6 **(a)** Starting from the definitions of sinh and cosh in terms of exponentials, prove that

$$2\sinh^2 x = \cosh 2x - 1.$$ [3]

(b) Find the solution to the differential equation

$$\frac{dy}{dx} + y\coth x = 4\sinh x$$

for which $y = 1$ when $x = \ln 3$. [7]

2 **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$\cosh 2x = 2\sinh^2 x + 1.$$ [3]

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(b) Find the set of values of k for which $\cosh 2x = k\sinh x$ has two distinct real roots. [5]

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5 **(a)** Starting from the definitions of cosh and sinh in terms of exponentials, prove that

$$2\cosh^2 x = \cosh 2x + 1.$$ [3]

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(b) Find the solution of the differential equation

$$\frac{dy}{dx} + 2y\tanh x = 1$$

for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$. [8]

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6 **(a)** Show that $\left(\cosh x + \sinh x\right)^{\frac{1}{2}} = e^{\frac{1}{2}x}$. [2]

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(b) Find the particular solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$$

given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$. [10]

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8 **(a)** Starting from the definition of cosh in terms of exponentials, prove that

$$2\cosh^2 A = \cosh 2A + 1. \qquad [3]$$

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The curve C has parametric equations

$$x = 2\cosh 2t + 3t, \quad y = \tfrac{3}{2}\cosh 2t - 4t, \quad \text{for } -\tfrac{1}{2} \leqslant t \leqslant \tfrac{1}{2}.$$

The area of the surface generated when C is rotated through 2π radians about the y-axis is denoted by A.

(b) **(i)** Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2\cosh 2t + 3t)\cosh 2t \, dt$. [4]

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