equation $\text{p}(z) = 0$, [5]

(b) the six roots of the equation $\text{p}(z^2) = 0$. [6]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2011**

1 hour 45 minutes

9424243601

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

JC11 11_9709_31/RP

UNIVERSITY of CAMBRIDGE
International Examinations

[Turn over

1 Using the substitution $u = e^x$, or otherwise, solve the equation

$$e^x = 1 + 6e^{-x},$$

giving your answer correct to 3 significant figures. [4]

2 The parametric equations of a curve are

$$x = 3(1 + \sin^2 t), \quad y = 2\cos^3 t.$$

Find $\frac{dy}{dx}$ in terms of t, simplifying your answer as far as possible. [5]

3 The polynomial $x^4 + 3x^3 + ax + 3$ is denoted by p(x). It is given that p(x) is divisible by $x^2 - x + 1$.

(i) Find the value of a. [4]

(ii) When a has this value, find the real roots of the equation p(x) = 0. [2]

4 The variables x and θ are related by the differential equation

$$\sin 2\theta \frac{dx}{d\theta} = (x + 1)\cos 2\theta,$$

where $0 < \theta < \frac{1}{2}\pi$. When $\theta = \frac{1}{12}\pi$, $x = 0$. Solve the differential equation, obtaining an expression for x in terms of θ, and simplifying your answer as far as possible. [7]

5 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - \tfrac{1}{2}x^2,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1 and 1.4. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{2}{6 - x^2}\right).$$ [1]

(iv) Use an iterative formula based on the equation in part **(iii)** to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(i)** Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $\cos 2\theta + 3\sin 2\theta = 2$, for $0° < \theta < 90°$. [5]

7 With respect to the origin O, the position vectors of two points A and B are given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line through A and B, and $\overrightarrow{AP} = \lambda\overrightarrow{AB}$.

(i) Show that $\overrightarrow{OP} = (1 + 2\lambda)\mathbf{i} + (2 + 2\lambda)\mathbf{j} + (2 - 2\lambda)\mathbf{k}$. [2]

(ii) By equating expressions for $\cos AOP$ and $\cos BOP$ in terms of λ, find the value of λ for which OP bisects the angle AOB. [5]

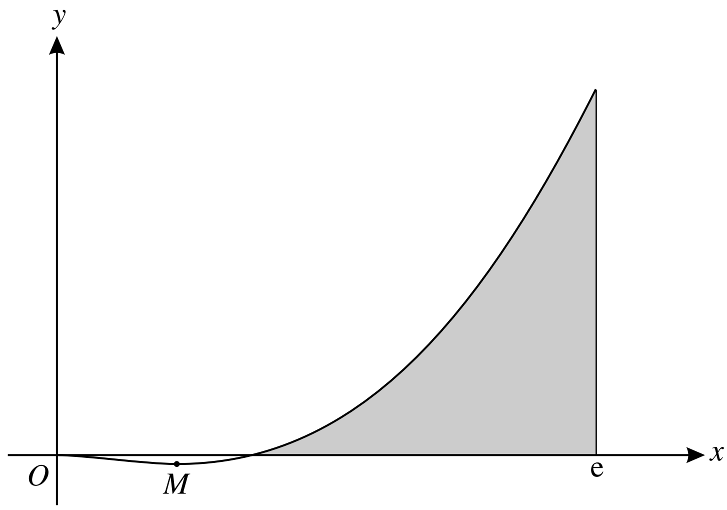
(iii) When λ has this value, verify that $AP : PB = OA : OB$. [1]

8 Let $\mathrm{f}(x) = \dfrac{12 + 8x - x^2}{(2 - x)(4 + x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{4 + x^2}$. [4]

(ii) Show that $\displaystyle\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \ln\left(\tfrac{25}{2}\right)$. [5]

9



The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = \mathrm{e}$. [5]

10 **(a)** Showing your working, find the two square roots of the complex number $1 - (2\sqrt{6})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent the complex numbers z which satisfy the inequality $|z - 3\mathrm{i}| \leqslant 2$. Find the greatest value of $\arg z$ for points in this region. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

0775793313

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JC11 11_9709_32/FP

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 Using the substitution $u = e^x$, or otherwise, solve the equation

$$e^x = 1 + 6e^{-x},$$

giving your answer correct to 3 significant figures. [4]

2 The parametric equations of a curve are

$$x = 3(1 + \sin^2 t), \quad y = 2\cos^3 t.$$

Find $\frac{dy}{dx}$ in terms of t, simplifying your answer as far as possible. [5]

3 The polynomial $x^4 + 3x^3 + ax + 3$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 1$.

(i) Find the value of a. [4]

(ii) When a has this value, find the real roots of the equation $p(x) = 0$. [2]

4 The variables x and θ are related by the differential equation

$$\sin 2\theta \frac{dx}{d\theta} = (x + 1)\cos 2\theta,$$

where $0 < \theta < \frac{1}{2}\pi$. When $\theta = \frac{1}{12}\pi$, $x = 0$. Solve the differential equation, obtaining an expression for x in terms of θ, and simplifying your answer as far as possible. [7]

5 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\sec x = 3 - \tfrac{1}{2}x^2,$$

where x is in radians, has a root in the interval $0 < x < \frac{1}{2}\pi$. [2]

(ii) Verify by calculation that this root lies between 1 and 1.4. [2]

(iii) Show that this root also satisfies the equation

$$x = \cos^{-1}\left(\frac{2}{6 - x^2}\right).$$ [1]

(iv) Use an iterative formula based on the equation in part **(iii)** to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(i)** Express $\cos x + 3\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving the exact value of R and the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $\cos 2\theta + 3\sin 2\theta = 2$, for $0° < \theta < 90°$. [5]

7 With respect to the origin O, the position vectors of two points A and B are given by $\overrightarrow{OA} = \mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$. The point P lies on the line through A and B, and $\overrightarrow{AP} = \lambda\overrightarrow{AB}$.

(i) Show that $\overrightarrow{OP} = (1 + 2\lambda)\mathbf{i} + (2 + 2\lambda)\mathbf{j} + (2 - 2\lambda)\mathbf{k}$. [2]

(ii) By equating expressions for $\cos AOP$ and $\cos BOP$ in terms of λ, find the value of λ for which OP bisects the angle AOB. [5]

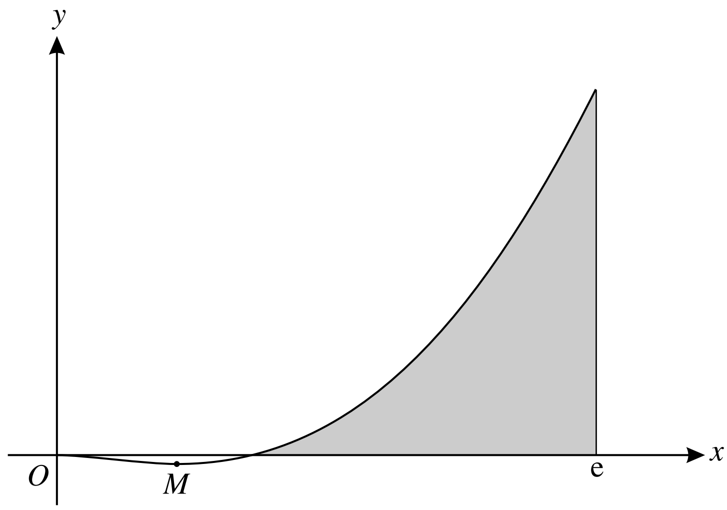
(iii) When λ has this value, verify that $AP : PB = OA : OB$. [1]

8 Let $\mathrm{f}(x) = \dfrac{12 + 8x - x^2}{(2 - x)(4 + x^2)}$.

(i) Express $\mathrm{f}(x)$ in the form $\dfrac{A}{2 - x} + \dfrac{Bx + C}{4 + x^2}$. [4]

(ii) Show that $\displaystyle\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \ln\left(\tfrac{25}{2}\right)$. [5]

9



The diagram shows the curve $y = x^2 \ln x$ and its minimum point M.

(i) Find the exact values of the coordinates of M. [5]

(ii) Find the exact value of the area of the shaded region bounded by the curve, the x-axis and the line $x = \mathrm{e}$. [5]

10 **(a)** Showing your working, find the two square roots of the complex number $1 - (2\sqrt{6})\mathrm{i}$. Give your answers in the form $x + \mathrm{i}y$, where x and y are exact. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent the complex numbers z which satisfy the inequality $|z - 3\mathrm{i}| \leqslant 2$. Find the greatest value of $\arg z$ for points in this region. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4082927740

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
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The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.

JC11 11_9709_33/RP

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 Expand $\dfrac{16}{(2+x)^2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 The equation of a curve is $y = \dfrac{e^{2x}}{1+e^{2x}}$. Show that the gradient of the curve at the point for which $x = \ln 3$ is $\frac{9}{50}$. [4]

3 **(i)** Express $8\cos\theta + 15\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $8\cos\theta + 15\sin\theta = 12$, giving all solutions in the interval $0° < \theta < 360°$. [4]

4 During an experiment, the number of organisms present at time t days is denoted by N, where N is treated as a continuous variable. It is given that

$$\frac{dN}{dt} = 1.2e^{-0.02t}N^{0.5}.$$

When $t = 0$, the number of organisms present is 100.

(i) Find an expression for N in terms of t. [6]

(ii) State what happens to the number of organisms present after a long time. [1]

5 It is given that $\int_1^a x\ln x\,dx = 22$, where a is a constant greater than 1.

(i) Show that $a = \sqrt{\left(\dfrac{87}{2\ln a - 1}\right)}$. [5]

(ii) Use an iterative formula based on the equation in part **(i)** to find the value of a correct to 2 decimal places. Use an initial value of 6 and give the result of each iteration to 4 decimal places. [3]

6 The complex number w is defined by $w = -1 + i$.

(i) Find the modulus and argument of w^2 and w^3, showing your working. [4]

(ii) The points in an Argand diagram representing w and w^2 are the ends of a diameter of a circle. Find the equation of the circle, giving your answer in the form $|z - (a + bi)| = k$. [4]

7 The polynomial $p(x)$ is defined by

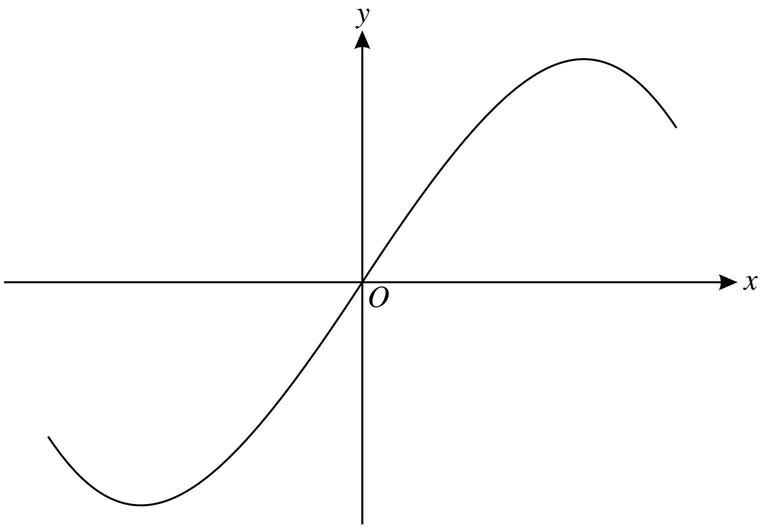
$$p(x) = ax^3 - x^2 + 4x - a,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $p(x)$.

(i) Find the value of a and hence factorise $p(x)$. [4]

(ii) When a has the value found in part **(i)**, express $\dfrac{8x - 13}{p(x)}$ in partial fractions. [5]

8



The diagram shows the curve with parametric equations

$$x = \sin t + \cos t, \quad y = \sin^3 t + \cos^3 t,$$

for $\frac{1}{4}\pi < t < \frac{5}{4}\pi$.

(i) Show that $\dfrac{dy}{dx} = -3 \sin t \cos t$. [3]

(ii) Find the gradient of the curve at the origin. [2]

(iii) Find the values of t for which the gradient of the curve is 1, giving your answers correct to 2 significant figures. [4]

9 The line l has equation $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$, where a is a constant. The plane p has equation $2x - 2y + z = 10$.

(i) Given that l does not lie in p, show that l is parallel to p. [2]

(ii) Find the value of a for which l lies in p. [2]

(iii) It is now given that the distance between l and p is 6. Find the possible values of a. [5]

10 **(i)** Use the substitution $u = \tan x$ to show that, for $n \neq -1$,

$$\int_0^{\frac{1}{4}\pi} (\tan^{n+2} x + \tan^n x)\, dx = \frac{1}{n+1}.$$ [4]

(ii) Hence find the exact value of

(a) $\int_0^{\frac{1}{4}\pi} (\sec^4 x - \sec^2 x)\, dx$, [3]

(b) $\int_0^{\frac{1}{4}\pi} (\tan^9 x + 5\tan^7 x + 5\tan^5 x + \tan^3 x)\, dx$. [3]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

3197066494

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JC12 11_9709_31/FP

UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

1 Find the set of values of x satisfying the inequality $3|x - 1| < |2x + 1|$. [4]

2 Solve the equation

$$5^{x-1} = 5^x - 5,$$

giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$\sin(\theta + 45^\circ) = 2\cos(\theta - 30^\circ),$$

giving all solutions in the interval $0^\circ < \theta < 180^\circ$. [5]

4 When $(1 + ax)^{-2}$, where a is a positive constant, is expanded in ascending powers of x, the coefficients of x and x^3 are equal.

