	$2\sinh^2 x = \cosh 2x - 1.$	[3]
b)	Find the solution to the differential equation	
	$\frac{\mathrm{d}y}{\mathrm{d}x} + y \coth x = 4 \sinh x$	
	$\frac{dy}{dx} + y \coth x = 4 \sinh x$ for which $y = 1$ when $x = \ln 3$.	[7]
		[7]
		[7]
		[7]
	for which $y = 1$ when $x = \ln 3$.	
	for which $y = 1$ when $x = \ln 3$.	
	for which $y = 1$ when $x = \ln 3$.	
	for which $y = 1$ when $x = \ln 3$.	
	for which $y = 1$ when $x = \ln 3$.	
	for which $y = 1$ when $x = \ln 3$.	

© UCLES 2021 9231/23/M/J/21

(a)	Starting from the definitions of cosh and sinn in terms of exponentials, prove that	
	$\cosh 2x = 2\sinh^2 x + 1.$	[3]
(b)	Find the set of values of k for which $\cosh 2x = k \sinh x$ has two distinct real roots.	[5]
		•••••
		•••••
		•••••
		•••••

$2\cosh^2 x = \cosh 2x + 1.$	[3]
Find the solution of the differential equation $\frac{dy}{dy} = 0$	
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$	
Find the solution of the differential equation	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$	[8]
Find the solution of the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = \mathrm{f}(x)$.	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[8]
Find the solution of the differential equation $\frac{dy}{dx} + 2y \tanh x = 1$ for which $y = 1$ when $x = 0$. Give your answer in the form $y = f(x)$.	[8]

© UCLES 2023 9231/23/M/J/23

(a)	Show that $\left(\cosh x + \sinh x\right)^{\frac{1}{2}} = e^{\frac{1}{2}x}$.	[2
(b)	Find the particular solution of the differential equation	
(b)	Find the particular solution of the differential equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$	
(b)	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$	[10
(b)		
(b)	$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$	
(b)	$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$ given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$.	
(b)	$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$ given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$.	
(b)	$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$ given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$.	
(b)	$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$ given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$.	
(b)	$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$ given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$.	
(b)	$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$ given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$.	
(b)	$\frac{d^2y}{dx^2} + \frac{dy}{dx} + 3y = 5\left(\cosh x + \sinh x\right)^{\frac{1}{2}},$ given that, when $x = 0$, $y = 1$ and $\frac{dy}{dx} = \frac{4}{3}$.	

© UCLES 2024 9231/21/M/J/24

	$2\cosh^2 A = \cosh 2A + 1.$	[3]
The curv	we C has parametric equations	
	$x = 2\cosh 2t + 3t$, $y = \frac{3}{2}\cosh 2t - 4t$, for $-\frac{1}{2} \le t \le \frac{1}{2}$.	
The area by A .	a of the surface generated when C is rotated through 2π radians about the	y-axis is denoted
-	Show that $A = 10\pi \int_{-\frac{1}{2}}^{\frac{1}{2}} (2\cosh 2t + 3t) \cosh 2t dt$.	[4]
(b) (1)	Show that $A = 10\pi \int_{-\frac{1}{2}}^{-\frac{1}{2}} (2\cos i(2t + 5t)) \cos i(2t + 4t)$	[ד]
		•••••

© UCLES 2021 9231/21/O/N/21