# 欧几里得数学竞赛

# 对数专题

(b) Determine the coordinates of the points of intersection of the graphs of $y=\log_{10}(x-2)$ and $y=1-\log_{10}(x+1)$.

(b) If $\log_{10} x = 3 + \log_{10} y$, what is the value of $\frac{x}{y}$?

(a) What is the value of $x$ such that $\log_2(\log_2(2x-2))=2$?

(a) What are all values of $x$ such that

$$\log_5(x+3)+\log_5(x-1)=1?$$

(b) Solve the system of equations:

$$\log_{10}\left(x^3\right) + \log_{10}\left(y^2\right) = 11$$

$$\log_{10}\left(x^2\right) - \log_{10}\left(y^3\right) = 3$$

(a) If $\log_2 x - 2\log_2 y = 2$, determine $y$ as a function of $x$, and sketch a graph of this function on the axes in the answer booklet.

(a) Determine all values of $x$ for which $(\sqrt{x})^{\log_{10} x} = 100$.

(b) Determine all real solutions to the system of equations

$$\begin{aligned} x &+ \log_{10} x &= y-1 \\ y &+ \log_{10}(y-1) &= z-1 \\ z &+ \log_{10}(z-2) &= x+2 \end{aligned}$$

and prove that there are no more solutions.

(a) If $\log_2 x$, $(1 + \log_4 x)$, and $\log_8 4x$ are consecutive terms of a geometric sequence, determine the possible values of $x$.

(A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a constant. For example, 3, 6, 12 is a geometric sequence with three terms.)

(b) Determine all points $(x, y)$ where the two curves $y = \log_{10}(x^4)$ and $y = (\log_{10} x)^3$ intersect.

(b) Determine all values of $x$ for which $2^{\log_{10}(x^2)} = 3(2^{1+\log_{10} x}) + 16$.

(b) Determine all real values of $x$ such that

$$\log_{5x+9}(x^2 + 6x + 9) + \log_{x+3}(5x^2 + 24x + 27) = 4$$

(b) Determine all real values of $x$ for which $\log_2(2^{x-1} + 3^{x+1}) = 2x - \log_2(3^x)$.

(b) Determine all real numbers $x$ for which

$$(\log_{10} x)^{\log_{10}(\log_{10} x)} = 10\,000$$

9. Consider the following system of equations in which all logarithms have base 10:

$$\begin{aligned}(\log x)(\log y) - 3\log 5y - \log 8x &= a \\ (\log y)(\log z) - 4\log 5y - \log 16z &= b \\ (\log z)(\log x) - 4\log 8x - 3\log 625z &= c\end{aligned}$$

(a) If $a = -4$, $b = 4$, and $c = -18$, solve the system of equations.

(b) Determine all triples $(a, b, c)$ of real numbers for which the system of equations has an infinite number of solutions $(x, y, z)$.

(b) Determine all real numbers $x > 0$ for which

$$\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$$

(b) Determine all pairs $(a, b)$ of real numbers that satisfy the following system of equations:

$$\sqrt{a} + \sqrt{b} = 8$$
$$\log_{10} a + \log_{10} b = 2$$

Give your answer(s) as pairs of simplified exact numbers.

(a) Determine all values of $x$ such that $\log_{2x}(48\sqrt[3]{3}) = \log_{3x}(162\sqrt[3]{2})$.

(a) Determine all real numbers $x$ for which $2\log_2(x-1) = 1 - \log_2(x+2)$.

(b) Determine all pairs of angles $(x, y)$ with $0° \leq x < 180°$ and $0° \leq y < 180°$ that satisfy the following system of equations:

$$\log_2(\sin x \cos y) = -\frac{3}{2}$$

$$\log_2\left(\frac{\sin x}{\cos y}\right) = \frac{1}{2}$$

(b) Suppose that $f(a) = 2a^2 - 3a + 1$ for all real numbers $a$ and $g(b) = \log_{\frac{1}{2}} b$ for all $b > 0$. Determine all $\theta$ with $0 \le \theta \le 2\pi$ for which $f(g(\sin\theta)) = 0$.

(b) Determine all real numbers $a$, $b$ and $c$ for which the graph of the function $y = \log_a(x + b) + c$ passes through the points $P(3, 5)$, $Q(5, 4)$ and $R(11, 3)$.

(a) A computer is programmed to choose an integer between 1 and 99, inclusive, so that the probability that it selects the integer $x$ is equal to $\log_{100}\left(1+\frac{1}{x}\right)$. Suppose that the probability that $81 \leq x \leq 99$ is equal to 2 times the probability that $x = n$ for some integer $n$. What is the value of $n$?

(b) Determine all real values of $x$ for which

$$\sqrt{\log_2 x \cdot \log_2(4x) + 1} + \sqrt{\log_2 x \cdot \log_2\left(\frac{x}{64}\right) + 9} = 4$$

(b) Determine all triples $(x, y, z)$ of real numbers that are solutions to the following system of equations:

$$\log_9 x + \log_9 y + \log_3 z = 2$$
$$\log_{16} x + \log_4 y + \log_{16} z = 1$$
$$\log_5 x + \log_{25} y + \log_{25} z = 0$$

(b) Determine the coordinates of the points of intersection of the graphs of $y = \log_{10}(x-2)$ and $y = 1 - \log_{10}(x+1)$.