(i) Find the exact value of a. [4]

(ii) When a has this value, obtain the expansion up to and including the term in x^2, simplifying the coefficients. [3]

5 **(i)** By differentiating $\frac{1}{\cos x}$, show that if $y = \sec x$ then $\frac{dy}{dx} = \sec x \tan x$. [2]

(ii) Show that $\frac{1}{\sec x - \tan x} \equiv \sec x + \tan x$. [1]

(iii) Deduce that $\frac{1}{(\sec x - \tan x)^2} \equiv 2\sec^2 x - 1 + 2\sec x \tan x$. [2]

(iv) Hence show that $\int_0^{\frac{1}{4}\pi} \frac{1}{(\sec x - \tan x)^2}\,dx = \frac{1}{4}(8\sqrt{2} - \pi)$. [3]

6 The variables x and y are related by the differential equation

$$x\frac{dy}{dx} = 1 - y^2.$$

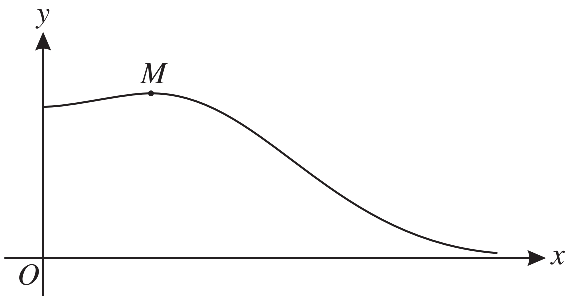
When $x = 2$, $y = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [8]

7 The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $\frac{dy}{dx} = \frac{y}{x(3y^3 - 1)}$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures. [4]

8



The diagram shows the curve $y = e^{-\frac{1}{2}x^2}\sqrt{(1 + 2x^2)}$ for $x \geqslant 0$, and its maximum point M.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) The sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(\ln(4 + 8x_n^2)\right)},$$

with initial value $x_1 = 2$, converges to a certain value α. State an equation satisfied by α and hence show that α is the x-coordinate of a point on the curve where $y = 0.5$. [3]

(iii) Use the iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9 The complex number $1 + (\sqrt{2})\mathrm{i}$ is denoted by u. The polynomial $x^4 + x^2 + 2x + 6$ is denoted by $\mathrm{p}(x)$.

(i) Showing your working, verify that u is a root of the equation $\mathrm{p}(x) = 0$, and write down a second complex root of the equation. [4]

(ii) Find the other two roots of the equation $\mathrm{p}(x) = 0$. [6]

10 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}.$$

The plane m is parallel to $\overrightarrow{OC}$ and contains A and B.

(i) Find the equation of m, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Find the length of the perpendicular from C to the line through A and B. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2012**

1 hour 45 minutes

4955884271

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC12 11_9709_32/RP

UNIVERSITY of CAMBRIDGE International Examinations

[Turn over

1 Find the set of values of x satisfying the inequality $3|x-1| < |2x+1|$. [4]

2 Solve the equation

$$5^{x-1} = 5^x - 5,$$

giving your answer correct to 3 significant figures. [4]

3 Solve the equation

$$\sin(\theta + 45°) = 2\cos(\theta - 30°),$$

giving all solutions in the interval $0° < \theta < 180°$. [5]

4 When $(1+ax)^{-2}$, where a is a positive constant, is expanded in ascending powers of x, the coefficients of x and x^3 are equal.

(i) Find the exact value of a. [4]

(ii) When a has this value, obtain the expansion up to and including the term in x^2, simplifying the coefficients. [3]

5 **(i)** By differentiating $\dfrac{1}{\cos x}$, show that if $y = \sec x$ then $\dfrac{dy}{dx} = \sec x \tan x$. [2]

(ii) Show that $\dfrac{1}{\sec x - \tan x} \equiv \sec x + \tan x$. [1]

(iii) Deduce that $\dfrac{1}{(\sec x - \tan x)^2} \equiv 2\sec^2 x - 1 + 2\sec x \tan x$. [2]

(iv) Hence show that $\displaystyle\int_0^{\frac{1}{4}\pi} \frac{1}{(\sec x - \tan x)^2}\,dx = \tfrac{1}{4}(8\sqrt{2} - \pi)$. [3]

6 The variables x and y are related by the differential equation

$$x\frac{dy}{dx} = 1 - y^2.$$

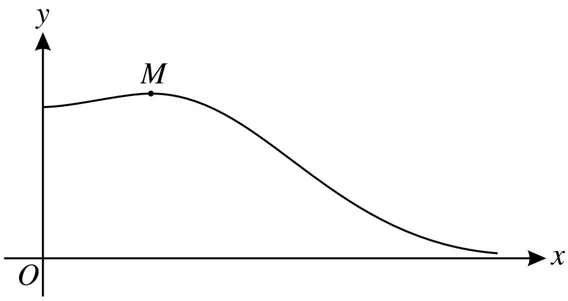
When $x = 2$, $y = 0$. Solve the differential equation, obtaining an expression for y in terms of x. [8]

7 The equation of a curve is $\ln(xy) - y^3 = 1$.

(i) Show that $\dfrac{dy}{dx} = \dfrac{y}{x(3y^3 - 1)}$. [4]

(ii) Find the coordinates of the point where the tangent to the curve is parallel to the y-axis, giving each coordinate correct to 3 significant figures. [4]

8



The diagram shows the curve $y = e^{-\frac{1}{2}x^2}\sqrt{(1 + 2x^2)}$ for $x \geqslant 0$, and its maximum point M.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) The sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(\ln(4 + 8x_n^2)\right)},$$

with initial value $x_1 = 2$, converges to a certain value α. State an equation satisfied by α and hence show that α is the x-coordinate of a point on the curve where $y = 0.5$. [3]

(iii) Use the iterative formula to determine α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

9 The complex number $1 + (\sqrt{2})i$ is denoted by u. The polynomial $x^4 + x^2 + 2x + 6$ is denoted by $p(x)$.

(i) Showing your working, verify that u is a root of the equation $p(x) = 0$, and write down a second complex root of the equation. [4]

(ii) Find the other two roots of the equation $p(x) = 0$. [6]

10 With respect to the origin O, the points A, B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -2 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -5 \\ -3 \end{pmatrix}.$$

The plane m is parallel to $\overrightarrow{OC}$ and contains A and B.

(i) Find the equation of m, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Find the length of the perpendicular from C to the line through A and B. [5]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9969803086

READ THESE INSTRUCTIONS FIRST

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UNIVERSITY *of* CAMBRIDGE International Examinations

[Turn over

1 Solve the equation

$$\ln(x+5) = 1 + \ln x,$$

giving your answer in terms of e. [3]

2 (i) Express $24\sin\theta - 7\cos\theta$ in the form $R\sin(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence find the smallest positive value of θ satisfying the equation

$$24\sin\theta - 7\cos\theta = 17.$$ [2]

3 The parametric equations of a curve are

$$x = \frac{4t}{2t+3}, \quad y = 2\ln(2t+3).$$

(i) Express $\frac{dy}{dx}$ in terms of t, simplifying your answer. [4]

(ii) Find the gradient of the curve at the point for which $x = 1$. [2]

4 The variables x and y are related by the differential equation

$$(x^2+4)\frac{dy}{dx} = 6xy.$$

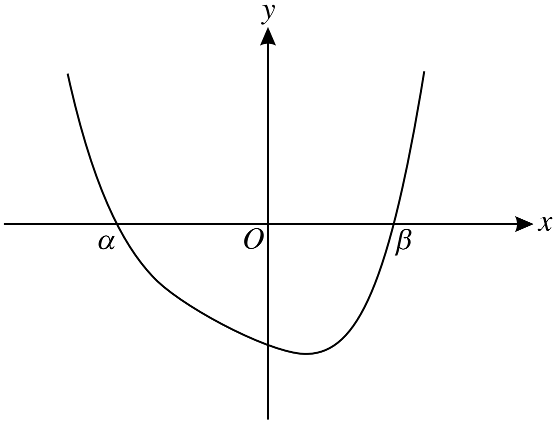
It is given that $y = 32$ when $x = 0$. Find an expression for y in terms of x. [6]

5 The expression $f(x)$ is defined by $f(x) = 3xe^{-2x}$.

(i) Find the exact value of $f'(-\frac{1}{2})$. [3]

(ii) Find the exact value of $\int_{-\frac{1}{2}}^{0} f(x)\,dx$. [5]

6



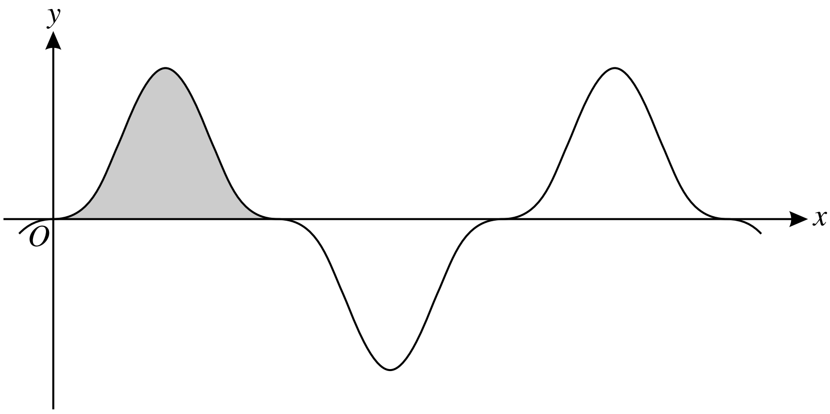
The diagram shows the curve $y = x^4 + 2x^3 + 2x^2 - 4x - 16$, which crosses the x-axis at the points $(\alpha, 0)$ and $(\beta, 0)$ where $\alpha < \beta$. It is given that α is an integer.

(i) Find the value of α. [2]

(ii) Show that β satisfies the equation $x = \sqrt[3]{(8 - 2x)}$. [3]

(iii) Use an iteration process based on the equation in part **(ii)** to find the value of β correct to 2 decimal places. Show the result of each iteration to 4 decimal places. [3]

7



The diagram shows part of the curve $y = \sin^3 2x \cos^3 2x$. The shaded region shown is bounded by the curve and the x-axis and its exact area is denoted by A.

(i) Use the substitution $u = \sin 2x$ in a suitable integral to find the value of A. [6]

(ii) Given that $\int_0^{k\pi} |\sin^3 2x \cos^3 2x| \, dx = 40A$, find the value of the constant k. [2]

[Questions 8, 9 and 10 are printed on the next page.]

8 Two lines have equations

$$\mathbf{r} = \begin{pmatrix} 5 \\ 1 \\ -4 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \quad \text{and} \quad \mathbf{r} = \begin{pmatrix} p \\ 4 \\ -2 \end{pmatrix} + t\begin{pmatrix} 2 \\ 5 \\ -4 \end{pmatrix},$$

where p is a constant. It is given that the lines intersect.

(i) Find the value of p and determine the coordinates of the point of intersection. [5]

(ii) Find the equation of the plane containing the two lines, giving your answer in the form $ax + by + cz = d$, where a, b, c and d are integers. [5]

9 **(i)** Express $\dfrac{9 - 7x + 8x^2}{(3 - x)(1 + x^2)}$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\dfrac{9 - 7x + 8x^2}{(3 - x)(1 + x^2)}$ in ascending powers of x, up to and including the term in x^3. [5]

10 **(a)** Without using a calculator, solve the equation $\mathrm{i}w^2 = (2 - 2\mathrm{i})^2$. [3]

(b) **(i)** Sketch an Argand diagram showing the region R consisting of points representing the complex numbers z where

$$|z - 4 - 4\mathrm{i}| \leqslant 2.$$ [2]

(ii) For the complex numbers represented by points in the region R, it is given that

$$p \leqslant |z| \leqslant q \quad \text{and} \quad \alpha \leqslant \arg z \leqslant \beta.$$

Find the values of p, q, α and β, giving your answers correct to 3 significant figures. [6]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

5238158802

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UNIVERSITY *of* CAMBRIDGE
International Examinations

1 The equation of a curve is $y = \dfrac{1+x}{1+2x}$ for $x > -\frac{1}{2}$. Show that the gradient of the curve is always negative. [3]

2 Solve the equation $2|3^x - 1| = 3^x$, giving your answers correct to 3 significant figures. [4]

3 Find the exact value of $\displaystyle\int_1^4 \frac{\ln x}{\sqrt{x}}\,dx$. [5]

4 The parametric equations of a curve are

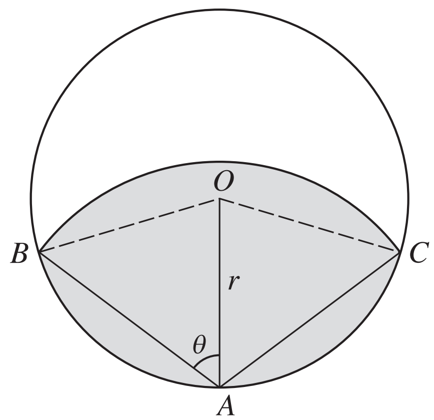
$$x = e^{-t}\cos t, \quad y = e^{-t}\sin t.$$

Show that $\dfrac{dy}{dx} = \tan\left(t - \frac{1}{4}\pi\right)$. [6]

5 **(i)** Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta\,d\theta = \frac{1}{2}\ln 3$. [4]

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r. A circular arc with centre A meets the circumference at B and C. The angle OAB is θ radians. The shaded region is bounded by the circumference of the circle and the arc with centre A joining B and C. The area of the shaded region is equal to half the area of the circle.

(i) Show that $\cos 2\theta = \dfrac{2\sin 2\theta - \pi}{4\theta}$. [5]

(ii) Use the iterative formula

$$\theta_{n+1} = \frac{1}{2}\cos^{-1}\left(\frac{2\sin 2\theta_n - \pi}{4\theta_n}\right),$$

with initial value $\theta_1 = 1$, to determine θ correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 Let $\mathrm{f}(x) = \dfrac{2x^2 - 7x - 1}{(x-2)(x^2+3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

8 **Throughout this question the use of a calculator is not permitted.**

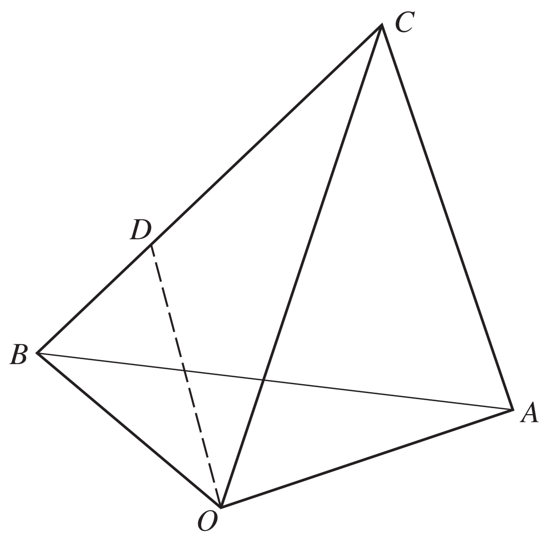
(a) The complex numbers u and v satisfy the equations

$$u + 2v = 2\mathrm{i} \quad \text{and} \quad \mathrm{i}u + v = 3.$$

Solve the equations for u and v, giving both answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers z satisfying $|z + \mathrm{i}| = 1$ and the locus representing complex numbers w satisfying $\arg(w - 2) = \frac{3}{4}\pi$. Find the least value of $|z - w|$ for points on these loci. [5]

9



The diagram shows three points A, B and C whose position vectors with respect to the origin O are given by $\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$, $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$ and $\overrightarrow{OC} = \begin{pmatrix} 3 \\ 0 \\ 4 \end{pmatrix}$. The point D lies on BC, between B and C, and is such that $CD = 2DB$.

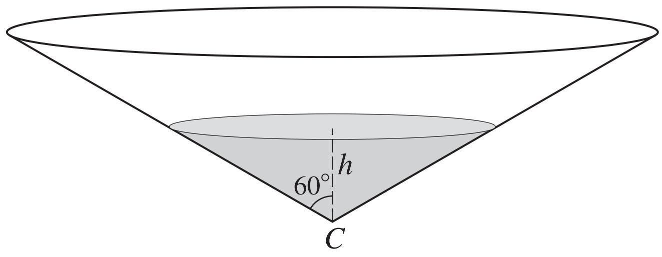
(i) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Find the position vector of D. [1]

(iii) Show that the length of the perpendicular from A to OD is $\frac{1}{3}\sqrt{(65)}$. [4]

[Question 10 is printed on the next page.]

10



A tank containing water is in the form of a cone with vertex C. The axis is vertical and the semi-vertical angle is 60°, as shown in the diagram. At time $t = 0$, the tank is full and the depth of water is H. At this instant, a tap at C is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to $\sqrt{h}$, where h is the depth of water at time t. The tank becomes empty when $t = 60$.

(i) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -Ah^{-\frac{3}{2}},$$

where A is a positive constant. [4]

(ii) Solve the differential equation given in part **(i)** and obtain an expression for t in terms of h and H. [6]

(iii) Find the time at which the depth reaches $\frac{1}{2}H$. [1]

[The volume V of a cone of vertical height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

4229833005

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UNIVERSITY *of* CAMBRIDGE
International Examinations

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2 Solve the equation $2|3^x - 1| = 3^x$, giving your answers correct to 3 significant figures. [4]

3 Find the exact value of $\displaystyle\int_1^4 \frac{\ln x}{\sqrt{x}}\,dx$. [5]

4 The parametric equations of a curve are

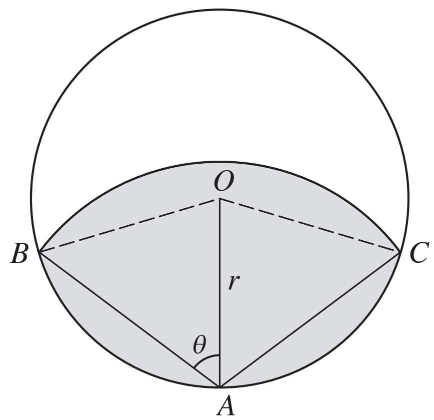
$$x = e^{-t}\cos t, \quad y = e^{-t}\sin t.$$

Show that $\dfrac{dy}{dx} = \tan\left(t - \frac{1}{4}\pi\right)$. [6]

5 **(i)** Prove that $\cot\theta + \tan\theta \equiv 2\operatorname{cosec} 2\theta$. [3]

(ii) Hence show that $\displaystyle\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \operatorname{cosec} 2\theta\,d\theta = \frac{1}{2}\ln 3$. [4]

6



In the diagram, A is a point on the circumference of a circle with centre O and radius r. A circular arc with centre A meets the circumference at B and C. The angle OAB is θ radians. The shaded region is bounded by the circumference of the circle and the arc with centre A joining B and C. The area of the shaded region is equal to half the area of the circle.

(i) Show that $\cos 2\theta = \dfrac{2\sin 2\theta - \pi}{4\theta}$. [5]

(ii) Use the iterative formula

$$\theta_{n+1} = \frac{1}{2}\cos^{-1}\left(\frac{2\sin 2\theta_n - \pi}{4\theta_n}\right),$$

with initial value $\theta_1 = 1$, to determine θ correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

7 Let $\mathrm{f}(x) = \dfrac{2x^2 - 7x - 1}{(x-2)(x^2+3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

8 **Throughout this question the use of a calculator is not permitted.**

(a) The complex numbers u and v satisfy the equations

$$u + 2v = 2\mathrm{i} \quad \text{and} \quad \mathrm{i}u + v = 3.$$

Solve the equations for u and v, giving both answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) On an Argand diagram, sketch the locus representing complex numbers z satisfying $|z + \mathrm{i}| = 1$ and the locus representing complex numbers w satisfying $\arg(w - 2) = \frac{3}{4}\pi$. Find the least value of $|z - w|$ for points on these loci. [5]

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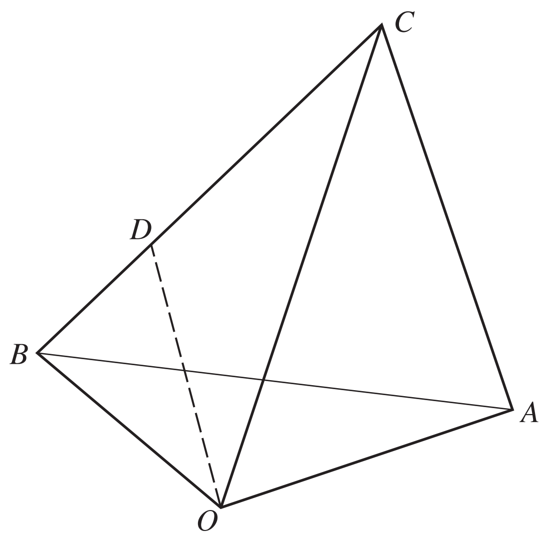

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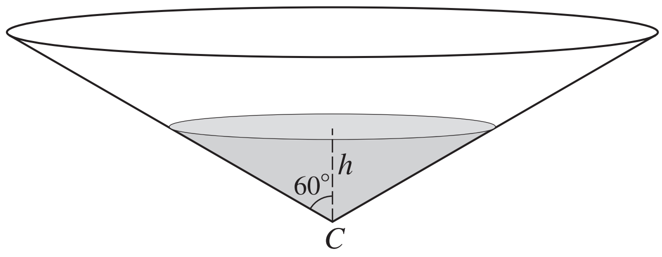
(i) Find the equation of the plane ABC, giving your answer in the form $ax + by + cz = d$. [6]

(ii) Find the position vector of D. [1]

(iii) Show that the length of the perpendicular from A to OD is $\frac{1}{3}\sqrt{(65)}$. [4]

[Question 10 is printed on the next page.]

10



A tank containing water is in the form of a cone with vertex C. The axis is vertical and the semi-vertical angle is 60°, as shown in the diagram. At time $t = 0$, the tank is full and the depth of water is H. At this instant, a tap at C is opened and water begins to flow out. The volume of water in the tank decreases at a rate proportional to $\sqrt{h}$, where h is the depth of water at time t. The tank becomes empty when $t = 60$.

(i) Show that h and t satisfy a differential equation of the form

$$\frac{dh}{dt} = -Ah^{-\frac{3}{2}},$$

where A is a positive constant. [4]

(ii) Solve the differential equation given in part **(i)** and obtain an expression for t in terms of h and H. [6]

(iii) Find the time at which the depth reaches $\frac{1}{2}H$. [1]

[The volume V of a cone of vertical height h and base radius r is given by $V = \frac{1}{3}\pi r^2 h$.]

UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

6391576228

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JC13 11_9709_33/FP

UNIVERSITY of CAMBRIDGE
International Examinations

1 Given that $2\ln(x+4) - \ln x = \ln(x+a)$, express x in terms of a. [4]

2 Use the substitution $u = 3x + 1$ to find $\displaystyle\int \frac{3x}{3x+1}\,dx$. [4]

3 The polynomial $f(x)$ is defined by

$$f(x) = x^3 + ax^2 - ax + 14,$$

where a is a constant. It is given that $(x+2)$ is a factor of $f(x)$.

(i) Find the value of a. [2]

(ii) Show that, when a has this value, the equation $f(x) = 0$ has only one real root. [3]

4 A curve has equation $3e^{2x}y + e^x y^3 = 14$. Find the gradient of the curve at the point $(0, 2)$. [5]

5 It is given that $\displaystyle\int_0^p 4xe^{-\frac{1}{2}x}\,dx = 9$, where p is a positive constant.

(i) Show that $p = 2\ln\left(\dfrac{8p+16}{7}\right)$. [5]

(ii) Use an iterative process based on the equation in part **(i)** to find the value of p correct to 3 significant figures. Use a starting value of 3.5 and give the result of each iteration to 5 significant figures. [3]

6 Two planes have equations $3x - y + 2z = 9$ and $x + y - 4z = -1$.

(i) Find the acute angle between the planes. [3]

(ii) Find a vector equation of the line of intersection of the planes. [6]

7 **(i)** Given that $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec}2\theta$, show that $2\sin\theta + 4\cos\theta = 3$. [3]

(ii) Express $2\sin\theta + 4\cos\theta$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$, giving the value of α correct to 2 decimal places. [3]

(iii) Hence solve the equation $\sec\theta + 2\operatorname{cosec}\theta = 3\operatorname{cosec}2\theta$ for $0° < \theta < 360°$. [4]

8 **(i)** Express $\dfrac{7x^2+8}{(1+x)^2(2-3x)}$ in partial fractions. [5]

(ii) Hence expand $\dfrac{7x^2+8}{(1+x)^2(2-3x)}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [5]

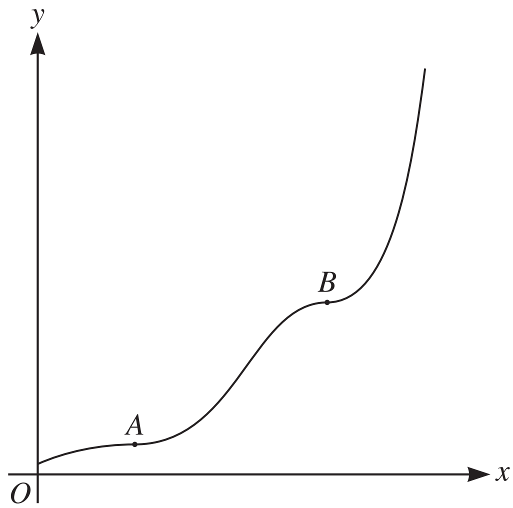
9 **(a)** Without using a calculator, use the formula for the solution of a quadratic equation to solve

$$(2 - \mathrm{i})z^2 + 2z + 2 + \mathrm{i} = 0.$$

Give your answers in the form $a + b\mathrm{i}$. [5]

(b) The complex number w is defined by $w = 2\mathrm{e}^{\frac{1}{4}\pi\mathrm{i}}$. In an Argand diagram, the points A, B and C represent the complex numbers w, w^3 and w^* respectively (where w^* denotes the complex conjugate of w). Draw the Argand diagram showing the points A, B and C, and calculate the area of triangle ABC. [5]

10



A particular solution of the differential equation

$$3y^2\frac{\mathrm{d}y}{\mathrm{d}x} = 4(y^3 + 1)\cos^2 x$$

is such that $y = 2$ when $x = 0$. The diagram shows a sketch of the graph of this solution for $0 \leqslant x \leqslant 2\pi$; the graph has stationary points at A and B. Find the y-coordinates of A and B, giving each coordinate correct to 1 decimal place. [10]

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Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2014**

1 hour 45 minutes

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0962340356

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JC14 11_9709_31/FP

CAMBRIDGE International Examinations

1 Use logarithms to solve the equation $e^x = 3^{x-2}$, giving your answer correct to 3 decimal places. [3]

2 **(i)** Use the trapezium rule with 3 intervals to estimate the value of

$$\int_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} \operatorname{cosec} x \, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) Using a sketch of the graph of $y = \operatorname{cosec} x$, explain whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part **(i)**. [2]

3 The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 2)$ the remainder is 21. Find the values of a and b. [5]

4 The parametric equations of a curve are

$$x = \frac{1}{\cos^3 t}, \quad y = \tan^3 t,$$

where $0 \leqslant t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \sin t$. [4]

(ii) Hence show that the equation of the tangent to the curve at the point with parameter t is $y = x \sin t - \tan t$. [3]

5 **Throughout this question the use of a calculator is not permitted.**

The complex numbers w and z satisfy the relation

$$w = \frac{z + i}{iz + 2}.$$

(i) Given that $z = 1 + i$, find w, giving your answer in the form $x + iy$, where x and y are real. [4]

(ii) Given instead that $w = z$ and the real part of z is negative, find z, giving your answer in the form $x + iy$, where x and y are real. [4]

6 It is given that $\int_1^a \ln(2x)\,dx = 1$, where $a > 1$.

(i) Show that $a = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a}\right)$, where $\exp(x)$ denotes e^x. [6]

(ii) Use the iterative formula

$$a_{n+1} = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a_n}\right)$$

to determine the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 In a certain country the government charges tax on each litre of petrol sold to motorists. The revenue per year is R million dollars when the rate of tax is x dollars per litre. The variation of R with x is modelled by the differential equation

$$\frac{dR}{dx} = R\left(\frac{1}{x} - 0.57\right),$$

where R and x are taken to be continuous variables. When $x = 0.5$, $R = 16.8$.

(i) Solve the differential equation and obtain an expression for R in terms of x. [6]

(ii) This model predicts that R cannot exceed a certain amount. Find this maximum value of R. [3]

8 **(i)** By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Show that, after making the substitution $x = \frac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \frac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

9 Let $f(x) = \frac{x^2 - 8x + 9}{(1-x)(2-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

10 The line l has equation $\mathbf{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. The point A has position vector $3\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$.

(i) Show that the length of the perpendicular from A to l is 15. [5]

(ii) The line l lies in the plane with equation $ax + by - 3z + 1 = 0$, where a and b are constants. Find the values of a and b. [5]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

9231113471

READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.
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CAMBRIDGE International Examinations

1 Use logarithms to solve the equation $e^x = 3^{x-2}$, giving your answer correct to 3 decimal places. [3]

2 **(i)** Use the trapezium rule with 3 intervals to estimate the value of

$$\int_{\frac{1}{6}\pi}^{\frac{2}{3}\pi} \operatorname{cosec} x \, dx,$$

giving your answer correct to 2 decimal places. [3]

(ii) Using a sketch of the graph of $y = \operatorname{cosec} x$, explain whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part **(i)**. [2]

3 The polynomial $ax^3 + bx^2 + x + 3$, where a and b are constants, is denoted by $p(x)$. It is given that $(3x + 1)$ is a factor of $p(x)$, and that when $p(x)$ is divided by $(x - 2)$ the remainder is 21. Find the values of a and b. [5]

4 The parametric equations of a curve are

$$x = \frac{1}{\cos^3 t}, \quad y = \tan^3 t,$$

where $0 \leqslant t < \frac{1}{2}\pi$.

(i) Show that $\frac{dy}{dx} = \sin t$. [4]

(ii) Hence show that the equation of the tangent to the curve at the point with parameter t is $y = x \sin t - \tan t$. [3]

5 **Throughout this question the use of a calculator is not permitted.**

The complex numbers w and z satisfy the relation

$$w = \frac{z + i}{iz + 2}.$$

(i) Given that $z = 1 + i$, find w, giving your answer in the form $x + iy$, where x and y are real. [4]

(ii) Given instead that $w = z$ and the real part of z is negative, find z, giving your answer in the form $x + iy$, where x and y are real. [4]

6 It is given that $\int_1^a \ln(2x)\,dx = 1$, where $a > 1$.

(i) Show that $a = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a}\right)$, where $\exp(x)$ denotes e^x. [6]

(ii) Use the iterative formula

$$a_{n+1} = \frac{1}{2}\exp\left(1 + \frac{\ln 2}{a_n}\right)$$

to determine the value of a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 In a certain country the government charges tax on each litre of petrol sold to motorists. The revenue per year is R million dollars when the rate of tax is x dollars per litre. The variation of R with x is modelled by the differential equation

$$\frac{dR}{dx} = R\left(\frac{1}{x} - 0.57\right),$$

where R and x are taken to be continuous variables. When $x = 0.5$, $R = 16.8$.

(i) Solve the differential equation and obtain an expression for R in terms of x. [6]

(ii) This model predicts that R cannot exceed a certain amount. Find this maximum value of R. [3]

8 **(i)** By first expanding $\sin(2\theta + \theta)$, show that

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta.$$ [4]

(ii) Show that, after making the substitution $x = \frac{2\sin\theta}{\sqrt{3}}$, the equation $x^3 - x + \frac{1}{6}\sqrt{3} = 0$ can be written in the form $\sin 3\theta = \frac{3}{4}$. [1]

(iii) Hence solve the equation

$$x^3 - x + \frac{1}{6}\sqrt{3} = 0,$$

giving your answers correct to 3 significant figures. [4]

9 Let $f(x) = \frac{x^2 - 8x + 9}{(1-x)(2-x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

10 The line l has equation $\mathbf{r} = 4\mathbf{i} - 9\mathbf{j} + 9\mathbf{k} + \lambda(-2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$. The point A has position vector $3\mathbf{i} + 8\mathbf{j} + 5\mathbf{k}$.

(i) Show that the length of the perpendicular from A to l is 15. [5]

(ii) The line l lies in the plane with equation $ax + by - 3z + 1 = 0$, where a and b are constants. Find the values of a and b. [5]

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Cambridge International Examinations
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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2014**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

8698806885

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JC14 11_9709_33/2R

CAMBRIDGE International Examinations

[Turn over

1 Solve the inequality $|3x - 1| < |2x + 5|$. [4]

2 A curve is defined for $0 < \theta < \frac{1}{2}\pi$ by the parametric equations

$$x = \tan\theta, \quad y = 2\cos^2\theta\sin\theta.$$

Show that $\dfrac{dy}{dx} = 6\cos^5\theta - 4\cos^3\theta$. [5]

3 The polynomial $4x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ and $(x + 2)$ are factors of $p(x)$.

(i) Find the values of a and b. [4]

(ii) When a and b have these values, find the remainder when $p(x)$ is divided by $(x^2 + 1)$. [3]

4 **(i)** Show that $\cos(\theta - 60°) + \cos(\theta + 60°) \equiv \cos\theta$. [3]

(ii) Given that $\dfrac{\cos(2x - 60°) + \cos(2x + 60°)}{\cos(x - 60°) + \cos(x + 60°)} = 3$, find the exact value of $\cos x$. [4]

5 The complex numbers w and z are defined by $w = 5 + 3i$ and $z = 4 + i$.

(i) Express $\dfrac{iw}{z}$ in the form $x + iy$, showing all your working and giving the exact values of x and y. [3]

(ii) Find wz and hence, by considering arguments, show that

$$\tan^{-1}\left(\tfrac{3}{5}\right) + \tan^{-1}\left(\tfrac{1}{4}\right) = \tfrac{1}{4}\pi.$$ [4]

6 It is given that $I = \displaystyle\int_0^{0.3} \left(1 + 3x^2\right)^{-2} dx$.

(i) Use the trapezium rule with 3 intervals to find an approximation to I, giving the answer correct to 3 decimal places. [3]

(ii) For small values of x, $\left(1 + 3x^2\right)^{-2} \approx 1 + ax^2 + bx^4$. Find the values of the constants a and b. Hence, by evaluating $\displaystyle\int_0^{0.3} (1 + ax^2 + bx^4)\, dx$, find a second approximation to I, giving the answer correct to 3 decimal places. [5]

7 The equations of two straight lines are

$$\mathbf{r} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{k}) \quad \text{and} \quad \mathbf{r} = a\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \mu(\mathbf{i} + 2\mathbf{j} + 3a\mathbf{k}),$$

where a is a constant.

(i) Show that the lines intersect for all values of a. [4]

(ii) Given that the point of intersection is at a distance of 9 units from the origin, find the possible values of a. [4]

8 The variables x and y are related by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \tfrac{1}{5}xy^{\frac{1}{2}}\sin\left(\tfrac{1}{3}x\right).$$

(i) Find the general solution, giving y in terms of x. [6]

(ii) Given that $y = 100$ when $x = 0$, find the value of y when $x = 25$. [3]

9 **(i)** Sketch the curve $y = \ln(x + 1)$ and hence, by sketching a second curve, show that the equation

$$x^3 + \ln(x + 1) = 40$$

has exactly one real root. State the equation of the second curve. [3]

(ii) Verify by calculation that the root lies between 3 and 4. [2]

(iii) Use the iterative formula

$$x_{n+1} = \sqrt[3]{\left(40 - \ln(x_n + 1)\right)},$$

with a suitable starting value, to find the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iv) Deduce the root of the equation

$$(\mathrm{e}^y - 1)^3 + y = 40,$$

giving the answer correct to 2 decimal places. [2]

10 By first using the substitution $u = \mathrm{e}^x$, show that

$$\int_0^{\ln 4} \frac{\mathrm{e}^{2x}}{\mathrm{e}^{2x} + 3\mathrm{e}^x + 2}\,\mathrm{d}x = \ln\left(\tfrac{8}{5}\right).$$ [10]

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MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2015**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

2390741568

READ THESE INSTRUCTIONS FIRST

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JC15 11_9709_31/RP

1 Solve the inequality $|2x - 5| > 3|2x + 1|$. [4]

2 Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

3 The angles θ and ϕ lie between 0° and 180°, and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan\theta + \tan\phi = 1.$$

Find the possible values of θ and ϕ. [6]

4 The equation $x^3 - x^2 - 6 = 0$ has one real root, denoted by α.

(i) Find by calculation the pair of consecutive integers between which α lies. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(x_n + \frac{6}{x_n}\right)}$$

converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

5 The equation of a curve is $y = e^{-2x}\tan x$, for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ and show that it can be written in the form $e^{-2x}(a + b\tan x)^2$, where a and b are constants. [5]

(ii) Explain why the gradient of the curve is never negative. [1]

(iii) Find the value of x for which the gradient is least. [1]

6 The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x + 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

7 The points A, B and C have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix}1\\2\\0\end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix}3\\0\\1\end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix}1\\1\\4\end{pmatrix}.$$

The plane m is perpendicular to AB and contains the point C.

(i) Find a vector equation for the line passing through A and B. [2]

(ii) Obtain the equation of the plane m, giving your answer in the form $ax + by + cz = d$. [2]

(iii) The line through A and B intersects the plane m at the point N. Find the position vector of N and show that $CN = \sqrt{(13)}$. [5]

8 The variables x and θ satisfy the differential equation

$$\frac{dx}{d\theta} = (x + 2)\sin^2 2\theta,$$

and it is given that $x = 0$ when $\theta = 0$. Solve the differential equation and calculate the value of x when $\theta = \frac{1}{4}\pi$, giving your answer correct to 3 significant figures. [9]

9 The complex number $3 - \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

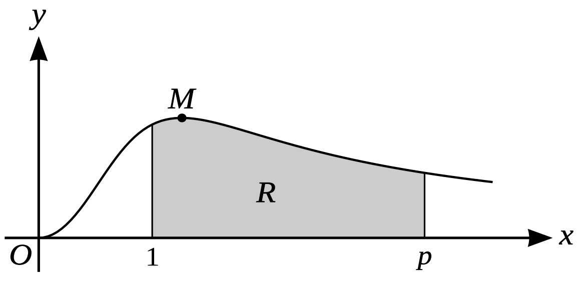
(i) On an Argand diagram with origin O, show the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]

(ii) Showing your working and without using a calculator, express $\frac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\frac{u^*}{u}$, prove that

$$\tan^{-1}\left(\tfrac{3}{4}\right) = 2\tan^{-1}\left(\tfrac{1}{3}\right).$$ [3]

10



The diagram shows the curve $y = \frac{x^2}{1 + x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2015**

1 hour 45 minutes

* 0 1 0 7 1 0 7 7 4 5 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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JC15 11_9709_32/FP

CAMBRIDGE International Examinations

[Turn over

1 Solve the inequality $|2x - 5| > 3|2x + 1|$. [4]

2 Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

3 The angles θ and ϕ lie between 0° and 180°, and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan\theta + \tan\phi = 1.$$

Find the possible values of θ and ϕ. [6]

4 The equation $x^3 - x^2 - 6 = 0$ has one real root, denoted by α.

(i) Find by calculation the pair of consecutive integers between which α lies. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(x_n + \frac{6}{x_n}\right)}$$

converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

5 The equation of a curve is $y = e^{-2x} \tan x$, for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ and show that it can be written in the form $e^{-2x}(a + b\tan x)^2$, where a and b are constants. [5]

(ii) Explain why the gradient of the curve is never negative. [1]

(iii) Find the value of x for which the gradient is least. [1]

6 The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x + 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

7 The points A, B and C have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$$

The plane m is perpendicular to AB and contains the point C.

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8 The variables x and θ satisfy the differential equation

$$\frac{dx}{d\theta} = (x + 2)\sin^2 2\theta,$$

and it is given that $x = 0$ when $\theta = 0$. Solve the differential equation and calculate the value of x when $\theta = \frac{1}{4}\pi$, giving your answer correct to 3 significant figures. [9]

9 The complex number $3 - \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

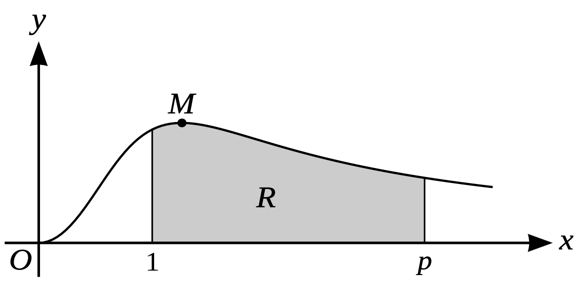
(i) On an Argand diagram with origin O, show the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]

(ii) Showing your working and without using a calculator, express $\dfrac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

(iii) By considering the argument of $\dfrac{u^*}{u}$, prove that

$$\tan^{-1}\left(\tfrac{3}{4}\right) = 2\tan^{-1}\left(\tfrac{1}{3}\right).$$ [3]

10



The diagram shows the curve $y = \dfrac{x^2}{1 + x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

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Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2015**

1 hour 45 minutes

* 8 9 4 0 4 5 7 8 9 7 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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1 Sketch the graph of $y = e^{ax} - 1$ where a is a positive constant. [2]

2 Given that $\sqrt[3]{(1 + 9x)} \approx 1 + 3x + ax^2 + bx^3$ for small values of x, find the values of the coefficients a and b. [3]

3 A curve has equation

$$y = \frac{2 - \tan x}{1 + \tan x}.$$

Find the equation of the tangent to the curve at the point for which $x = \frac{1}{4}\pi$, giving the answer in the form $y = mx + c$ where c is correct to 3 significant figures. [6]

4 A curve has parametric equations

$$x = t^2 + 3t + 1, \qquad y = t^4 + 1.$$

The point P on the curve has parameter p. It is given that the gradient of the curve at P is 4.

(i) Show that $p = \sqrt[3]{(2p + 3)}$. [3]

(ii) Verify by calculation that the value of p lies between 1.8 and 2.0. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

5 Use the substitution $u = 4 - 3\cos x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{2}\pi} \frac{9 \sin 2x}{\sqrt{(4 - 3\cos x)}}\,dx$. [8]

6 The angles A and B are such that

$$\sin(A + 45°) = (2\sqrt{2})\cos A \quad \text{and} \quad 4\sec^2 B + 5 = 12\tan B.$$

Without using a calculator, find the exact value of $\tan(A - B)$. [8]

7 **(i)** Show that $(x + 1)$ is a factor of $4x^3 - x^2 - 11x - 6$. [2]

(ii) Find $\displaystyle\int \frac{4x^2 + 9x - 1}{4x^3 - x^2 - 11x - 6}\,dx$. [8]

8 A plane has equation $4x - y + 5z = 39$. A straight line is parallel to the vector $\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and passes through the point $A(0, 2, -8)$. The line meets the plane at the point B.

(i) Find the coordinates of B. [3]

(ii) Find the acute angle between the line and the plane. [4]

(iii) The point C lies on the line and is such that the distance between C and B is twice the distance between A and B. Find the coordinates of each of the possible positions of the point C. [3]

9 **(a)** It is given that $(1 + 3i)w = 2 + 4i$. Showing all necessary working, prove that the exact value of $|w^2|$ is 2 and find $\arg(w^2)$ correct to 3 significant figures. [6]

(b) On a single Argand diagram sketch the loci $|z| = 5$ and $|z - 5| = |z|$. Hence determine the complex numbers represented by points common to both loci, giving each answer in the form $re^{i\theta}$. [4]

10 Naturalists are managing a wildlife reserve to increase the number of plants of a rare species. The number of plants at time t years is denoted by N, where N is treated as a continuous variable.

(i) It is given that the rate of increase of N with respect to t is proportional to $(N - 150)$. Write down a differential equation relating N, t and a constant of proportionality. [1]

(ii) Initially, when $t = 0$, the number of plants was 650. It was noted that, at a time when there were 900 plants, the number of plants was increasing at a rate of 60 per year. Express N in terms of t. [7]

(iii) The naturalists had a target of increasing the number of plants from 650 to 2000 within 15 years. Will this target be met? [2]

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Cambridge International A Level

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Cambridge International Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

5711204026

READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
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The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **3** printed pages, **1** blank page and **1** insert.

JC16 11_9709_31/2R

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1 Solve the equation $\dfrac{3^x + 2}{3^x - 2} = 8$, giving your answer correct to 3 decimal places. [3]

2 Expand $(2 - x)(1 + 2x)^{-\frac{3}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

3 Express the equation $\sec\theta = 3\cos\theta + \tan\theta$ as a quadratic equation in $\sin\theta$. Hence solve this equation for $-90° < \theta < 90°$. [5]

4 The equation of a curve is $xy(x - 6y) = 9a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [7]

5 **(i)** Prove the identity $\tan 2\theta - \tan\theta \equiv \tan\theta \sec 2\theta$. [4]

(ii) Hence show that $\int_0^{\frac{1}{6}\pi} \tan\theta \sec 2\theta \, d\theta = \frac{1}{2}\ln\frac{3}{2}$. [4]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

$$\operatorname{cosec}\tfrac{1}{2}x = \tfrac{1}{3}x + 1$$

has one root in the interval $0 < x \leqslant \pi$. [2]

(ii) Show by calculation that this root lies between 1.4 and 1.6. [2]

(iii) Show that, if a sequence of values in the interval $0 < x \leqslant \pi$ given by the iterative formula

$$x_{n+1} = 2\sin^{-1}\left(\frac{3}{x_n + 3}\right)$$

converges, then it converges to the root of the equation in part **(i)**. [2]

(iv) Use this iterative formula to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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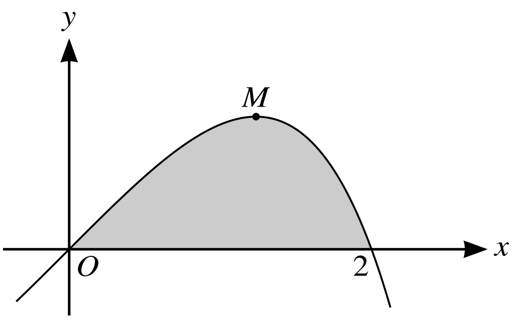

The diagram shows part of the curve $y = (2x - x^2)e^{\frac{1}{2}x}$ and its maximum point M.

(i) Find the exact x-coordinate of M. [4]

(ii) Find the exact value of the area of the shaded region bounded by the curve and the positive x-axis. [5]

8 Two planes have equations $3x + y - z = 2$ and $x - y + 2z = 3$.

(i) Show that the planes are perpendicular. [3]

(ii) Find a vector equation for the line of intersection of the two planes. [6]

9 **Throughout this question the use of a calculator is not permitted.**

(a) Solve the equation $(1 + 2i)w^2 + 4w - (1 - 2i) = 0$, giving your answers in the form $x + iy$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 1 - i| \leqslant 2$ and $-\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{4}\pi$. [5]

10 A large field of area 4 km^2 is becoming infected with a soil disease. At time t years the area infected is x km^2 and the rate of growth of the infected area is given by the differential equation $\frac{dx}{dt} = kx(4 - x)$, where k is a positive constant. It is given that when $t = 0$, $x = 0.4$ and that when $t = 2$, $x = 2$.

(i) Solve the differential equation and show that $k = \frac{1}{4}\ln 3$. [9]

(ii) Find the value of t when 90% of the area of the field is infected. [2]

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Cambridge International A Level

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2016**

1 hour 45 minutes

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Additional Materials: List of Formulae (MF9)

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[Turn over

1 Solve the equation $\dfrac{3^x + 2}{3^x - 2} = 8$, giving your answer correct to 3 decimal places. [3]

2 Expand $(2 - x)(1 + 2x)^{-\frac{3}{2}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

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4 The equation of a curve is $xy(x - 6y) = 9a^3$, where a is a non-zero constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis, and find the coordinates of this point. [7]

5 **(i)** Prove the identity $\tan 2\theta - \tan\theta \equiv \tan\theta \sec 2\theta$. [4]

(ii) Hence show that $\displaystyle\int_0^{\frac{1}{6}\pi} \tan\theta \sec 2\theta \, d\theta = \tfrac{1}{2}\ln\tfrac{3}{2}$. [4]

6 **(i)** By sketching a suitable pair of graphs, show that the equation

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has one root in the interval $0 < x \leqslant \pi$. [2]

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(iii) Show that, if a sequence of values in the interval $0 < x \leqslant \pi$ given by the iterative formula

$$x_{n+1} = 2\sin^{-1}\left(\frac{3}{x_n + 3}\right)$$

converges, then it converges to the root of the equation in part **(i)**. [2]

(iv) Use this iterative formula to calculate the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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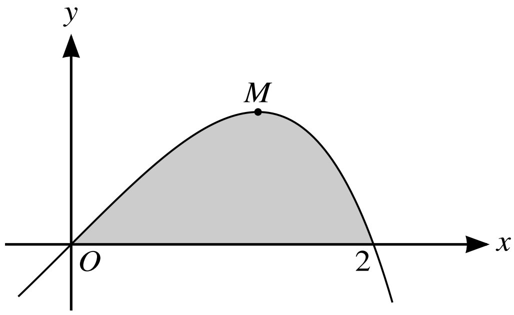

The diagram shows part of the curve $y = (2x - x^2)e^{\frac{1}{2}x}$ and its maximum point M.

(i) Find the exact x-coordinate of M. [4]

(ii) Find the exact value of the area of the shaded region bounded by the curve and the positive x-axis. [5]

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(i) Show that the planes are perpendicular. [3]

(ii) Find a vector equation for the line of intersection of the two planes. [6]

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(a) Solve the equation $(1 + 2i)w^2 + 4w - (1 - 2i) = 0$, giving your answers in the form $x + iy$, where x and y are real. [5]

(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 1 - i| \leqslant 2$ and $-\frac{1}{4}\pi \leqslant \arg z \leqslant \frac{1}{4}\pi$. [5]

10 A large field of area 4 km^2 is becoming infected with a soil disease. At time t years the area infected is x km^2 and the rate of growth of the infected area is given by the differential equation $\frac{dx}{dt} = kx(4 - x)$, where k is a positive constant. It is given that when $t = 0$, $x = 0.4$ and that when $t = 2$, $x = 2$.

(i) Solve the differential equation and show that $k = \frac{1}{4}\ln 3$. [9]

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MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2016**

1 hour 45 minutes

Additional Materials: List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

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The total number of marks for this paper is 75.

JC16 11_9709_33/2R

1 It is given that $z = \ln(y + 2) - \ln(y + 1)$. Express y in terms of z. [3]

2 The equation of a curve is $y = \dfrac{\sin x}{1 + \cos x}$, for $-\pi < x < \pi$. Show that the gradient of the curve is positive for all x in the given interval. [4]

3 Express the equation $\cot 2\theta = 1 + \tan\theta$ as a quadratic equation in $\tan\theta$. Hence solve this equation for $0° < \theta < 180°$. [6]

4 The polynomial $4x^4 + ax^2 + 11x + b$, where a and b are constants, is denoted by $p(x)$. It is given that $p(x)$ is divisible by $x^2 - x + 2$.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, find the real roots of the equation $p(x) = 0$. [2]

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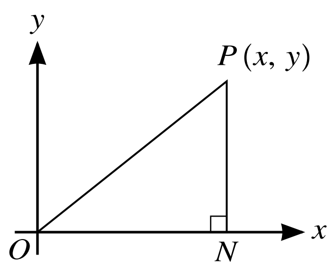


The diagram shows a variable point P with coordinates (x, y) and the point N which is the foot of the perpendicular from P to the x-axis. P moves on a curve such that, for all $x \geqslant 0$, the gradient of the curve is equal in value to the area of the triangle OPN, where O is the origin.

(i) State a differential equation satisfied by x and y. [1]

The point with coordinates $(0, 2)$ lies on the curve.

(ii) Solve the differential equation to obtain the equation of the curve, expressing y in terms of x. [5]

(iii) Sketch the curve. [1]

6 Let $I = \displaystyle\int_1^4 \frac{(\sqrt{x}) - 1}{2(x + \sqrt{x})}\,dx$.

(i) Using the substitution $u = \sqrt{x}$, show that $I = \displaystyle\int_1^2 \frac{u - 1}{u + 1}\,du$. [3]

(ii) Hence show that $I = 1 + \ln\frac{4}{9}$. [6]

7 **Throughout this question the use of a calculator is not permitted.**

The complex number z is defined by $z = (\sqrt{2}) - (\sqrt{6})\mathrm{i}$. The complex conjugate of z is denoted by z^*.

(i) Find the modulus and argument of z. [2]

(ii) Express each of the following in the form $x + \mathrm{i}y$, where x and y are real and exact:

(a) $z + 2z^*$;

(b) $\dfrac{z^*}{\mathrm{i}z}$.

[4]

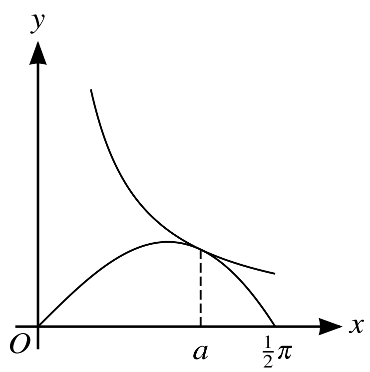
(iii) On a sketch of an Argand diagram with origin O, show the points A and B representing the complex numbers z^* and $\mathrm{i}z$ respectively. Prove that angle AOB is equal to $\frac{1}{6}\pi$. [3]

8 Let $\mathrm{f}(x) = \dfrac{3x^2 + x + 6}{(x+2)(x^2+4)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $\mathrm{f}(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9



The diagram shows the curves $y = x\cos x$ and $y = \dfrac{k}{x}$, where k is a constant, for $0 < x \leqslant \frac{1}{2}\pi$. The curves touch at the point where $x = a$.

(i) Show that a satisfies the equation $\tan a = \dfrac{2}{a}$. [5]

(ii) Use the iterative formula $a_{n+1} = \tan^{-1}\left(\dfrac{2}{a_n}\right)$ to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

(iii) Hence find the value of k correct to 2 decimal places. [2]

[Question 10 is printed on the next page.]

10 The line l has vector equation $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k})$.

(i) Find the position vectors of the two points on the line whose distance from the origin is $\sqrt{(10)}$. [5]

(ii) The plane p has equation $ax + y + z = 5$, where a is a constant. The acute angle between the line l and the plane p is equal to $\sin^{-1}\left(\frac{2}{3}\right)$. Find the possible values of a. [5]

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

8927566197

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC17 11_9709_31/FP

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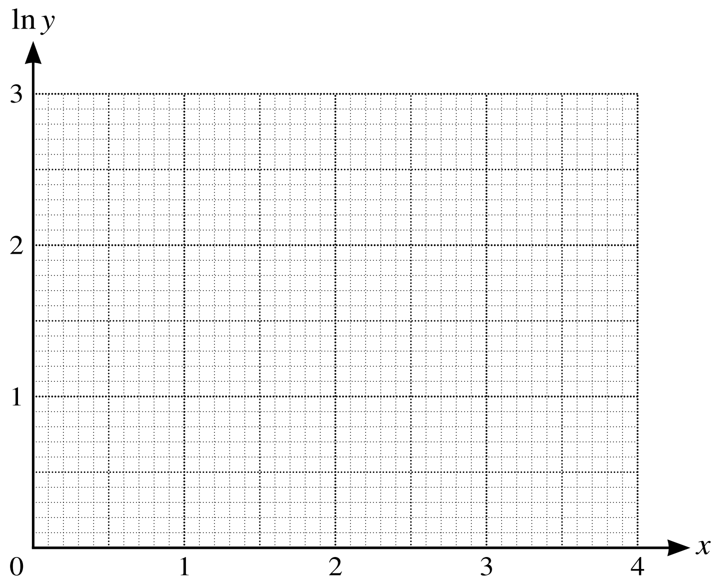
1 Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$. [3]

2 Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y. The table below gives the corresponding values of x and $\ln y$.

x	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a. Give your answers correct to 2 significant figures. [5]



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3 The equation $x^3 = 3x + 7$ has one real root, denoted by α.

(i) Show by calculation that α lies between 2 and 3. [2]

Two iterative formulae, A and B, derived from this equation are as follows:

$$x_{n+1} = (3x_n + 7)^{\frac{1}{3}}, \qquad (A)$$

$$x_{n+1} = \frac{x_n^3 - 7}{3}. \qquad (B)$$

Each formula is used with initial value $x_1 = 2.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [4]

4 **(i)** Prove the identity $\tan(45° + x) + \tan(45° - x) \equiv 2\sec 2x$. [4]

(ii) Hence sketch the graph of $y = \tan(45° + x) + \tan(45° - x)$ for $0° \leqslant x \leqslant 90°$. [3]

5 The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i) Show that $\frac{dy}{dx} = -\frac{8x^3 + y^3}{3xy^2 + 4y^3}$. [4]

(ii) Hence show that there are two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = 4\cos^2 y \tan x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and $x = 0$ when $y = \frac{1}{4}\pi$. Solve this differential equation and find the value of x when $y = \frac{1}{3}\pi$. [8]

7 **(a)** The complex number u is given by $u = 8 - 15\text{i}$. Showing all necessary working, find the two square roots of u. Give answers in the form $a + \text{i}b$, where the numbers a and b are real and exact. [5]

(b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - \mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z - \mathrm{i}) \leqslant \frac{1}{4}\pi$. [4]

8 Let $\mathrm{f}(x) = \dfrac{4x^2 + 9x - 8}{(x+2)(2x-1)}$.

(i) Express $\mathrm{f}(x)$ in the form $A + \dfrac{B}{x+2} + \dfrac{C}{2x-1}$. [4]

(ii) Hence show that $\int_1^4 f(x)\,dx = 6 + \frac{1}{2}\ln\left(\frac{16}{7}\right)$. [5]

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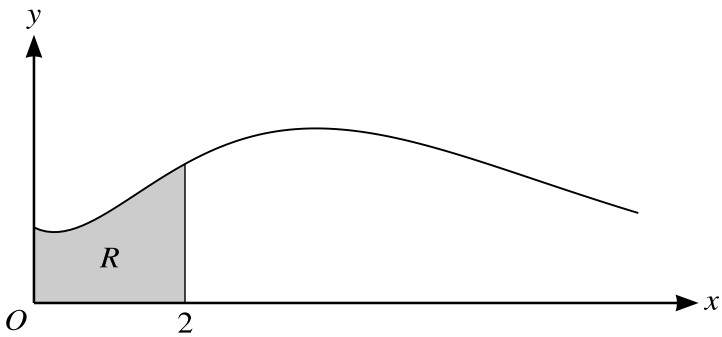

The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \geqslant 0$. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 2$.

(i) Find the exact values of the x-coordinates of the stationary points of the curve. [4]

(ii) Show that the exact value of the area of R is $18 - \dfrac{42}{e}$. [5]

10 The equations of two lines l and m are $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ respectively.

(i) Show that the lines do not intersect. [3]

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(ii) Calculate the acute angle between the directions of the lines. [3]

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(iii) Find the equation of the plane which passes through the point $(3, -2, -1)$ and which is parallel to both l and m. Give your answer in the form $ax + by + cz = d$. [5]

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MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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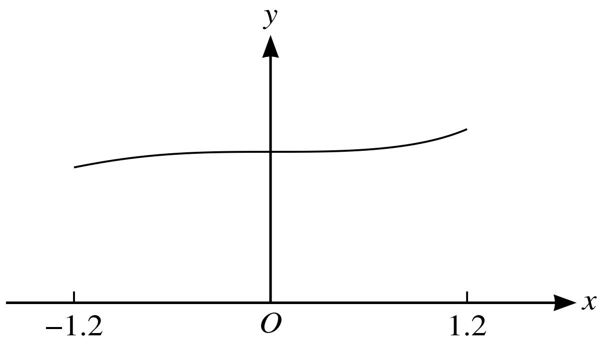

The diagram shows a sketch of the curve $y = \dfrac{3}{\sqrt{(9 - x^3)}}$ for values of x from −1.2 to 1.2.

(i) Use the trapezium rule, with two intervals, to estimate the value of

$$\int_{-1.2}^{1.2} \frac{3}{\sqrt{(9 - x^3)}}\, dx,$$

giving your answer correct to 2 decimal places. [3]

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(ii) Explain, with reference to the diagram, why the trapezium rule may be expected to give a good approximation to the true value of the integral in this case. [1]

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2 Showing all necessary working, solve the equation $2\log_2 x = 3 + \log_2(x + 1)$, giving your answer correct to 3 significant figures. [5]

3 By expressing the equation $\tan(\theta + 60°) + \tan(\theta - 60°) = \cot\theta$ in terms of $\tan\theta$ only, solve the equation for $0° < \theta < 90°$. [5]

4 The curve with equation $y = \dfrac{2 - \sin x}{\cos x}$ has one stationary point in the interval $-\frac{1}{2}\pi < x < \frac{1}{2}\pi$.

(i) Find the exact coordinates of this point. [5]

(ii) Determine whether this point is a maximum or a minimum point. [2]

5 The variables x and y satisfy the differential equation

$$(x+1)\frac{dy}{dx} = y(x+2),$$

and it is given that $y = 2$ when $x = 1$. Solve the differential equation and obtain an expression for y in terms of x. [7]

6 The equation of a curve is $x^3y - 3xy^3 = 2a^4$, where a is a non-zero constant.

(i) Show that $\dfrac{dy}{dx} = \dfrac{3x^2y - 3y^3}{9xy^2 - x^3}$. [4]

(ii) Hence show that there are only two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

7 **Throughout this question the use of a calculator is not permitted.**

The complex number $1 - (\sqrt{3})\mathrm{i}$ is denoted by u.

(i) Find the modulus and argument of u. [2]

(ii) Show that $u^3 + 8 = 0$. [2]

(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - u| \leqslant 2$ and $\text{Re}\, z \geqslant 2$, where $\text{Re}\, z$ denotes the real part of z. [4]

8 Let $f(x) = \dfrac{8x^2 + 9x + 8}{(1 - x)(2x + 3)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

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(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 It is given that $\int_1^a x^{\frac{1}{2}} \ln x \, dx = 2$, where $a > 1$.

(i) Show that $a^{\frac{3}{2}} = \dfrac{7 + 2a^{\frac{3}{2}}}{3 \ln a}$. [5]

(ii) Show by calculation that a lies between 2 and 4. [2]

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(iii) Use the iterative formula

$$a_{n+1} = \left(\frac{7 + 2a_n^{\frac{3}{2}}}{3\ln a_n}\right)^{\frac{2}{3}}$$

to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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10 Two planes p and q have equations $x + y + 3z = 8$ and $2x - 2y + z = 3$ respectively.

(i) Calculate the acute angle between the planes p and q. [4]

(ii) The point A on the line of intersection of p and q has y-coordinate equal to 2. Find the equation of the plane which contains the point A and is perpendicular to both the planes p and q. Give your answer in the form $ax + by + cz = d$. [7]

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Cambridge International A Level

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CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

9388870911

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2017**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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Answer **all** the questions.
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At the end of the examination, fasten all your work securely together.
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The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

JC17 11_9709_33/RP

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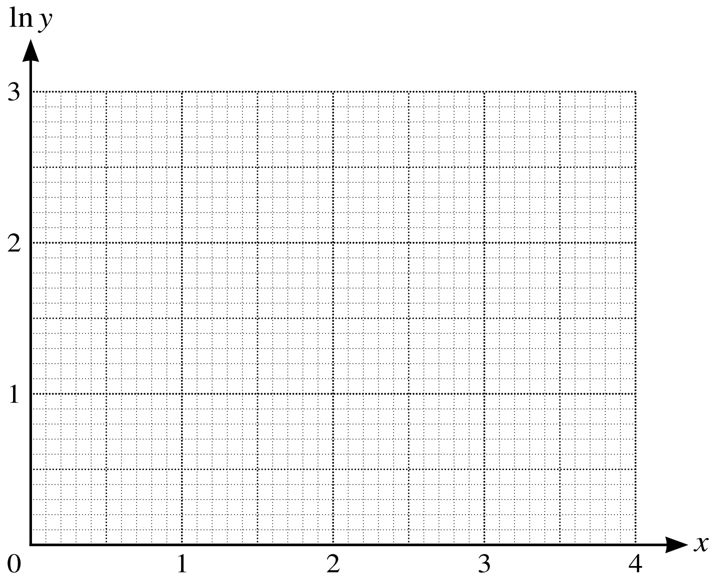
[Turn over

1 Find the quotient and remainder when x^4 is divided by $x^2 + 2x - 1$. [3]

2 Two variable quantities x and y are believed to satisfy an equation of the form $y = C(a^x)$, where C and a are constants. An experiment produced four pairs of values of x and y. The table below gives the corresponding values of x and $\ln y$.

x	0.9	1.6	2.4	3.2
$\ln y$	1.7	1.9	2.3	2.6

By plotting $\ln y$ against x for these four pairs of values and drawing a suitable straight line, estimate the values of C and a. Give your answers correct to 2 significant figures. [5]



3 The equation $x^3 = 3x + 7$ has one real root, denoted by α.

(i) Show by calculation that α lies between 2 and 3. [2]

Two iterative formulae, A and B, derived from this equation are as follows:

$$x_{n+1} = (3x_n + 7)^{\frac{1}{3}}, \qquad (A)$$

$$x_{n+1} = \frac{x_n^3 - 7}{3}. \qquad (B)$$

Each formula is used with initial value $x_1 = 2.5$.

(ii) Show that one of these formulae produces a sequence which fails to converge, and use the other formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [4]

4 **(i)** Prove the identity $\tan(45° + x) + \tan(45° - x) \equiv 2\sec 2x$. [4]

(ii) Hence sketch the graph of $y = \tan(45° + x) + \tan(45° - x)$ for $0° \leqslant x \leqslant 90°$. [3]

5 The equation of a curve is $2x^4 + xy^3 + y^4 = 10$.

(i) Show that $\dfrac{dy}{dx} = -\dfrac{8x^3 + y^3}{3xy^2 + 4y^3}$. [4]

(ii) Hence show that there are two points on the curve at which the tangent is parallel to the x-axis and find the coordinates of these points. [4]

6 The variables x and y satisfy the differential equation

$$\frac{dy}{dx} = 4\cos^2 y \tan x,$$

for $0 \leqslant x < \frac{1}{2}\pi$, and $x = 0$ when $y = \frac{1}{4}\pi$. Solve this differential equation and find the value of x when $y = \frac{1}{3}\pi$. [8]

7 (a) The complex number u is given by $u = 8 - 15\mathrm{i}$. Showing all necessary working, find the two square roots of u. Give answers in the form $a + \mathrm{i}b$, where the numbers a and b are real and exact. [5]

(b) On an Argand diagram, shade the region whose points represent complex numbers satisfying both the inequalities $|z - 2 - \mathrm{i}| \leqslant 2$ and $0 \leqslant \arg(z - \mathrm{i}) \leqslant \frac{1}{4}\pi$. [4]

8 Let $f(x) = \dfrac{4x^2 + 9x - 8}{(x+2)(2x-1)}$.

(i) Express $f(x)$ in the form $A + \dfrac{B}{x+2} + \dfrac{C}{2x-1}$. [4]

(ii) Hence show that $\int_1^4 f(x)\,dx = 6 + \frac{1}{2}\ln\left(\frac{16}{7}\right)$. [5]

[Turn over

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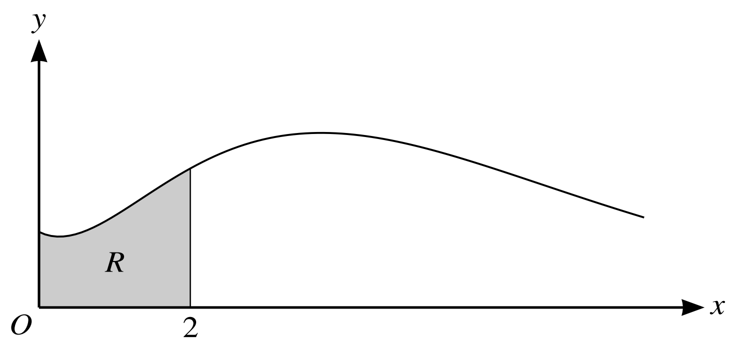

The diagram shows the curve $y = (1 + x^2)e^{-\frac{1}{2}x}$ for $x \geqslant 0$. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 0$ and $x = 2$.

(i) Find the exact values of the x-coordinates of the stationary points of the curve. [4]

(ii) Show that the exact value of the area of R is $18 - \dfrac{42}{e}$. [5]

10 The equations of two lines l and m are $\mathbf{r} = 3\mathbf{i} - \mathbf{j} - 2\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} + \mathbf{j} - 2\mathbf{k})$ respectively.

(i) Show that the lines do not intersect. [3]

(ii) Calculate the acute angle between the directions of the lines. [3]

(iii) Find the equation of the plane which passes through the point $(3, -2, -1)$ and which is parallel to both l and m. Give your answer in the form $ax + by + cz = d$. [5]

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Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 3 2 8 3 7 1 5 3 2 3 *

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **20** printed pages.

JC18 11_9709_31/FP

CAMBRIDGE International Examinations

1 Find the set of values of x satisfying the inequality $2|2x - a| < |x + 3a|$, where a is a positive constant. [4]

2 Showing all necessary working, solve the equation $\dfrac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

3 **(i)** By sketching a suitable pair of graphs, show that the equation $x^3 = 3 - x$ has exactly one real root. [2]

(ii) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 3}{3x_n^2 + 1}$$

converges, then it converges to the root of the equation in part **(i)**. [2]

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(iii) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

4 The parametric equations of a curve are

$$x = 2\sin\theta + \sin 2\theta, \quad y = 2\cos\theta + \cos 2\theta,$$

where $0 < \theta < \pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ in terms of θ. [3]

(ii) Hence find the exact coordinates of the point on the curve at which the tangent is parallel to the y-axis. [4]

5 The coordinates (x, y) of a general point on a curve satisfy the differential equation

$$x\frac{dy}{dx} = (2 - x^2)y.$$

The curve passes through the point $(1, 1)$. Find the equation of the curve, obtaining an expression for y in terms of x. [7]

6 **(i)** Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0° < x < 180°$. [4]

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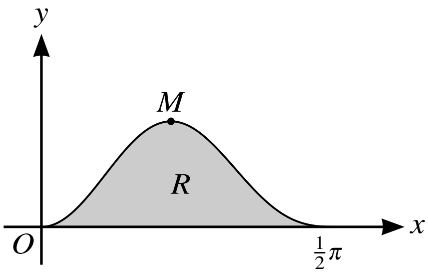

The diagram shows the curve $y = 5\sin^2 x \cos^3 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [5]

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R. [4]

8 **(a)** Showing all necessary working, express the complex number $\frac{2+3\mathrm{i}}{1-2\mathrm{i}}$ in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the values of r and θ correct to 3 significant figures. [5]

(b) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 + 2i| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]

9 Let $f(x) = \dfrac{6x^2 + 8x + 9}{(2 - x)(3 + 2x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

(ii) Hence, showing all necessary working, show that $\int_{-1}^{0} \mathrm{f}(x)\,\mathrm{d}x = 1 + \frac{1}{2}\ln\left(\frac{3}{4}\right)$. [5]

10 The planes m and n have equations $3x + y - 2z = 10$ and $x - 2y + 2z = 5$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l is parallel to m. [3]

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(ii) Calculate the acute angle between the planes m and n. [3]

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(iii) A point P lies on the line l. The perpendicular distance of P from the plane n is equal to 2. Find the position vectors of the two possible positions of P. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

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Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

9782733059

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC18 11_9709_32/RP

CAMBRIDGE International Examinations

1 Solve the inequality $3|2x - 1| > |x + 4|$. [4]

2 Showing all necessary working, solve the equation $\sin(\theta - 30°) + \cos\theta = 2\sin\theta$, for $0° < \theta < 180°$. [4]

3 **(i)** Find $\int \frac{\ln x}{x^3}\,dx$. [3]

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(ii) Hence show that $\int_1^2 \frac{\ln x}{x^3}\,dx = \frac{1}{16}(3 - \ln 4)$. [2]

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4 Showing all necessary working, solve the equation

$$\frac{e^x + e^{-x}}{e^x + 1} = 4,$$

giving your answer correct to 3 decimal places. [5]

5 The equation of a curve is $y = x\ln(8 - x)$. The gradient of the curve is equal to 1 at only one point, when $x = a$.

(i) Show that a satisfies the equation $x = 8 - \dfrac{8}{\ln(8 - x)}$. [3]

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(ii) Verify by calculation that a lies between 2.9 and 3.1. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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6 A certain curve is such that its gradient at a general point with coordinates (x, y) is proportional to $\frac{y^2}{x}$. The curve passes through the points with coordinates $(1, 1)$ and $(e, 2)$. By setting up and solving a differential equation, find the equation of the curve, expressing y in terms of x. [8]

7 A curve has equation $y = \dfrac{3\cos x}{2 + \sin x}$, for $-\frac{1}{2}\pi \leqslant x \leqslant \frac{1}{2}\pi$.

(i) Find the exact coordinates of the stationary point of the curve. [6]

(ii) The constant a is such that $\displaystyle\int_0^a \frac{3\cos x}{2+\sin x}\,dx = 1$. Find the value of a, giving your answer correct to 3 significant figures. [4]

8 Let $\mathrm{f}(x) = \dfrac{7x^2 - 15x + 8}{(1 - 2x)(2 - x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9 **(a)** **(i)** Without using a calculator, express the complex number $\dfrac{2+6i}{1-2i}$ in the form $x + iy$, where x and y are real. [2]

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(ii) Hence, without using a calculator, express $\dfrac{2+6i}{1-2i}$ in the form $r(\cos\theta + i\sin\theta)$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. [3]

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(b) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying both the inequalities $|z - 3\mathrm{i}| \leqslant 1$ and $\mathrm{Re}\, z \leqslant 0$, where $\mathrm{Re}\, z$ denotes the real part of z. Find the greatest value of $\arg z$ for points in this region, giving your answer in radians correct to 2 decimal places. [5]

10 The line l has equation $\mathbf{r} = 5\mathbf{i} - 3\mathbf{j} - \mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + \mathbf{k})$. The plane p has equation

$$(\mathbf{r} - \mathbf{i} - 2\mathbf{j}).(3\mathbf{i} + \mathbf{j} + \mathbf{k}) = 0.$$

The line l intersects the plane p at the point A.

(i) Find the position vector of A. [3]

(ii) Calculate the acute angle between l and p. [4]

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[Question 10 (iii) is printed on the next page.]

(iii) Find the equation of the line which lies in p and intersects l at right angles. [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 3 4 4 2 6 7 1 2 6 7 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2018**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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1 Find the set of values of x satisfying the inequality $2|2x - a| < |x + 3a|$, where a is a positive constant. [4]

2 Showing all necessary working, solve the equation $\frac{2e^x + e^{-x}}{e^x - e^{-x}} = 4$, giving your answer correct to 2 decimal places. [4]

3 **(i)** By sketching a suitable pair of graphs, show that the equation $x^3 = 3 - x$ has exactly one real root. [2]

(ii) Show that if a sequence of real values given by the iterative formula

$$x_{n+1} = \frac{2x_n^3 + 3}{3x_n^2 + 1}$$

converges, then it converges to the root of the equation in part **(i)**. [2]

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(iii) Use this iterative formula to determine the root correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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4 The parametric equations of a curve are

$$x = 2\sin\theta + \sin 2\theta, \quad y = 2\cos\theta + \cos 2\theta,$$

where $0 < \theta < \pi$.

(i) Obtain an expression for $\dfrac{dy}{dx}$ in terms of θ. [3]

(ii) Hence find the exact coordinates of the point on the curve at which the tangent is parallel to the y-axis. [4]

5 The coordinates (x, y) of a general point on a curve satisfy the differential equation

$$x\frac{dy}{dx} = (2 - x^2)y.$$

The curve passes through the point $(1, 1)$. Find the equation of the curve, obtaining an expression for y in terms of x. [7]

6 **(i)** Show that the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$ can be expressed in the form $R\sin(x - \alpha) = \sqrt{2}$, where $R > 0$ and $0° < \alpha < 90°$. [4]

(ii) Hence solve the equation $(\sqrt{2})\operatorname{cosec} x + \cot x = \sqrt{3}$, for $0° < x < 180°$. [4]

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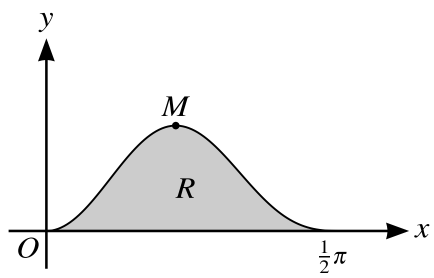

The diagram shows the curve $y = 5\sin^2 x \cos^3 x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M, giving your answer correct to 3 decimal places. [5]

(ii) Using the substitution $u = \sin x$ and showing all necessary working, find the exact area of R. [4]

8 **(a)** Showing all necessary working, express the complex number $\dfrac{2+3\text{i}}{1-2\text{i}}$ in the form $r\text{e}^{\text{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. Give the values of r and θ correct to 3 significant figures. [5]

(b) On an Argand diagram sketch the locus of points representing complex numbers z satisfying the equation $|z - 3 + 2\mathrm{i}| = 1$. Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [4]

9 Let $f(x) = \dfrac{6x^2 + 8x + 9}{(2 - x)(3 + 2x)^2}$.

(i) Express $f(x)$ in partial fractions. [5]

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(ii) Hence, showing all necessary working, show that $\int_{-1}^{0} f(x)\,dx = 1 + \frac{1}{2}\ln\left(\frac{3}{4}\right)$. [5]

10 The planes m and n have equations $3x + y - 2z = 10$ and $x - 2y + 2z = 5$ respectively. The line l has equation $\mathbf{r} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + 2\mathbf{k})$.

(i) Show that l is parallel to m. [3]

(ii) Calculate the acute angle between the planes m and n. [3]

(iii) A point P lies on the line l. The perpendicular distance of P from the plane n is equal to 2. Find the position vectors of the two possible positions of P. [4]

Additional Page

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Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

6182489980

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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JC19 11_9709_31/2R

Cambridge Assessment International Education

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1 Given that $\ln(1 + e^{2y}) = x$, express y in terms of x. [3]

2 Solve the inequality $|2x - 3| > 4|x + 1|$. [4]

3 The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = \ln(1 - \cos 2t).$$

Show that $\frac{dy}{dx} = \operatorname{cosec} 2t$. [5]

[Turn over

4 The number of insects in a population t weeks after the start of observations is denoted by N. The population is decreasing at a rate proportional to $Ne^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when $t = 0$, $N = 1000$ and $\frac{dN}{dt} = -10$.

(i) Show that N and t satisfy the differential equation

$$\frac{dN}{dt} = -0.01e^{-0.02t}N.$$ [1]

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(ii) Solve the differential equation and find the value of t when $N = 800$. [6]

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(iii) State what happens to the value of N as t becomes large. [1]

5 The curve with equation $y = e^{-2x}\ln(x - 1)$ has a stationary point when $x = p$.

(i) Show that p satisfies the equation $x = 1 + \exp\left(\frac{1}{2(x-1)}\right)$, where $\exp(x)$ denotes e^x. [3]

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(ii) Verify by calculation that p lies between 2.2 and 2.6. [2]

(iii) Use an iterative formula based on the equation in part **(i)** to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 **(i)** By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [2]

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(ii) Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} x \operatorname{cosec}^2 x \, dx = \frac{1}{4}(\pi + \ln 4)$. [6]

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7 Two lines l and m have equations $\mathbf{r} = a\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ respectively, where a is a constant. It is given that the lines intersect.

(i) Find the value of a. [4]

(ii) When a has this value, find the equation of the plane containing l and m. [5]

8 Let $f(x) = \dfrac{x^2 + x + 6}{x^2(x + 2)}$.

(i) Express f(x) in partial fractions. [5]

(ii) Hence, showing full working, show that the exact value of $\int_1^4 \mathrm{f}(x)\,\mathrm{d}x$ is $\frac{9}{4}$. [5]

9 **(i)** By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4]

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(ii) Hence solve the equation $\cos 3x + 3\cos x + 1 = 0$, for $0 \leqslant x \leqslant \pi$. [2]

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(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x \, dx$. [4]

10 **(a)** The complex number u is given by $u = -3 - (2\sqrt{10})\text{i}$. Showing all necessary working and without using a calculator, find the square roots of u. Give your answers in the form $a + \text{i}b$, where the numbers a and b are real and exact. [5]

(b) On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - \mathrm{i}| \leqslant 3$, $\arg z \geqslant \frac{1}{4}\pi$ and $\operatorname{Im} z \geqslant 2$, where $\operatorname{Im} z$ denotes the imaginary part of the complex number z. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME	

CENTRE NUMBER		CANDIDATE NUMBER	

4678410875

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

Cambridge Assessment International Education

[Turn over

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1 Solve the equation $5\ln(4 - 3^x) = 6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

2 The curve with equation $y = \dfrac{e^{-2x}}{1 - x^2}$ has a stationary point in the interval $-1 < x < 1$. Find $\dfrac{dy}{dx}$ and hence find the x-coordinate of this stationary point, giving the answer correct to 3 decimal places. [5]

3 The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $\text{p}(x)$. When $\text{p}(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b. [5]

4 **(i)** Express $(\sqrt{6})\sin x + \cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. State the exact value of R and give α correct to 3 decimal places. [3]

(ii) Hence solve the equation $(\sqrt{6})\sin 2\theta + \cos 2\theta = 2$, for $0° < \theta < 180°$. [4]

5 The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x-axis and find the y-coordinate of this point. [7]

6 The variables x and θ satisfy the differential equation

$$\sin\tfrac{1}{2}\theta \frac{dx}{d\theta} = (x+2)\cos\tfrac{1}{2}\theta$$

for $0 < \theta < \pi$. It is given that $x = 1$ when $\theta = \frac{1}{3}\pi$. Solve the differential equation and obtain an expression for x in terms of $\cos\theta$. [8]

7 (a) Find the complex number z satisfying the equation

$$z + \frac{\mathrm{i}z}{z^*} - 2 = 0,$$

where z^* denotes the complex conjugate of z. Give your answer in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) **(i)** On a single Argand diagram sketch the loci given by the equations $|z - 2\mathrm{i}| = 2$ and $\operatorname{Im} z = 3$, where $\operatorname{Im} z$ denotes the imaginary part of z. [2]

(ii) In the first quadrant the two loci intersect at the point P. Find the exact argument of the complex number represented by P. [2]

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8 Let $f(x) = \dfrac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}$.

(i) Express f(x) in partial fractions. [5]

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(ii) Hence, showing full working, find $\int_1^5 \mathrm{f}(x)\,\mathrm{d}x$, giving the answer in the form $\ln c$, where c is an integer. [5]

9 It is given that $\int_0^a x \cos \frac{1}{3}x \, dx = 3$, where the constant a is such that $0 < a < \frac{3}{2}\pi$.

(i) Show that a satisfies the equation

$$a = \frac{4 - 3\cos\frac{1}{3}a}{\sin\frac{1}{3}a}.$$ [5]

(ii) Verify by calculation that a lies between 2.5 and 3. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to calculate a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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10 The line l has equation $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$. The plane p has equation $2x + y - 3z = 5$.

(i) Find the position vector of the point of intersection of l and p. [3]

(ii) Calculate the acute angle between l and p. [3]

(iii) A second plane q is perpendicular to the plane p and contains the line l. Find the equation of q, giving your answer in the form $ax + by + cz = d$. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 4 3 3 9 1 0 7 6 7 9 *

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

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Cambridge Assessment International Education

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1 Solve the inequality $2|x+2| > |3x-1|$. [4]

2 The polynomial $6x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $\mathrm{p}(x)$. It is given that $(2x + 1)$ is a factor of $\mathrm{p}(x)$ and that when $\mathrm{p}(x)$ is divided by $(x + 2)$ the remainder is -24. Find the values of a and b. [5]

3 Showing all necessary working, solve the equation $\dfrac{3^{2x} + 3^{-x}}{3^{2x} - 3^{-x}} = 4$. Give your answer correct to 3 decimal places. [4]

4 **(i)** By first expanding $\tan(2x + x)$, show that the equation $\tan 3x = 3\cot x$ can be written in the form $\tan^4 x - 12\tan^2 x + 3 = 0$. [4]

(ii) Hence solve the equation $\tan 3x = 3\cot x$ for $0° < x < 90°$. [3]

5 **(i)** By sketching a suitable pair of graphs, show that the equation $\ln(x + 2) = 4e^{-x}$ has exactly one real root. [2]

(ii) Show by calculation that this root lies between $x = 1$ and $x = 1.5$. [2]

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(iii) Use the iterative formula $x_{n+1} = \ln\left(\frac{4}{\ln(x_n + 2)}\right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

6 Throughout this question the use of a calculator is not permitted.

The complex number with modulus 1 and argument $\frac{1}{3}\pi$ is denoted by w.

(i) Express w in the form $x + \mathrm{i}y$, where x and y are real and exact. [1]

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The complex number $1 + 2\mathrm{i}$ is denoted by u. The complex number v is such that $|v| = 2|u|$ and $\arg v = \arg u + \frac{1}{3}\pi$.

(ii) Sketch an Argand diagram showing the points representing u and v. [2]

(iii) Explain why v can be expressed as $2uw$. Hence find v, giving your answer in the form $a + \mathrm{i}b$, where a and b are real and exact. [4]

[Turn over

7 The plane m has equation $x + 4y - 8z = 2$. The plane n is parallel to m and passes through the point P with coordinates $(5, 2, -2)$.

(i) Find the equation of n, giving your answer in the form $ax + by + cz = d$. [2]

(ii) Calculate the perpendicular distance between m and n. [3]

(iii) The line l lies in the plane n, passes through the point P and is perpendicular to OP, where O is the origin. Find a vector equation for l. [4]

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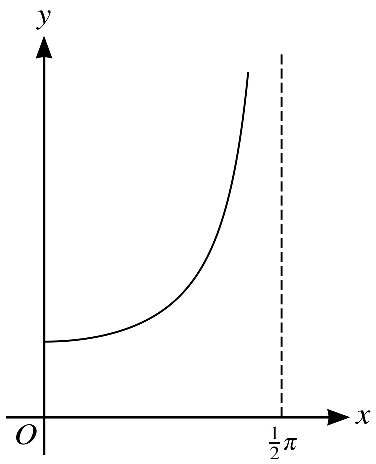

The diagram shows the graph of $y = \sec x$ for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Use the trapezium rule with 2 intervals to estimate the value of $\int_0^{1.2} \sec x \, dx$, giving your answer correct to 2 decimal places. [3]

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(ii) Explain, with reference to the diagram, whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part **(i)**. [1]

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(iii) P is the point on the part of the curve $y = \sec x$ for $0 \leqslant x < \frac{1}{2}\pi$ at which the gradient is 2. By first differentiating $\dfrac{1}{\cos x}$, find the x-coordinate of P, giving your answer correct to 3 decimal places. [6]

9 The variables x and t satisfy the differential equation $5\frac{dx}{dt} = (20 - x)(40 - x)$. It is given that $x = 10$ when $t = 0$.

(i) Using partial fractions, solve the differential equation, obtaining an expression for x in terms of t. [9]

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(ii) State what happens to the value of x when t becomes large. [1]

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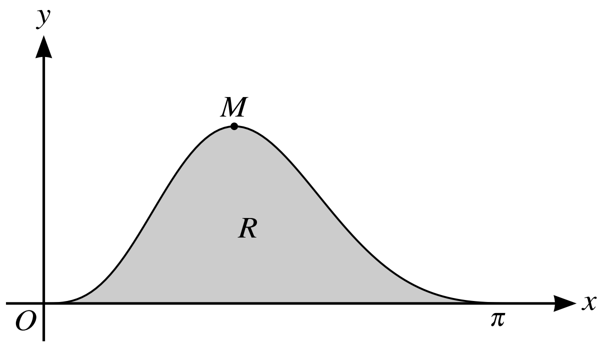

The diagram shows the graph of $y = e^{\cos x} \sin^3 x$ for $0 \leqslant x \leqslant \pi$, and its maximum point M. The shaded region R is bounded by the curve and the x-axis.

(i) Find the x-coordinate of M. Show all necessary working and give your answer correct to 2 decimal places. [5]

(ii) By first using the substitution $u = \cos x$, find the exact value of the area of R. [7]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

0123456789

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **For examination from 2020**

SPECIMEN PAPER **1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

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1 Find the set of values of x for which $3(2^{3x+1}) < 8$. Give your answer in a simplified exact form. [3]

2 **(a)** Expand $(1 + 3x)^{-\frac{1}{3}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

(b) State the set of values of x for which the expansion is valid. [1]

3 **(a)** Sketch the graph of $y = |2x - 3|$. [1]

(b) Solve the inequality $3x + 1 > |2x - 3|$. [3]

4 The parametric equations of a curve are

$$x = e^{2t-3}, \quad y = 4\ln t,$$

where $t > 0$. When $t = a$ the gradient of the curve is 2.

(a) Show that a satisfies the equation $a = \frac{1}{2}(3 - \ln a)$. [4]

(b) Verify by calculation that this equation has a root between 1 and 2. [2]

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(c) Use the iterative formula $a_{n+1} = \frac{1}{2}(3 - \ln a_n)$ to calculate a correct to 2 decimal places, showing the result of each iteration to 4 decimal places. [3]

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5 **(a)** Show that $\dfrac{d}{dx}(x - \tan^{-1}x) = \dfrac{x^2}{1+x^2}$. [2]

(b) Show that $\int_0^{\sqrt{3}} x\tan^{-1}x \, dx = \frac{2}{3}\pi - \frac{1}{2}\sqrt{3}$. [5]

6 The complex numbers 1 + 3i and 4 + 2i are denoted by u and v respectively.

(a) Find $\frac{u}{v}$ in the form $x + iy$, where x and y are real. [3]

(b) State the argument of $\frac{u}{v}$. [1]

In an Argand diagram, with origin O, the points A, B and C represent the complex numbers u, v and $u - v$ respectively.

(c) State fully the geometrical relationship between OC and BA. [2]

(d) Show that angle $AOB = \frac{1}{4}\pi$ radians. [2]

7 **(a)** By first expanding $\cos(x + 45°)$, express $\cos(x + 45°) - \sqrt{2}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of R correct to 4 significant figures and the value of α correct to 2 decimal places. [5]

(b) Hence solve the equation

$$\cos(x + 45°) - \sqrt{2}\sin x = 2$$

for $0° < x < 360°$. [4]

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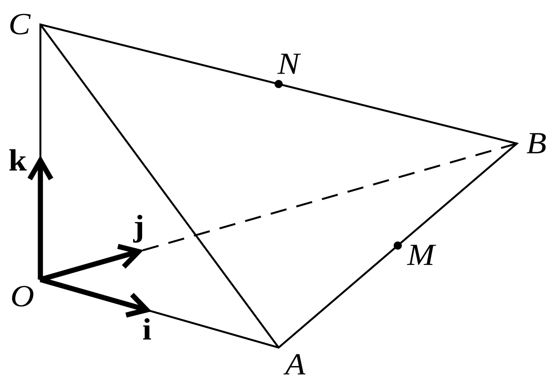

In the diagram, $OABC$ is a pyramid in which $OA = 2$ units, $OB = 4$ units and $OC = 2$ units. The edge OC is vertical, the base OAB is horizontal and angle $AOB = 90°$. Unit vectors **i**, **j** and **k** are parallel to OA, OB and OC respectively. The midpoints of AB and BC are M and N respectively.

(a) Express the vectors $\overrightarrow{ON}$ and $\overrightarrow{CM}$ in terms of **i**, **j** and **k**. [3]

(b) Calculate the angle between the directions of $\overrightarrow{ON}$ and $\overrightarrow{CM}$. [3]

(c) Show that the length of the perpendicular from M to ON is $\frac{3}{5}\sqrt{5}$. [4]

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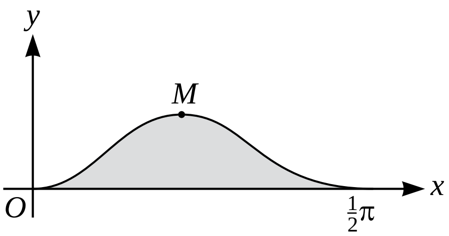

The diagram shows the curve $y = \sin^2 2x \cos x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$, and its maximum point M.

(a) Find the x-coordinate of M. [6]

(b) Using the substitution $u = \sin x$, find the area of the shaded region bounded by the curve and the x-axis. [4]

10 In a chemical reaction a compound X is formed from two compounds Y and Z.

The masses in grams of X, Y and Z present at time t seconds after the start of the reaction are x, $10 - x$ and $20 - x$ respectively. At any time the rate of formation of X is proportional to the product of the masses of Y and Z present at the time. When $t = 0$, $x = 0$ and $\frac{dx}{dt} = 2$.

(a) Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = 0.01(10 - x)(20 - x).$$ [1]

(b) Solve this differential equation and obtain an expression for x in terms of t. [9]

(c) State what happens to the value of x when t becomes large. [1]

Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

MATHEMATICS **9709/03**

Paper 3 Pure Mathematics 3 **(P3)** **For Examination from 2017**

SPECIMEN PAPER **1 hour 45 minutes**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
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You may use an HB pencil for any diagrams or graphs.
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Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
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You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.

CAMBRIDGE International Examinations

1 Solve the inequality $|2x - 5| > 3|2x + 1|$. [4]

2 Using the substitution $u = 3^x$, solve the equation $3^x + 3^{2x} = 3^{3x}$ giving your answer correct to 3 significant figures. [5]

3 The angles θ and ϕ lie between 0° and 180°, and are such that

$$\tan(\theta - \phi) = 3 \quad \text{and} \quad \tan\theta + \tan\phi = 1.$$

Find the possible values of θ and ϕ. [6]

4 The equation $x^3 - x^2 - 6 = 0$ has one real root, denoted by α.

(i) Find by calculation the pair of consecutive integers between which α lies. [2]

(ii) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \sqrt{\left(x_n + \frac{6}{x_n}\right)}$$

converges, then it converges to α. [2]

(iii) Use this iterative formula to determine α correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

5 The equation of a curve is $y = e^{-2x} \tan x$, for $0 \leqslant x < \frac{1}{2}\pi$.

(i) Obtain an expression for $\frac{dy}{dx}$ and show that it can be written in the form $e^{-2x}(a + b \tan x)^2$, where a and b are constants. [5]

(ii) Explain why the gradient of the curve is never negative. [1]

(iii) Find the value of x for which the gradient is least. [1]

6 The polynomial $8x^3 + ax^2 + bx - 1$, where a and b are constants, is denoted by $p(x)$. It is given that $(x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(2x + 1)$ the remainder is 1.

(i) Find the values of a and b. [5]

(ii) When a and b have these values, factorise $p(x)$ completely. [3]

7 The points A, B and C have position vectors, relative to the origin O, given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}.$$

The plane m is perpendicular to AB and contains the point C.

(i) Find a vector equation for the line passing through A and B. [2]

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(ii) Obtain the equation of the plane m, giving your answer in the form $ax + by + cz = d$. [2]

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(iii) The line through A and B intersects the plane m at the point N. Find the position vector of N and show that $CN = \sqrt{(13)}$. [5]

8 The variables x and θ satisfy the differential equation

$$\frac{dx}{d\theta} = (x+2)\sin^2 2\theta,$$

and it is given that $x = 0$ when $\theta = 0$. Solve the differential equation and calculate the value of x when $\theta = \frac{1}{4}\pi$, giving your answer correct to 3 significant figures. [9]

9 The complex number $3 - \mathrm{i}$ is denoted by u. Its complex conjugate is denoted by u^*.

(i) On an Argand diagram with origin O, show the points A, B and C representing the complex numbers u, u^* and $u^* - u$ respectively. What type of quadrilateral is $OABC$? [4]

(ii) Showing your working and without using a calculator, express $\dfrac{u^*}{u}$ in the form $x + \mathrm{i}y$, where x and y are real. [3]

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(iii) By considering the argument of $\frac{u^*}{u}$, prove that

$$\tan^{-1}\left(\tfrac{3}{4}\right) = 2\tan^{-1}\left(\tfrac{1}{3}\right).$$ [3]

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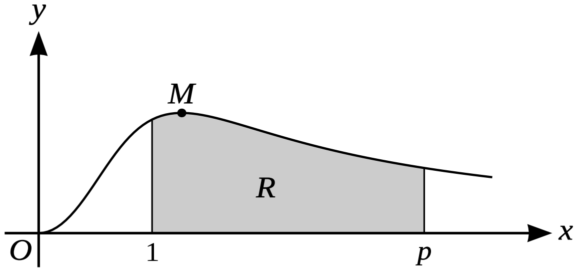

The diagram shows the curve $y = \dfrac{x^2}{1 + x^3}$ for $x \geqslant 0$, and its maximum point M. The shaded region R is enclosed by the curve, the x-axis and the lines $x = 1$ and $x = p$.

(i) Find the exact value of the x-coordinate of M. [4]

(ii) Calculate the value of p for which the area of R is equal to 1. Give your answer correct to 3 significant figures. [6]

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